# Finding a Funicular Curve Through Two Points

This is the glass pyramid at the Louvre Museum in Paris, designed by architect I.M. Pei. It is supported from beneath by steel cables.

In designing a structure such as this, it is often most useful to select a cable of a certain size and tensile strength, and then to find a shape for it that will utilize fully the given tensile strength.

In this lesson we will learn to find the form for a cable or arch that passes through any two points and experiences a designated maximum tensile or compressive force.

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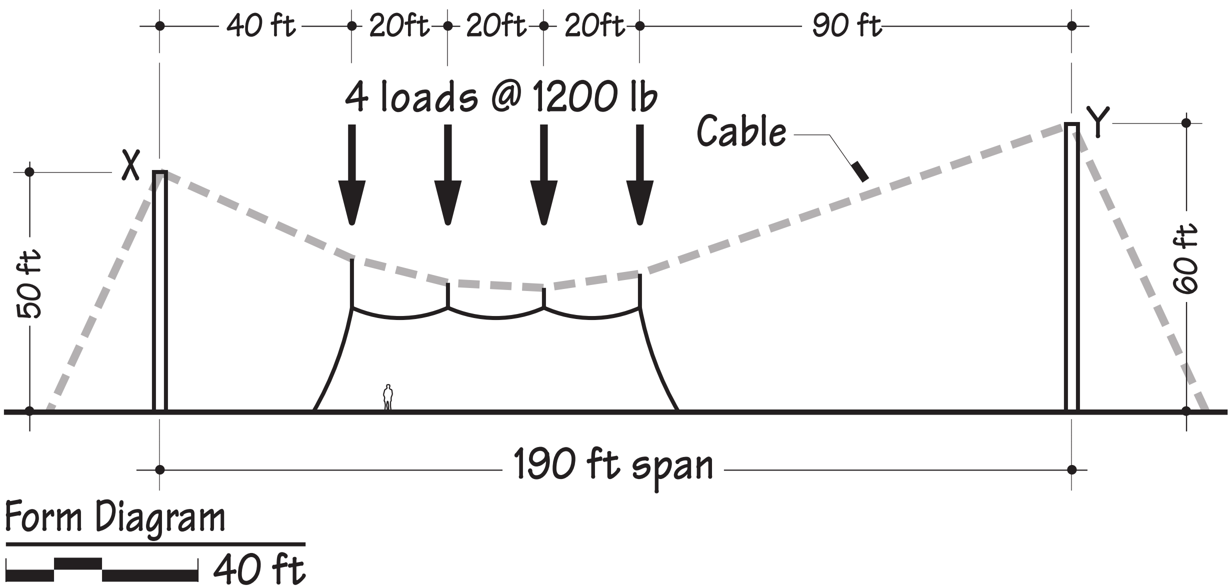
# Finding a Funicular Curve Through Two Points




**The Problem:** Suppose that we are designing a cable to support the ridge of an exhibition tent. The tent exerts four loads on the cable of 1200 lb each.

Although the loads are spaced at 20-foot intervals, the ends of the cable are 190 feet apart at locations *X* and *Y*, and their vertical elevations differ by 10 feet.

The maximum safe tensile force in the cable is 6600 lb.

We must find the form for the cable that meets these criteria.

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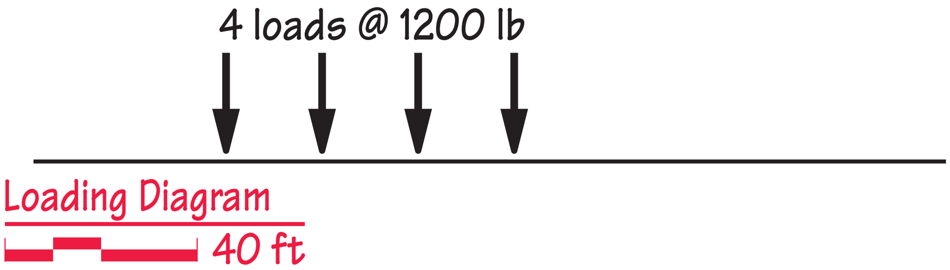


**Step 1: Set up the solution.**

We begin our work by constructing the Loading Diagram. The diagram is placed near the top of the page to leave room for the graphical constructions below.

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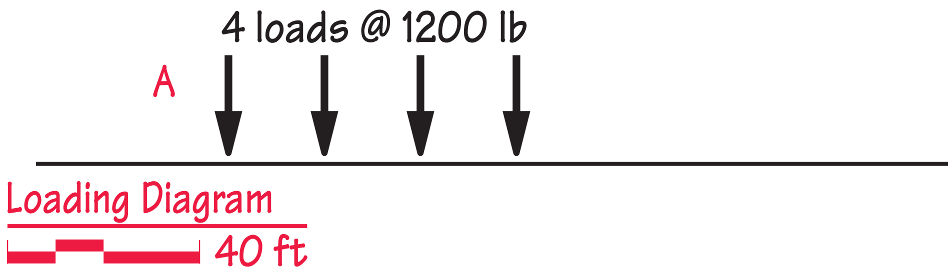


We apply interval notation to the Loading Diagram.

Beginning at the left, we place capital letters in the intervals between the loads.

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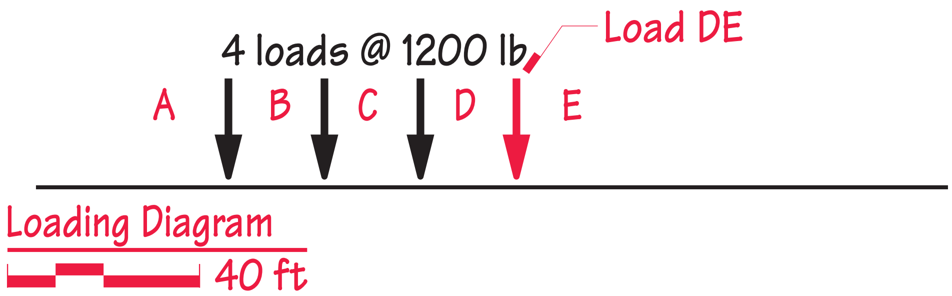


Working from left to right, we continue labeling the intervals between each pair of forces.

Loads are named by the letters on either side. For example, the rightmost load is named *DE*.

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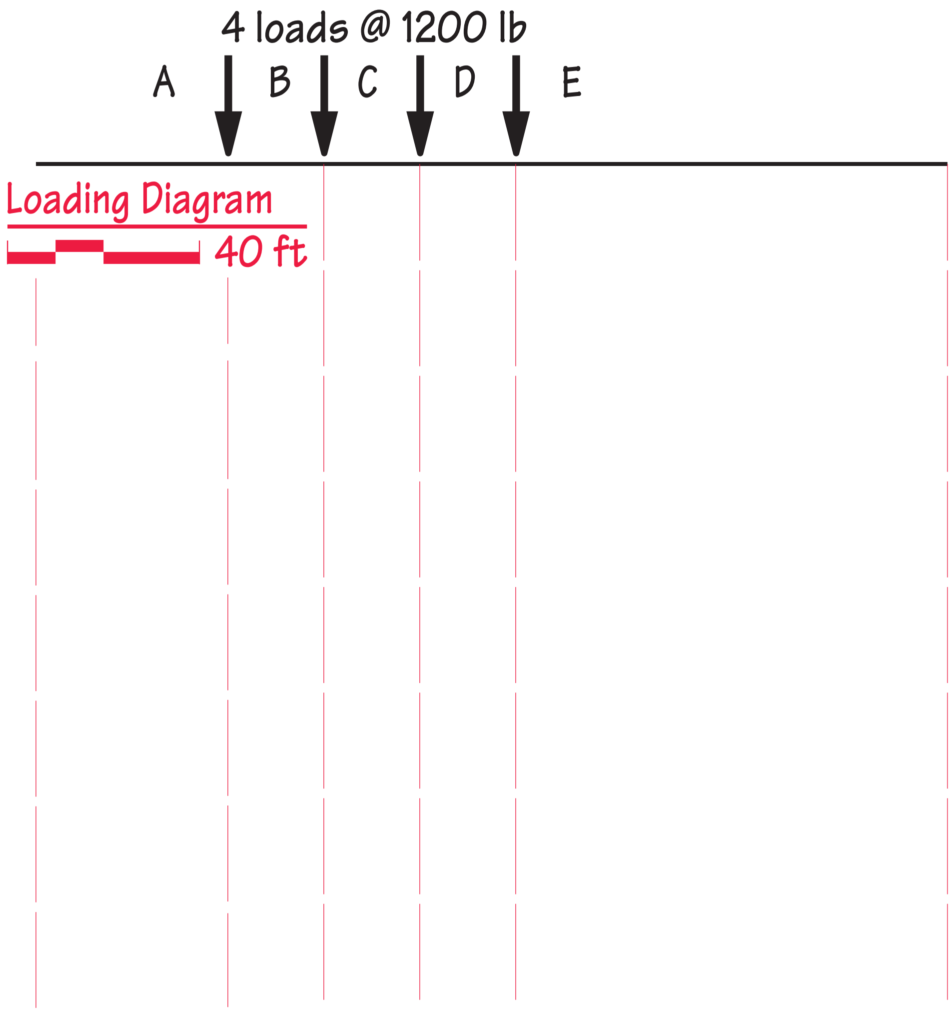


We extend vertical lines of action from the four load vectors downward on the page. Vertical lines are extended from the end points of the Loading Diagram as well.

