MATH 2202 Assignment No.2, October 14, 2009

The assignment is due Wednesday, October 21, 2009, in class. Late assignments receive mark zero. (**Show all of your work**.)

1. Using the Field axioms and theorems proven in class, show that for a, b in $\mathbb{R}$:
   a) $-(a+b) = (-a) + (-b)$. [3]
   b) If $a \neq 0, b \neq 0$, then $(ab)^{-1} = a^{-1}b^{-1}$. [4]

2. Use the **Principal of Mathematical Induction** to prove that:
   a) $5^{2n} - 1$ is divisible by 8, for every n in $\mathbb{N}$. [4]
   b) $2^n < n!$, for every n ≥ 4. [4]
   c) Let $a > 0$, $b > 0$. Show that $a < b$ if and only if $\forall n \in \mathbb{N}$ we have that $a^n < b^n$. [4]

3. a) Find all x in $\mathbb{R}$ satisfying $|x| + |x+1| < 2$. [4]
   b) If $a < x < b$ and $a < y < b$, show that $|x-y| < b-a$. Interpret this by using the notion of distance. [4]
   c) Show that if $a, b \in \mathbb{R}$, and $a \neq b$, then there exist ε neighbourhoods U of a and V of b such that $U \cap V = \varnothing$. [4]

4. a) Prove that for a nonempty, bounded subsets A and B of $\mathbb{R}$, inf(A+B)=infA+infB. [4]
   b) Prove that a supremum of a bounded nonempty set A is unique, i.e. prove that if s_1=supA and s_2=supA, then $s_1 = s_2$. [3]
   c) Prove that if A is a bounded non empty subset of $\mathbb{R}$, there exists a **smallest** closed interval I containing A. (In other words, I has the property that if J is a closed interval containing A, then J contains I.) [5]
   d) Give an example of a non empty bounded set A, subset of $\mathbb{R}$, such that there is no smallest open interval containing A. [2]

5. Find A first, by using a corollary to the Archimedean theorem, and then find supA and infA, whenever they exist. (You can use theorems proven in class on sup of an interval, on inf and sup of: sum of sets; product of a number with a set. You **cannot use limits**!)
   a) $A = \bigcap_{n=0}^{\infty} I_n$, were $I_n = (1, 2 + \frac{1}{n}]$. [4]
   b) $A = \bigcap_{n=0}^{\infty} I_n$, were $I_n = (1, 1 + \frac{1}{n})$. [3]
   c) $A = \{\frac{1}{n} - \frac{1}{m} : n, m \text{ in } \mathbb{N}\}$. (Hint: find inf and sup of $\{\frac{1}{n} : n \in \mathbb{N}\}$ first.) [5]

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total [57/54 marks]