MATH 2200 Assignment No.2, October 6, 2006

The assignment is due Friday, October 13, 2006, in class. Late assignments receive mark zero.

1. Use the **Principal of Mathematical Induction** to prove that:

   a) $5^{2n} - 1$ is divisible by 8, for every n in $\mathbb{N}$. [5]

   b) $2^n < n!$, for every n $\geq$4. [5]

2. a) If a < x < b and a < y < b, show that | x-y |< b-a. Interpret this geometrically. [5]

   b) Determine and sketch the set of pairs (x,y) in $\mathbb{R} \times \mathbb{R}$ that satisfy |x| = |y| .[5]

   c) Show that if a, b $\in \mathbb{R}$, and a$\neq$ b, then there exists ε neighbourhoods U of a and V of b such that U $\cap$ V = $\varnothing$. [6]

3. a) Prove that for a nonempty, bounded subsets A and B of $\mathbb{R}$, inf(A+B) = infA+infB. [6]

   b) Prove that a supremum of a bounded nonempty set A is unique, i.e. prove that if s_1=supA and s_2=supA, then s_1=s_2. [6]

4. For the given set A , find supA and infA, whenever they exist:

   a) $A = \bigcap_{n=0}^{\infty} I_n$ with $I_n = (1, 2 + \frac{1}{n}]$, [6]

   b) $A = \bigcap_{n=0}^{\infty} I_n$ with $I_n = (1, 1 + \frac{1}{n})$, [4]

   c) $A = \{\frac{1}{n} - \frac{1}{m} : n, m \text{ in } \mathbb{N}\}$. [7]

   (**Show all of your work**.)

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total [55/54 marks]