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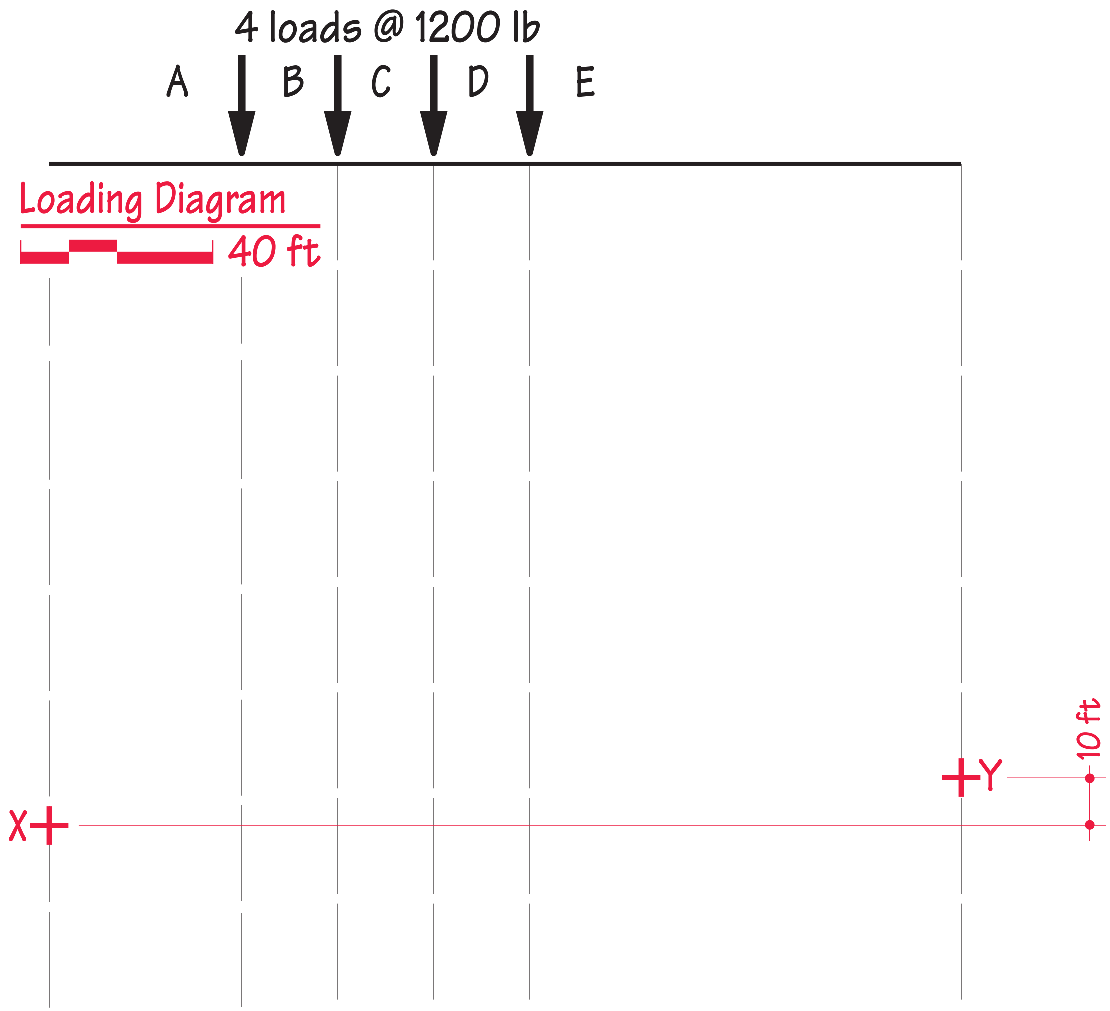
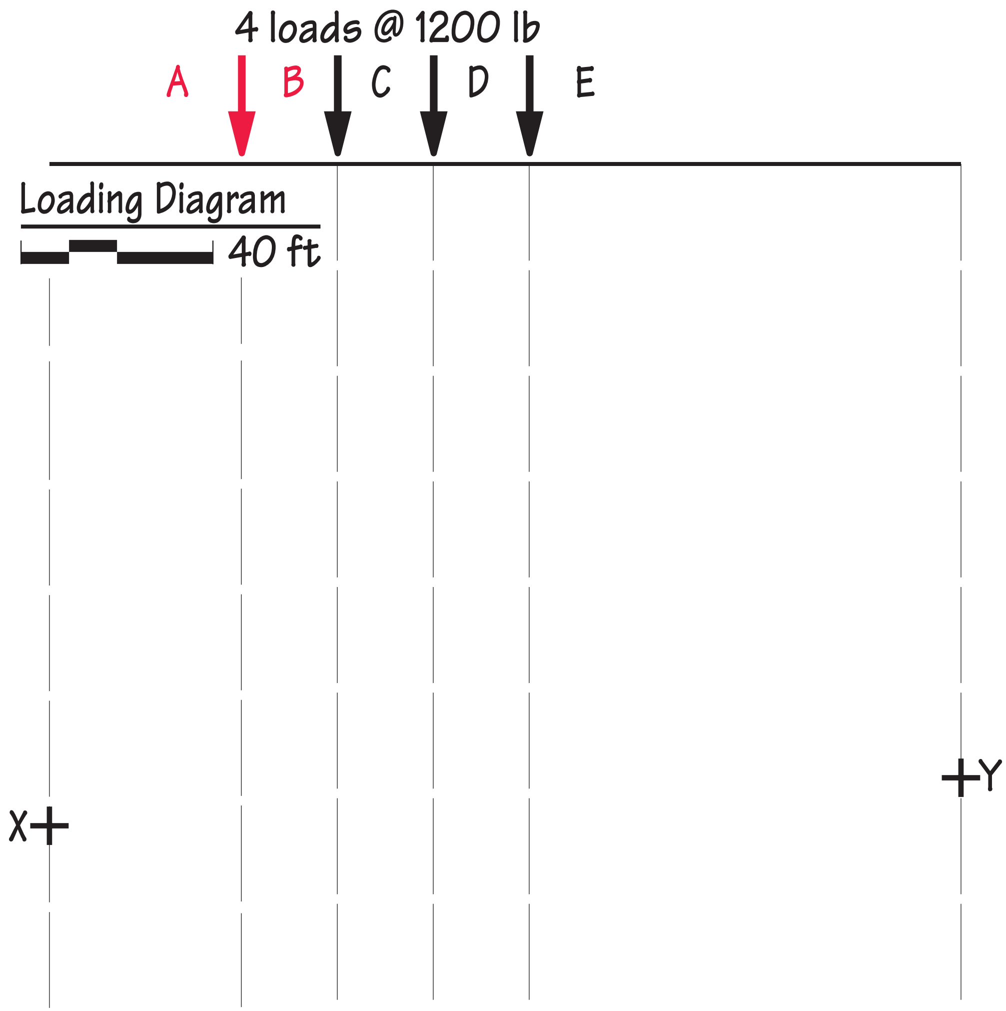



The locations of the endpoints of the cable, *X* and *Y*, are placed toward the bottom of the diagram, leaving space above for construction of a Trial Funicular Polygon.

The relative difference in height between *X* and *Y* is maintained.

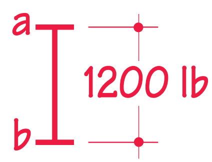
# Finding a Funicular Curve Through Two Points




**Step 2: Construct the Load Line.**

Next we construct a Load Line to any scale that fits comfortably on the page. The Load Line is a tip-to-tail addition of the loads acting on the structure. Lower case letters on the Load Line correspond to the capital letters on the Loading Diagram.

We begin by plotting *ab*, a vector of length 1200 lb, on the Load Line, corresponding to force *AB* on the Loading Diagram.

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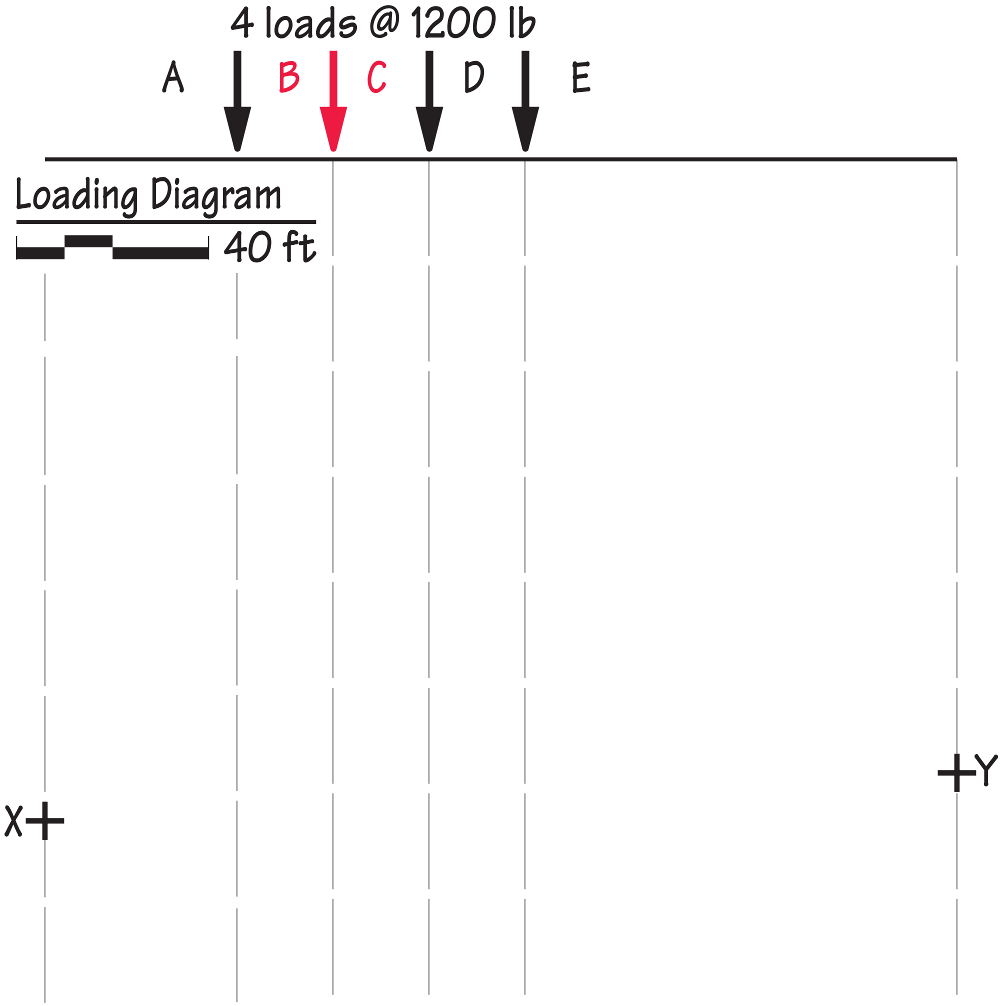
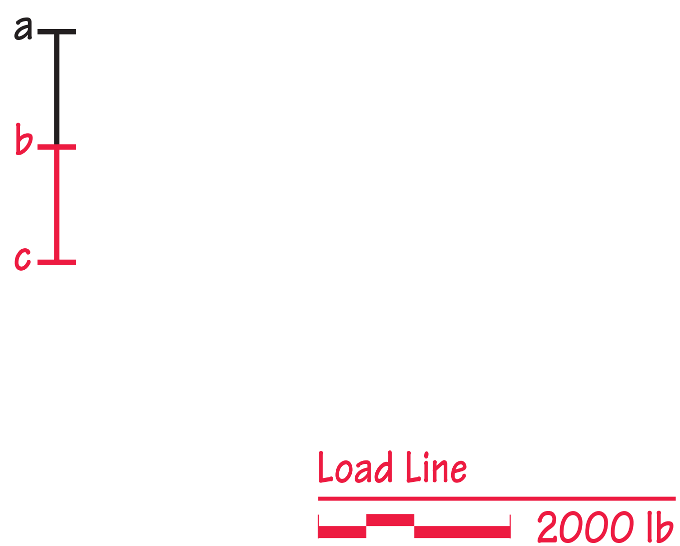



Working from left to right, we continue to plot the loads from the Loading Diagram onto the Load Line.



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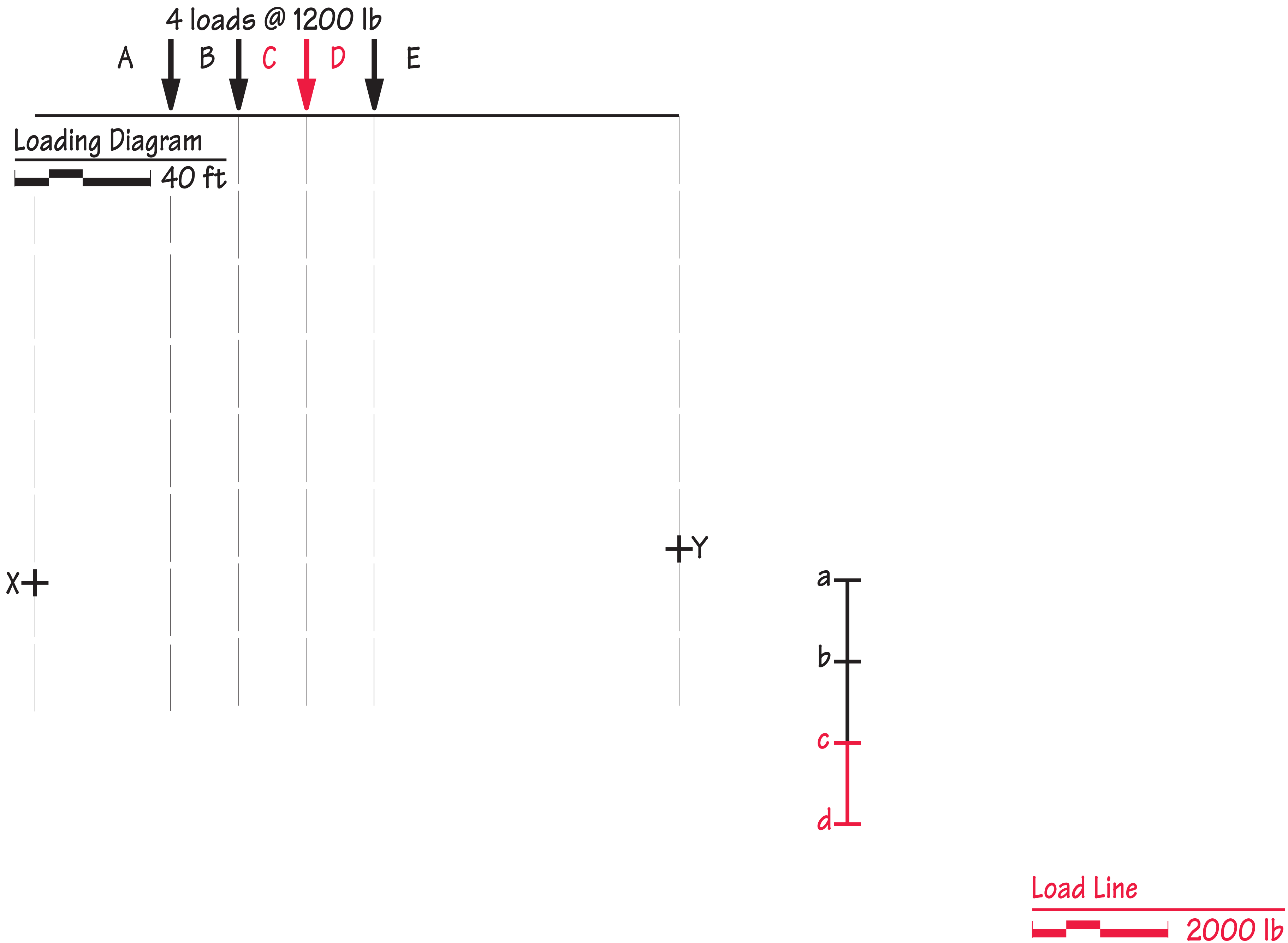

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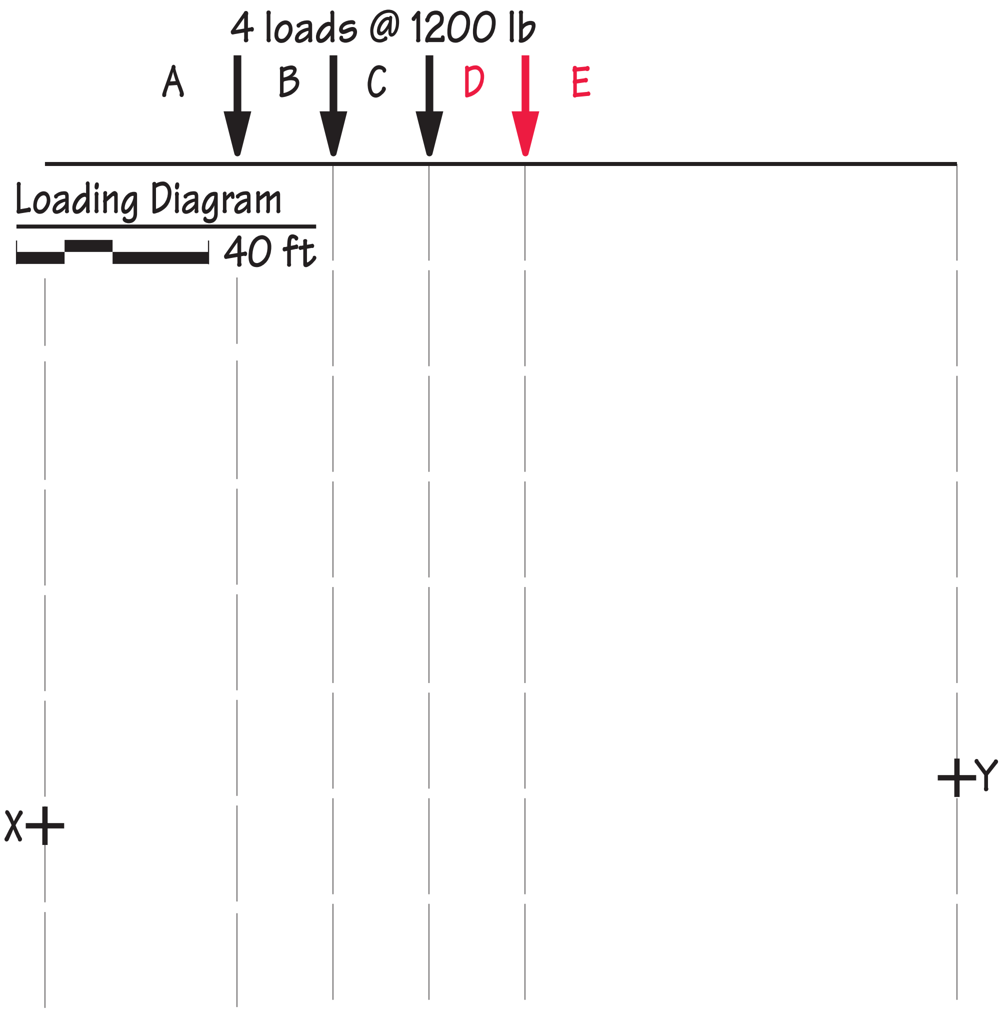
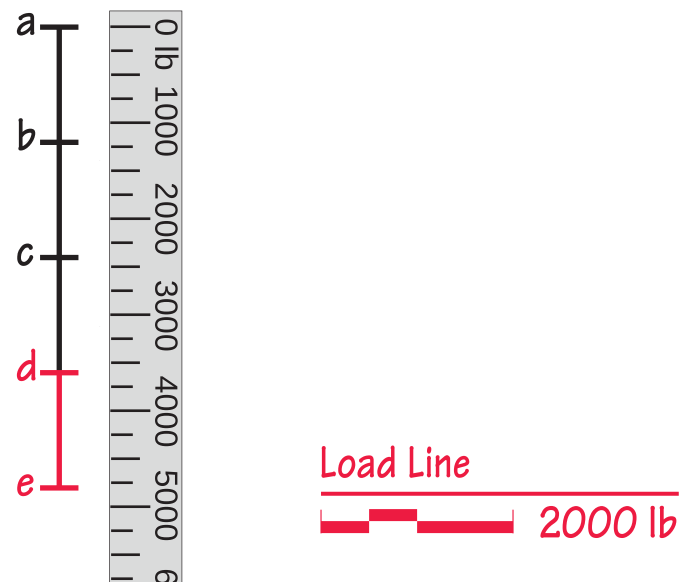



Once all the loads have been plotted, the overall length of the Load Line scales to 4800 lb, the total load on the structure.



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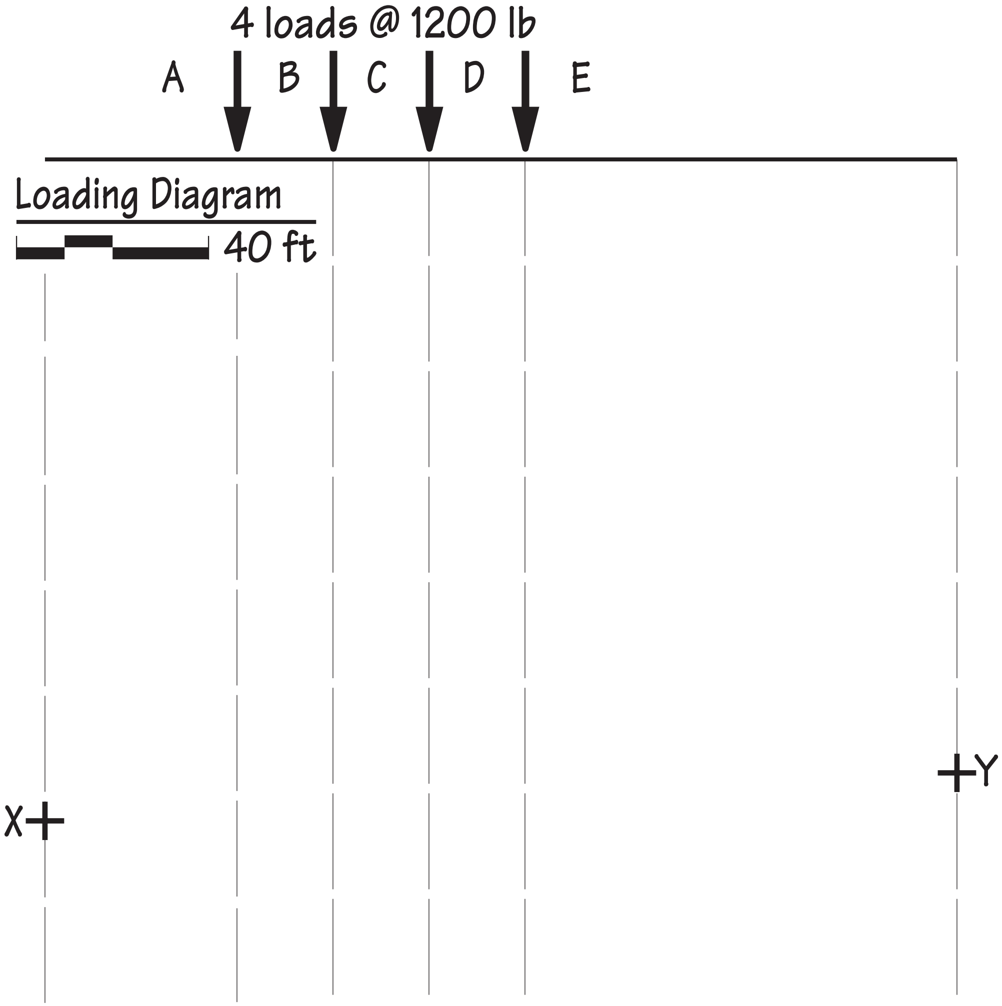
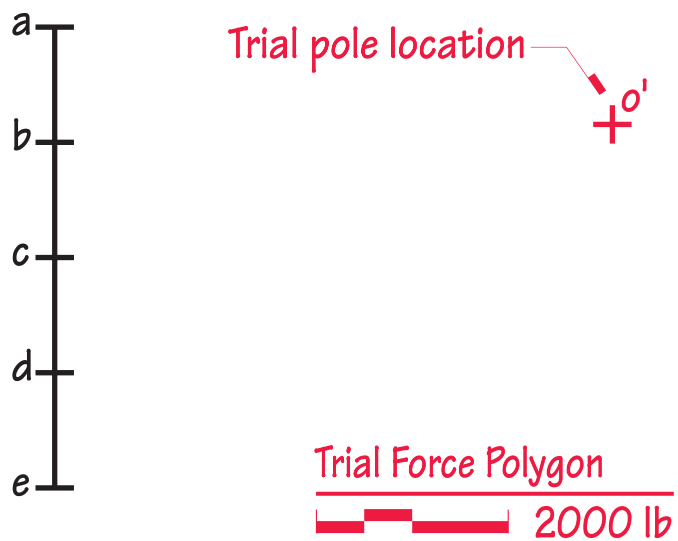



## Step 3: Construct the Trial Funicular Polygon.

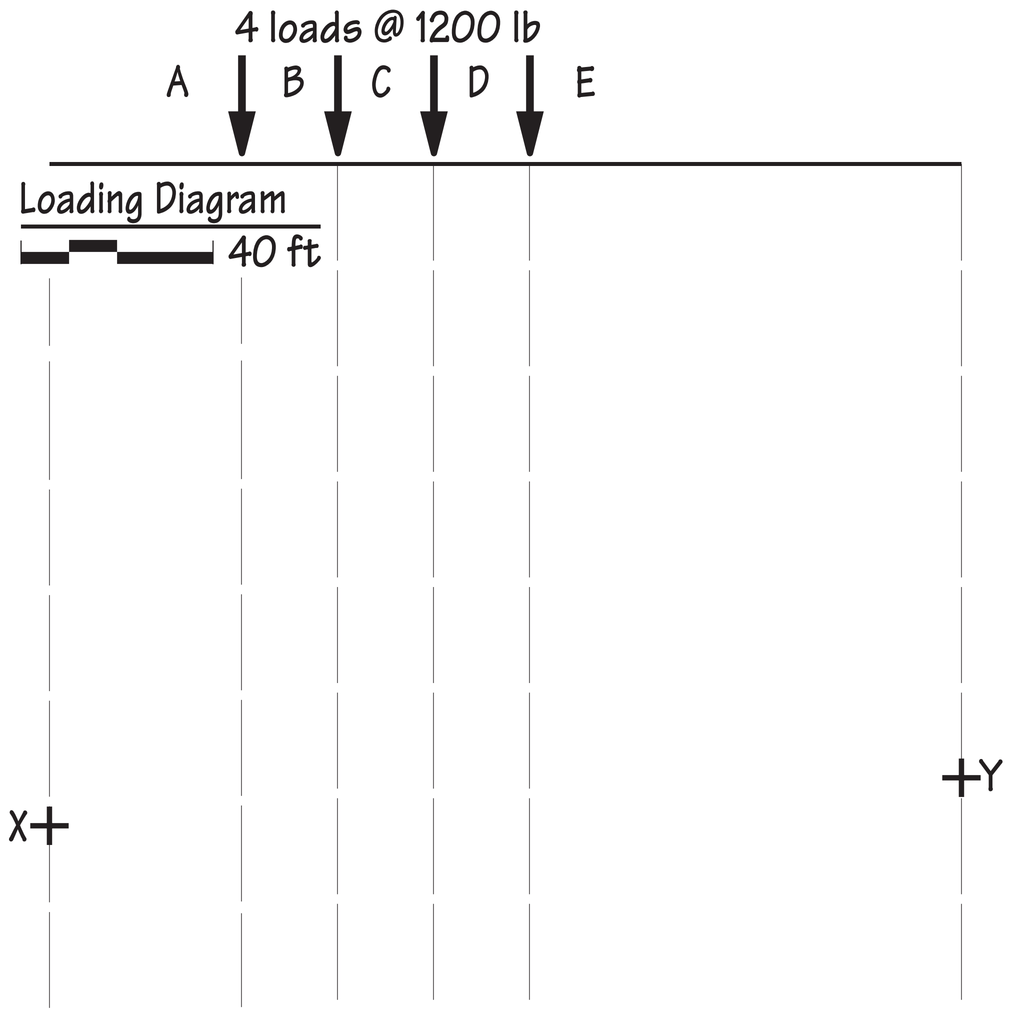
The solution proceeds in two steps, a Trial Step and a Final Step. For the Trial, we select a trial pole location, *o'*, arbitrarily.

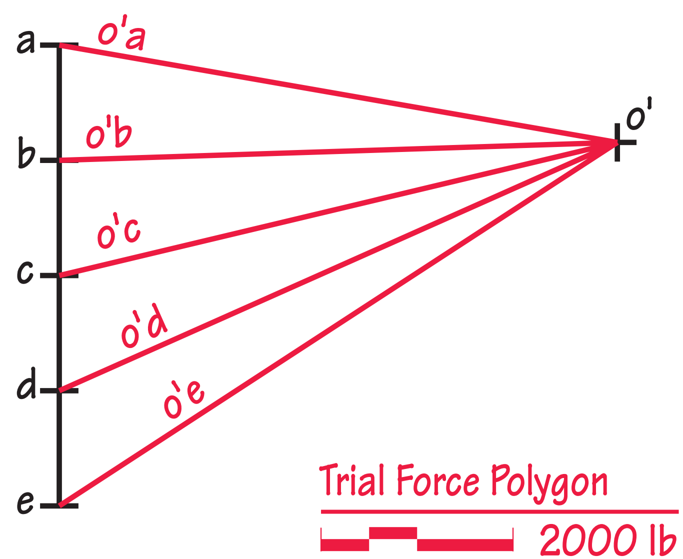
The Load Line now becomes part of a Trial Force Polygon.



# Finding a Funicular Curve Through Two Points

We construct a set of rays, *o′a* through *o′e*, on the Trial Force Polygon.



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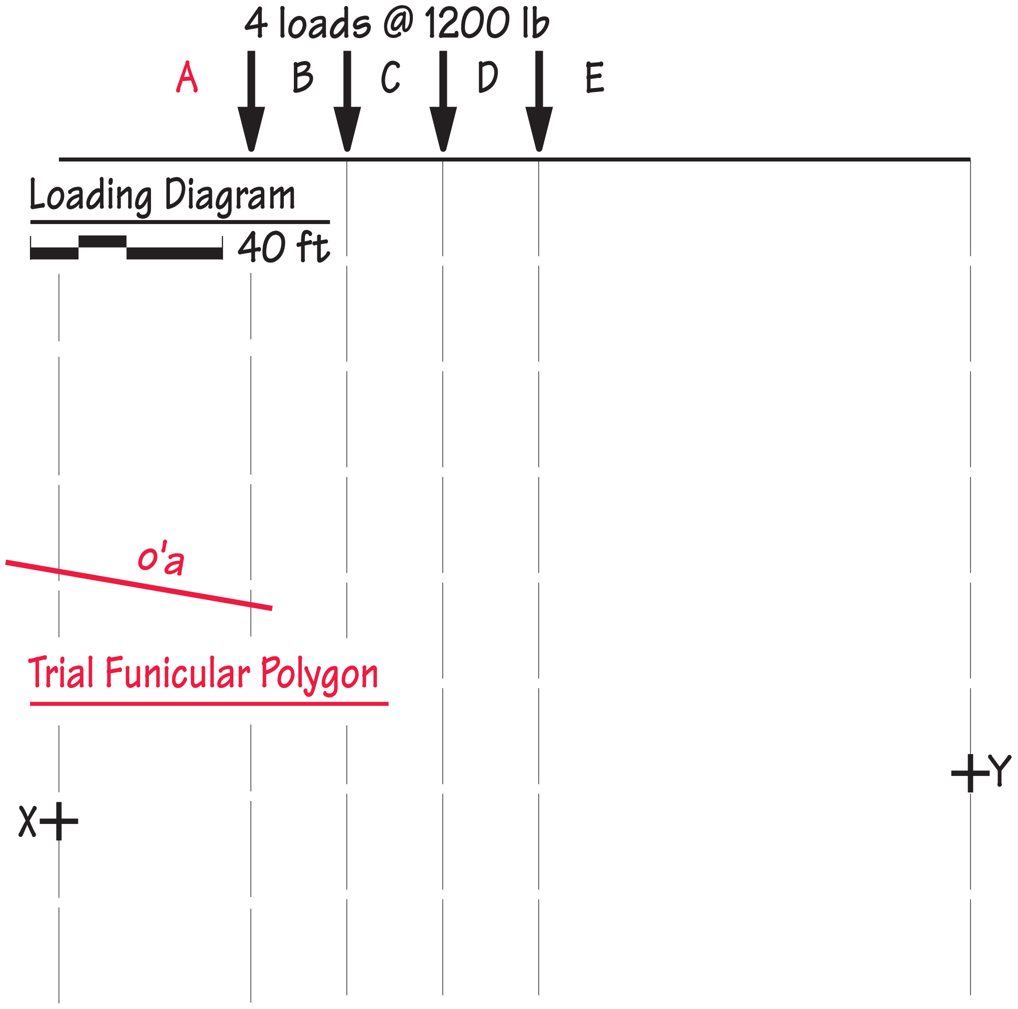
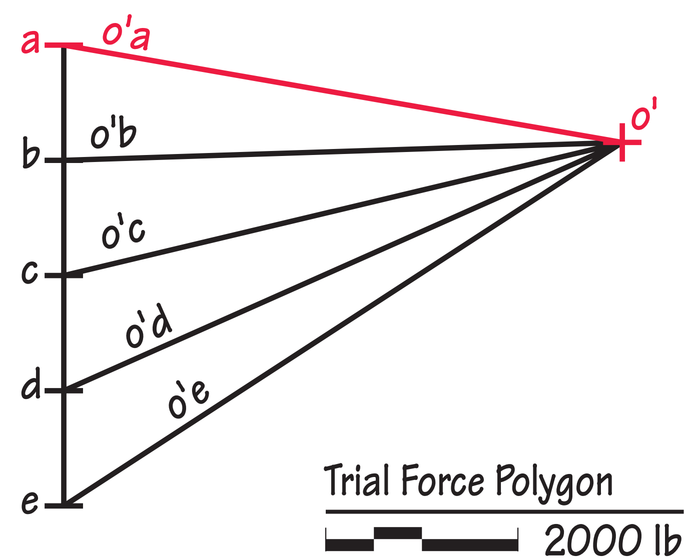


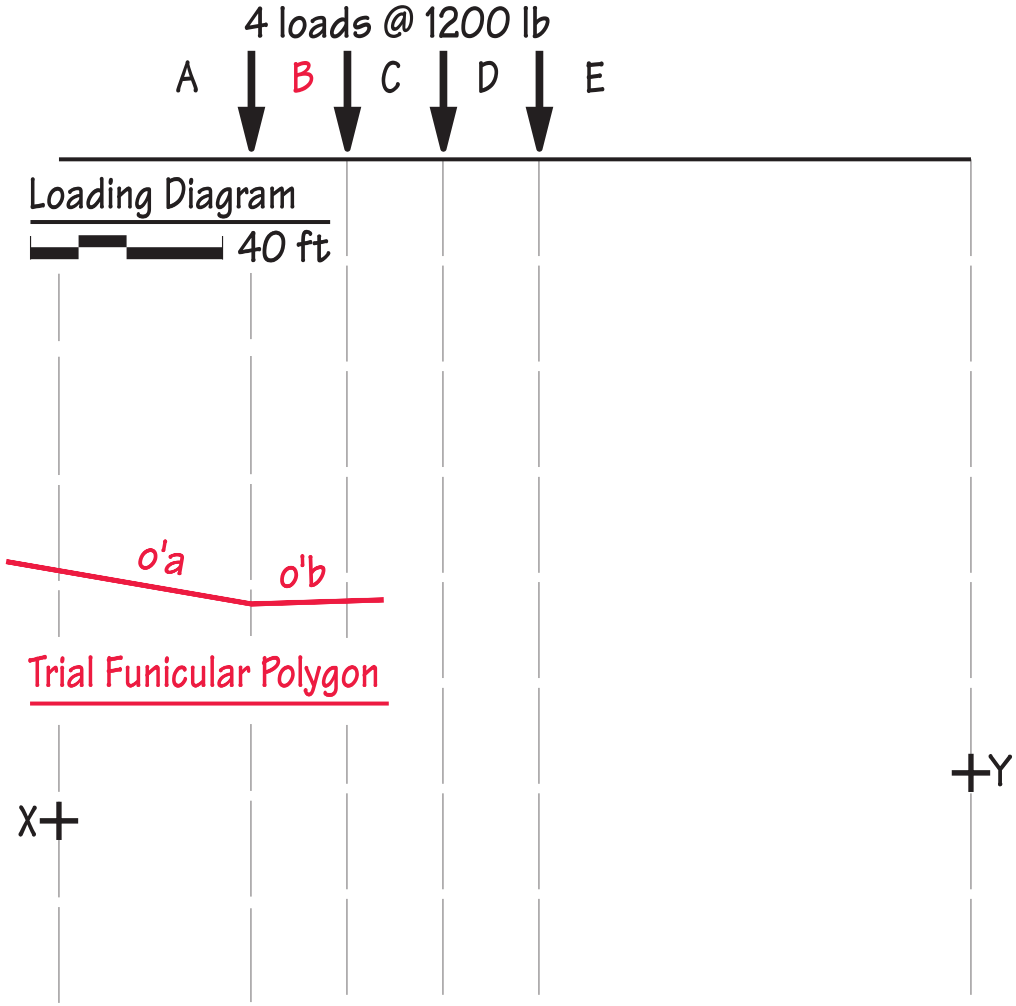

Parallel to each ray on the Trial Force Polygon, we draw the corresponding segment of the Trial Funicular Polygon. This is constructed in the empty space between the Loading Diagram and the Final Funicular Diagram.

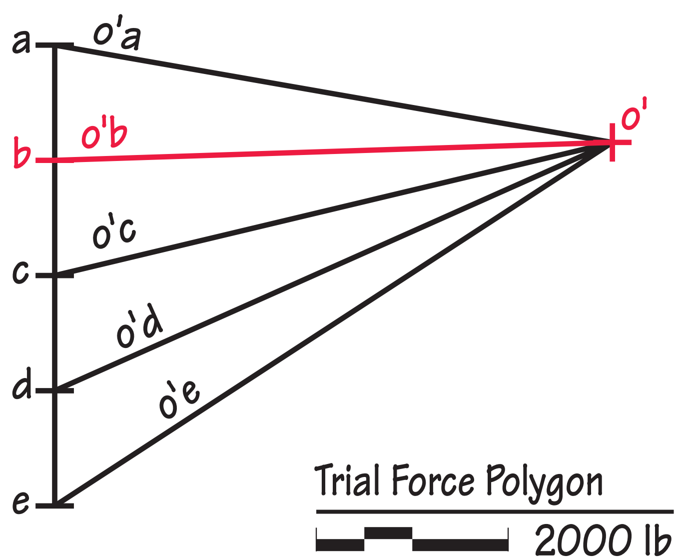
Segment *o'a* lies below interval *A* of the Loading Diagram.



# Finding a Funicular Curve Through Two Points

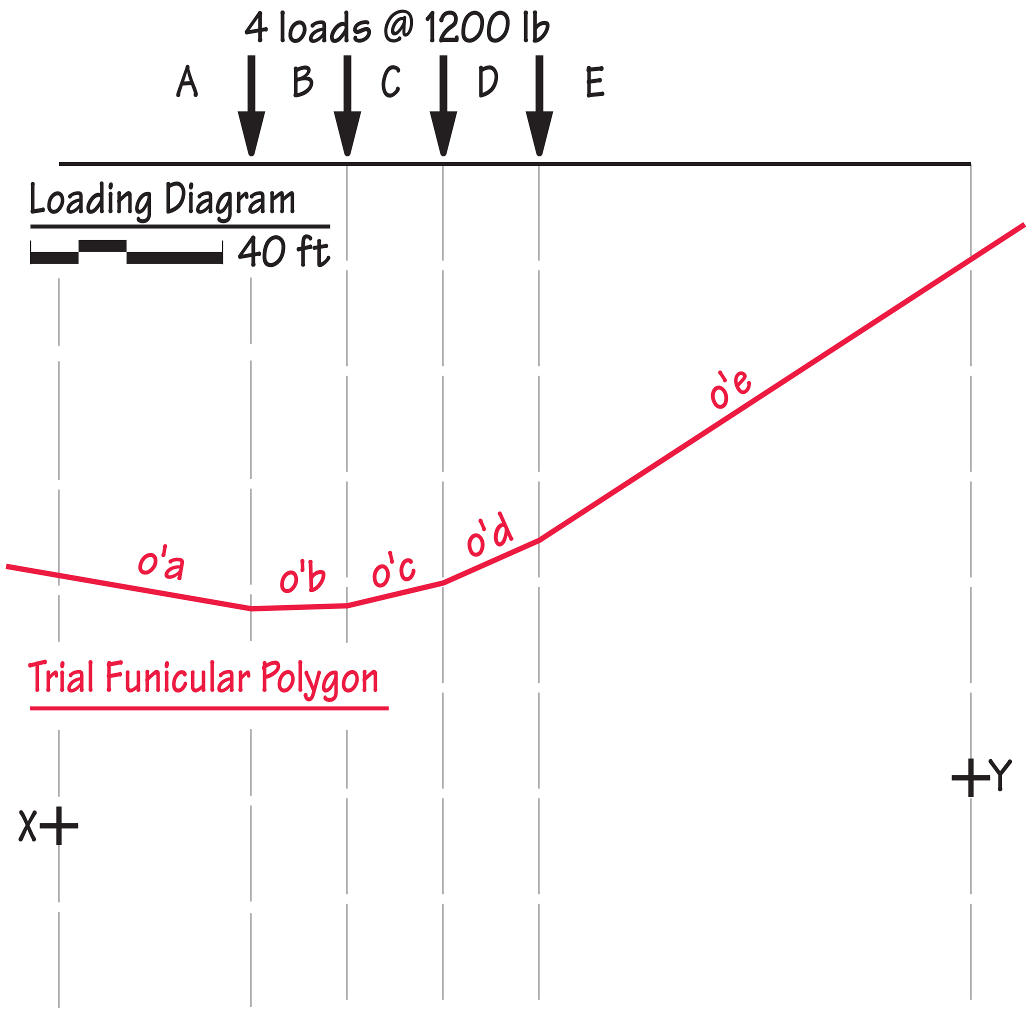
Segment *o'b* lies below interval *B* of the Loading Diagram.

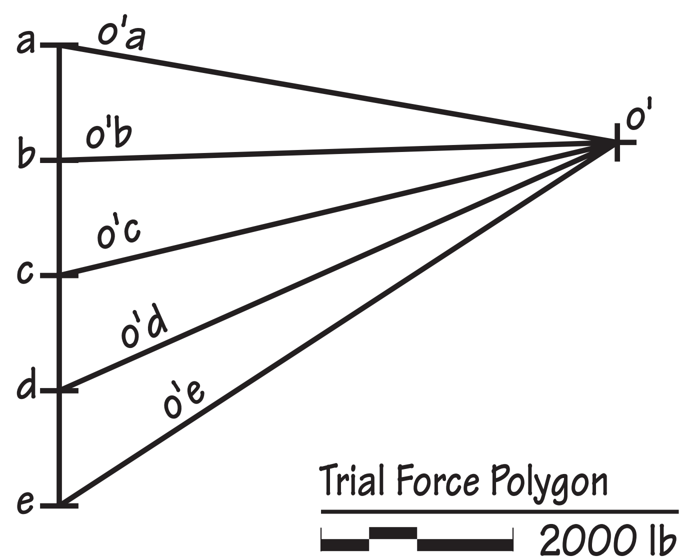


# Finding a Funicular Curve Through Two Points

By constructing segments *o′c*, *o′d*, and *o′e*, we complete the Trial Funicular Polygon.



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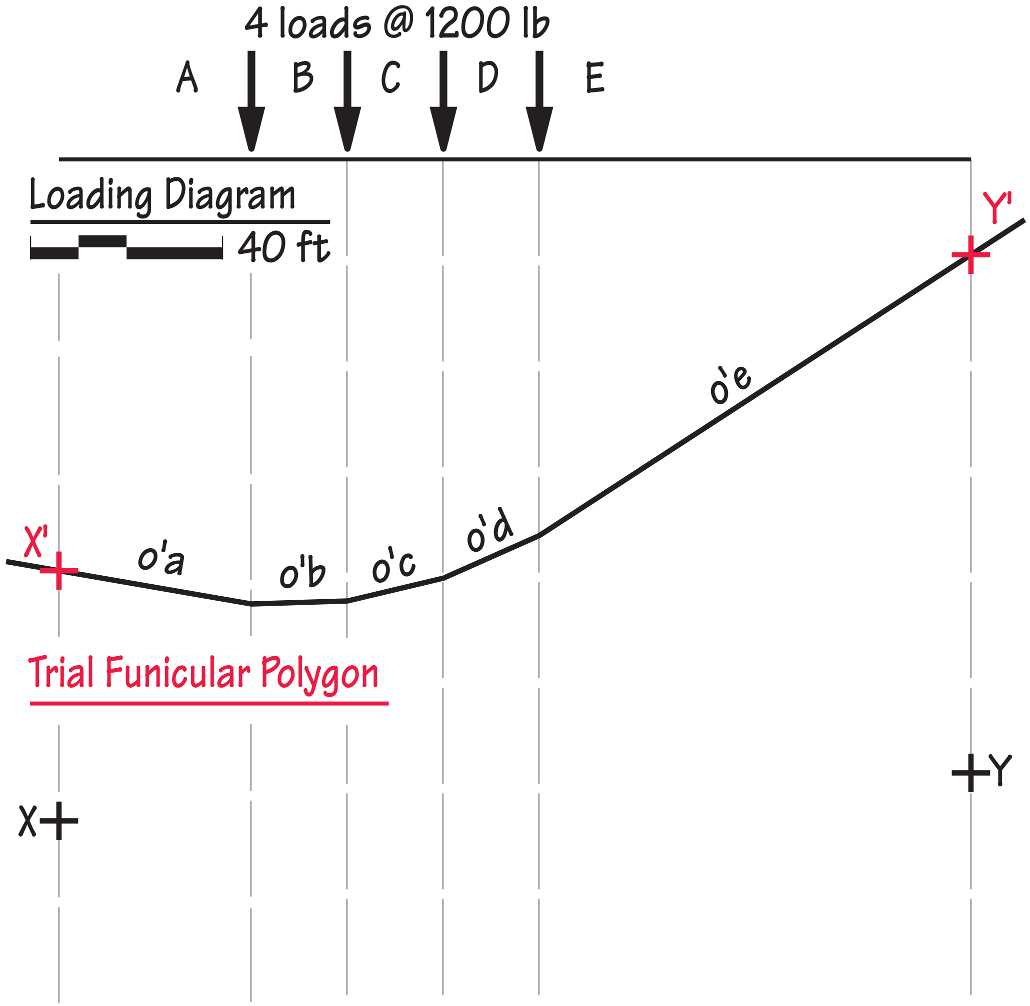
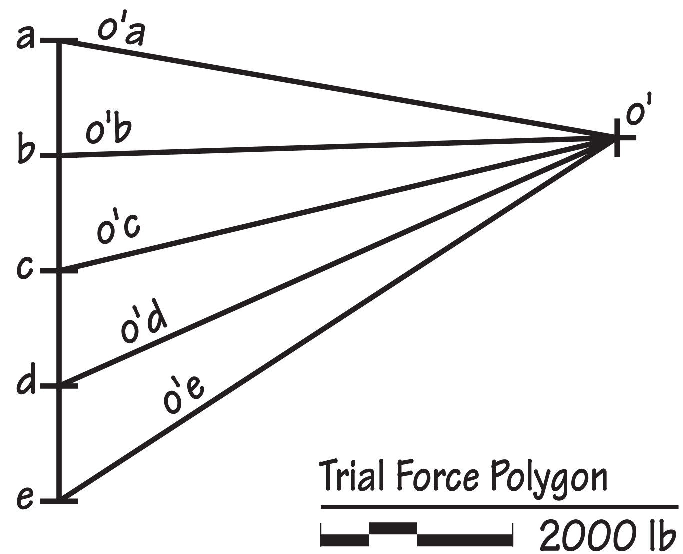



The ends of the Trial Funicular Polygon are labeled *X′* and *Y′*. *X′* and *Y′* align vertically with end points *X* and *Y* of the Final Funicular Polygon.



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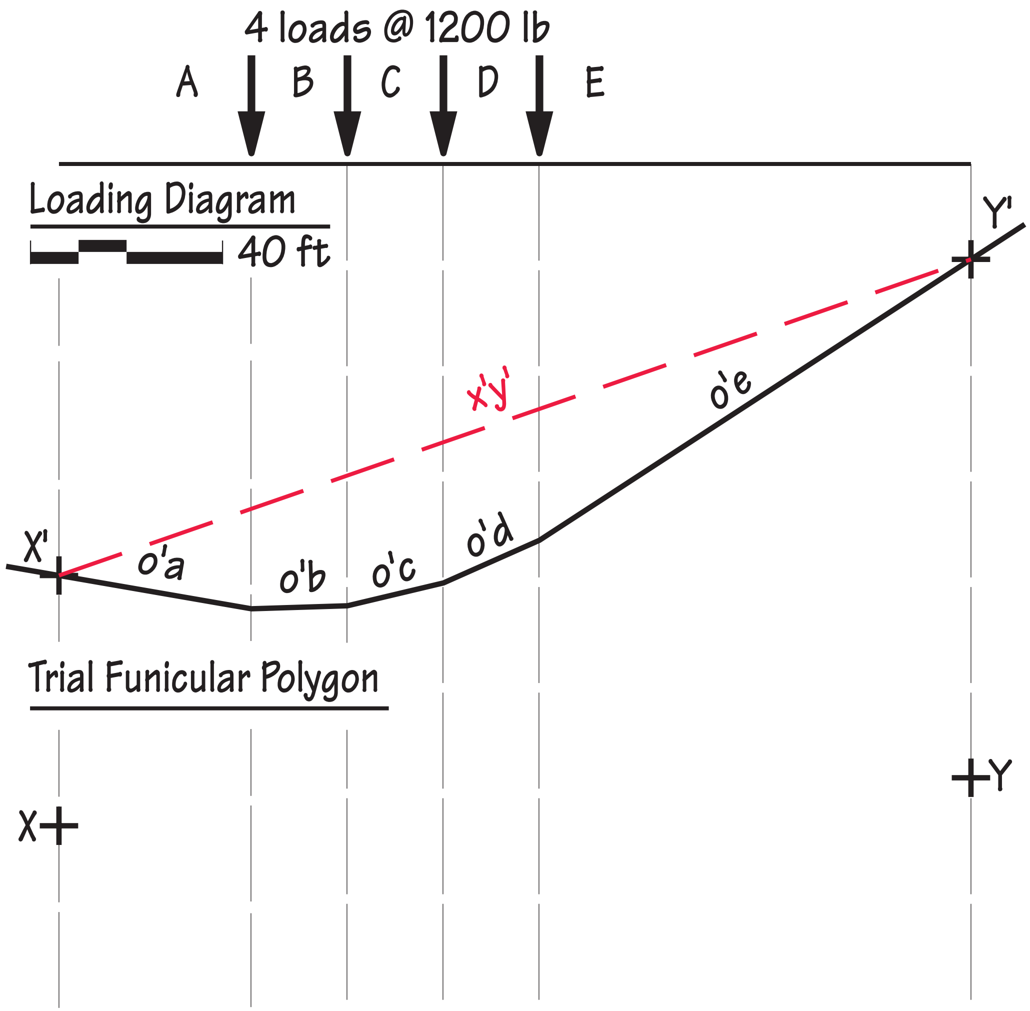
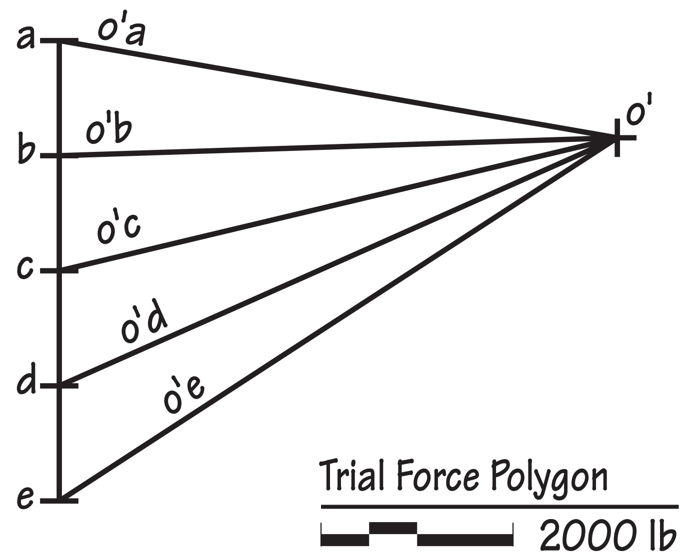



## Step 4: Find the Final Pole

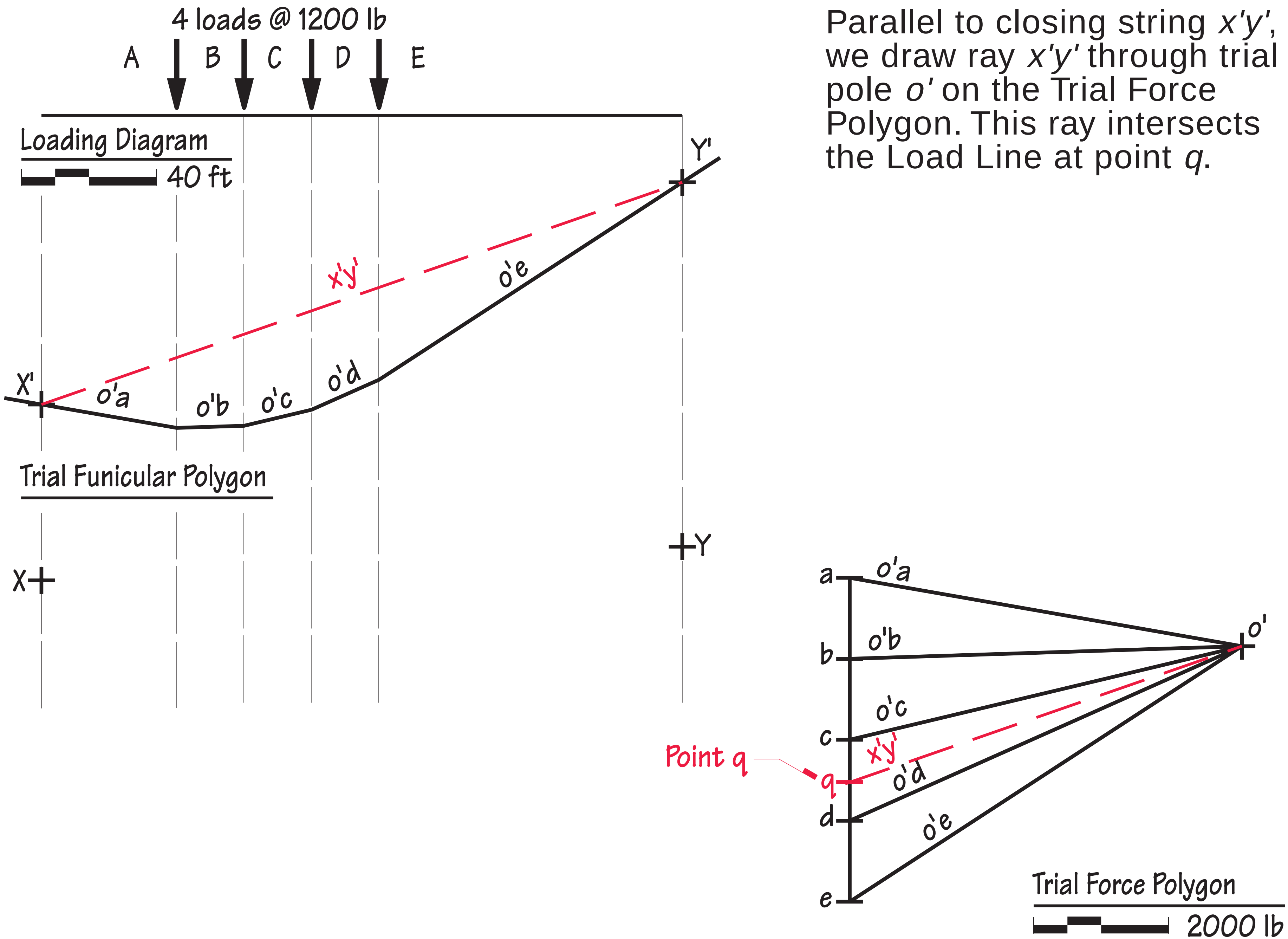
Now we are in a position to find the pole location that will generate the Final Funicular Polygon that passes through *X* and *Y* with a maximum internal force of 6600 lb.

First we draw the closing string *x'y'*, of the Trial Funicular Polygon.



# Finding a Funicular Curve Through Two Points

Parallel to closing string *x′y′*, we draw ray *x′y′* through trial pole *o′* on the Trial Force Polygon. This ray intersects the Load Line at point *q*.



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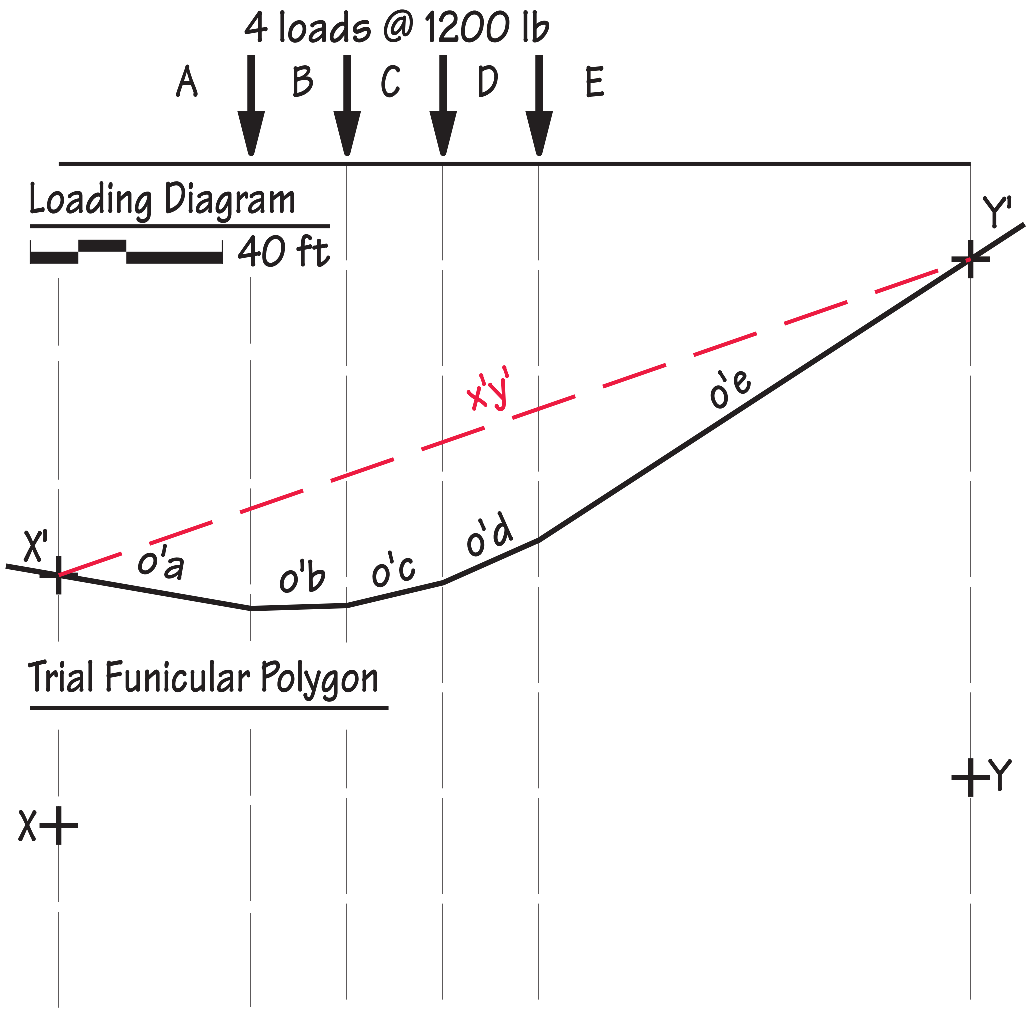
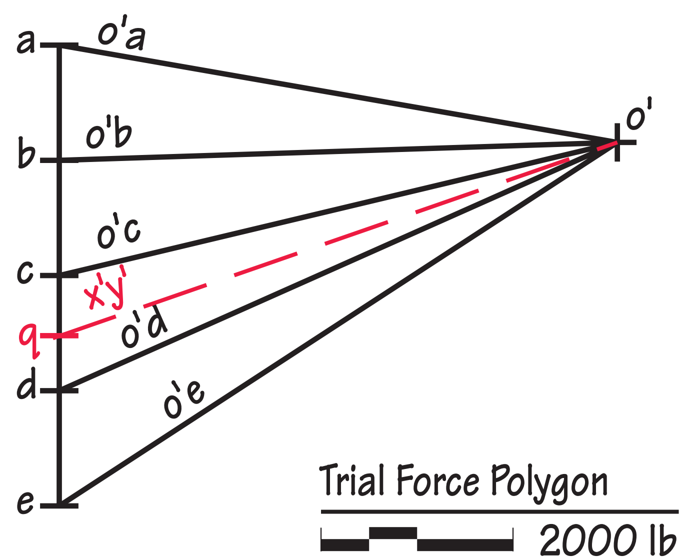



For any polygon that is funicular for this loading pattern, a ray parallel to its closing string must pass through point *q*.

Thus, the ray parallel to closing string *xy* of the Final Funicular Polygon must pass through *q*. We can use this fact to locate the pole of the Final Force Polygon, and from there, to construct the Final Funicular Polygon.



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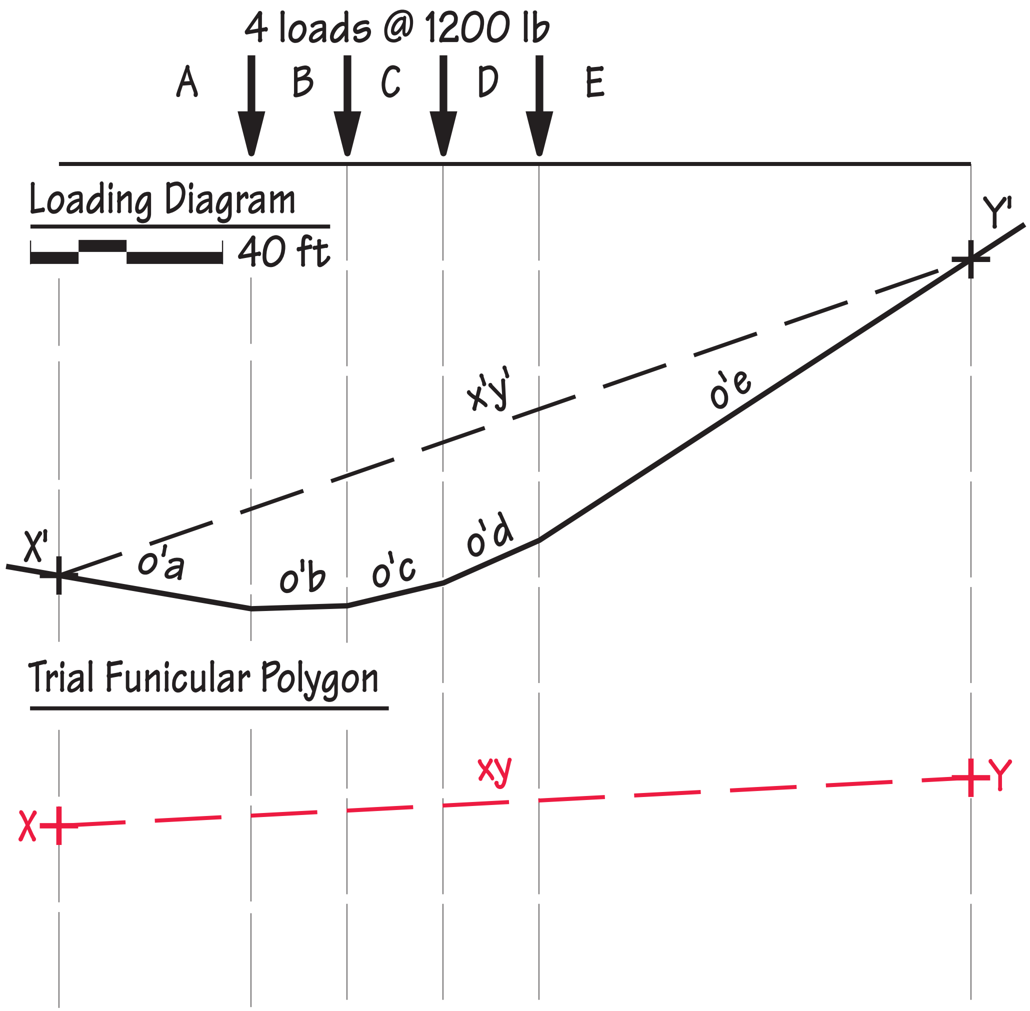
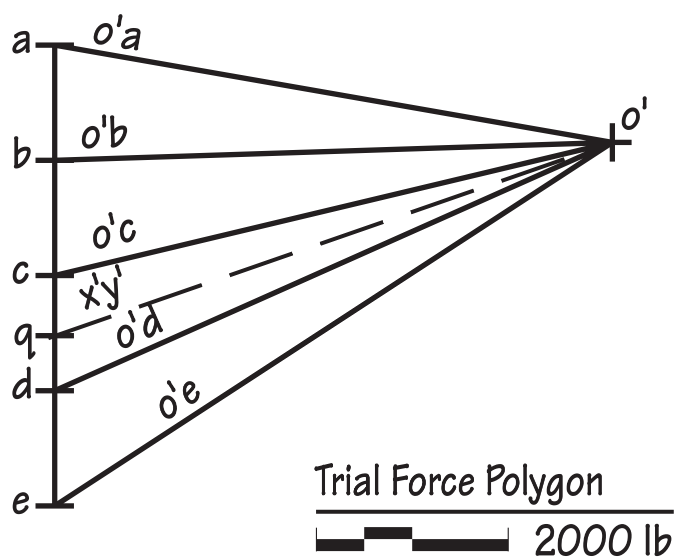



We construct line *xy* between *X* and *Y*, the ends of the Final Funicular Polygon that we seek. The slope of *xy* is determined by the relative heights of points *X* and *Y* established at the beginning of the problem.

*xy* will be the closing string of this Funicular Polygon.



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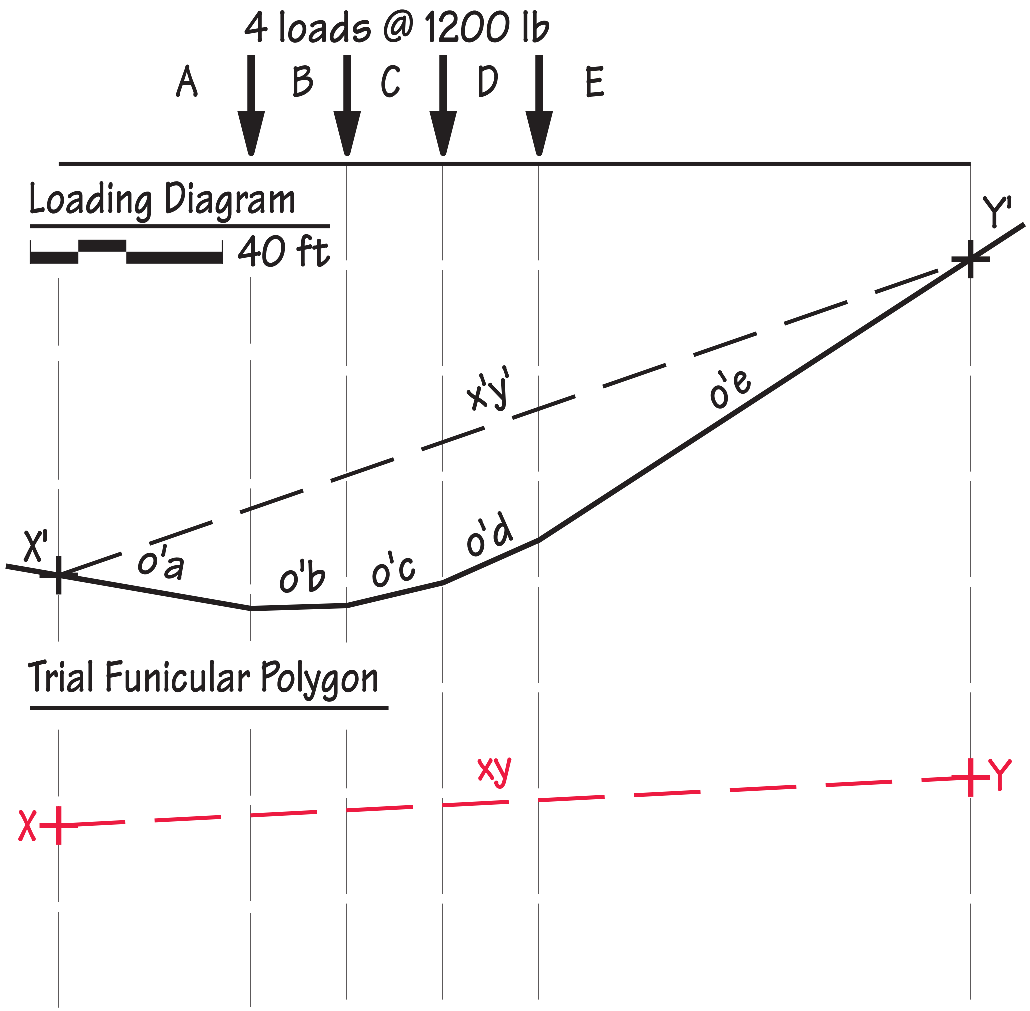
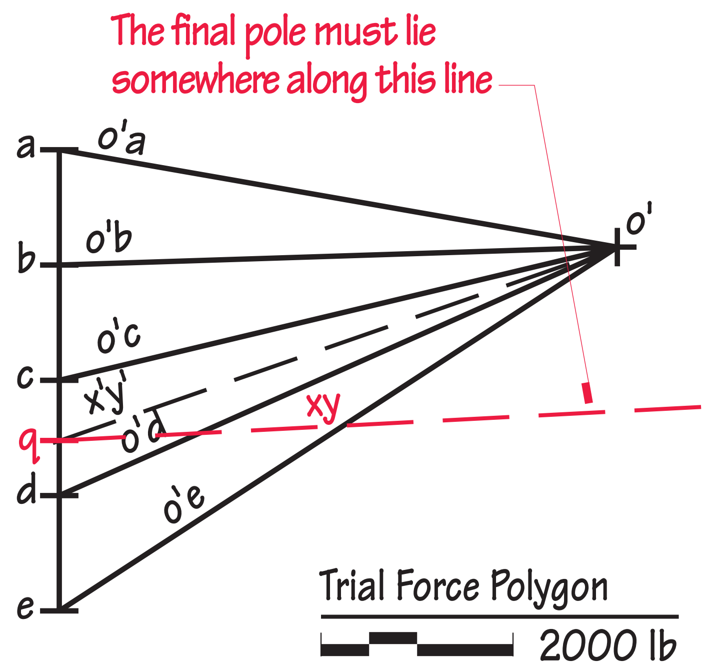



Parallel to closing string *xy*, we draw ray *xy* through point *q* on the Load Line. The final pole must lie somewhere on this line.



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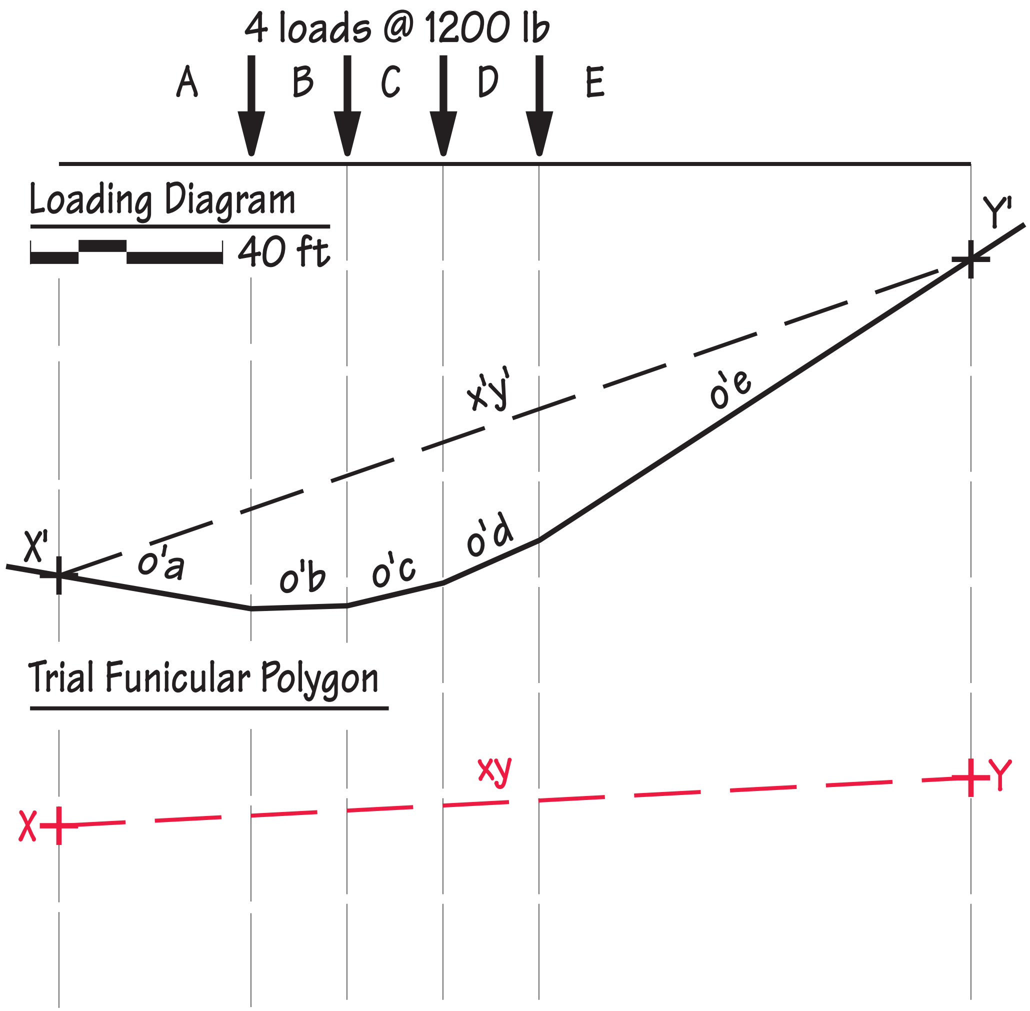
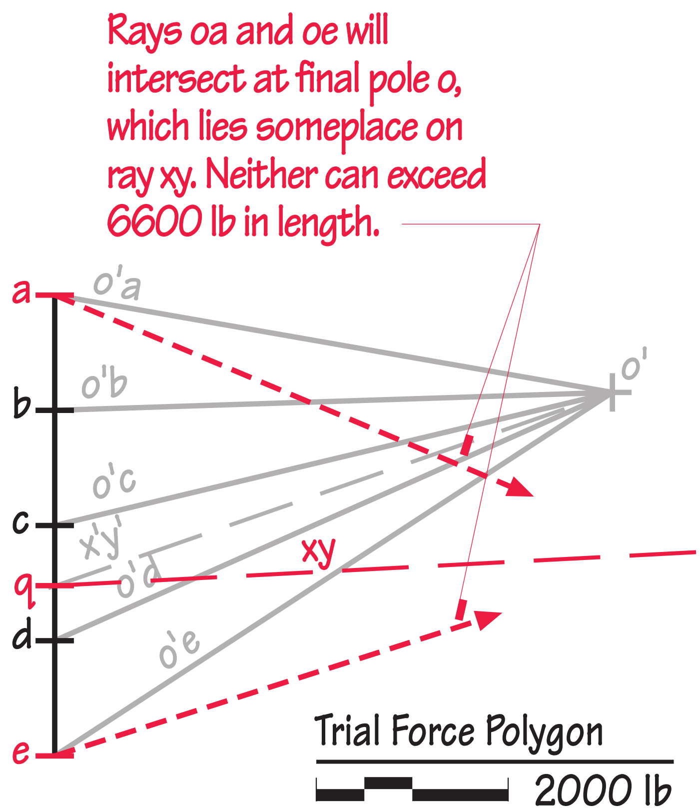



The maximum force in the cable was specified as 6600 lb. This means that the longest ray on the Final Force Polygon must be 6600 lb long.

The longest ray will be either *oa* or *oe*, it is not yet clear which.



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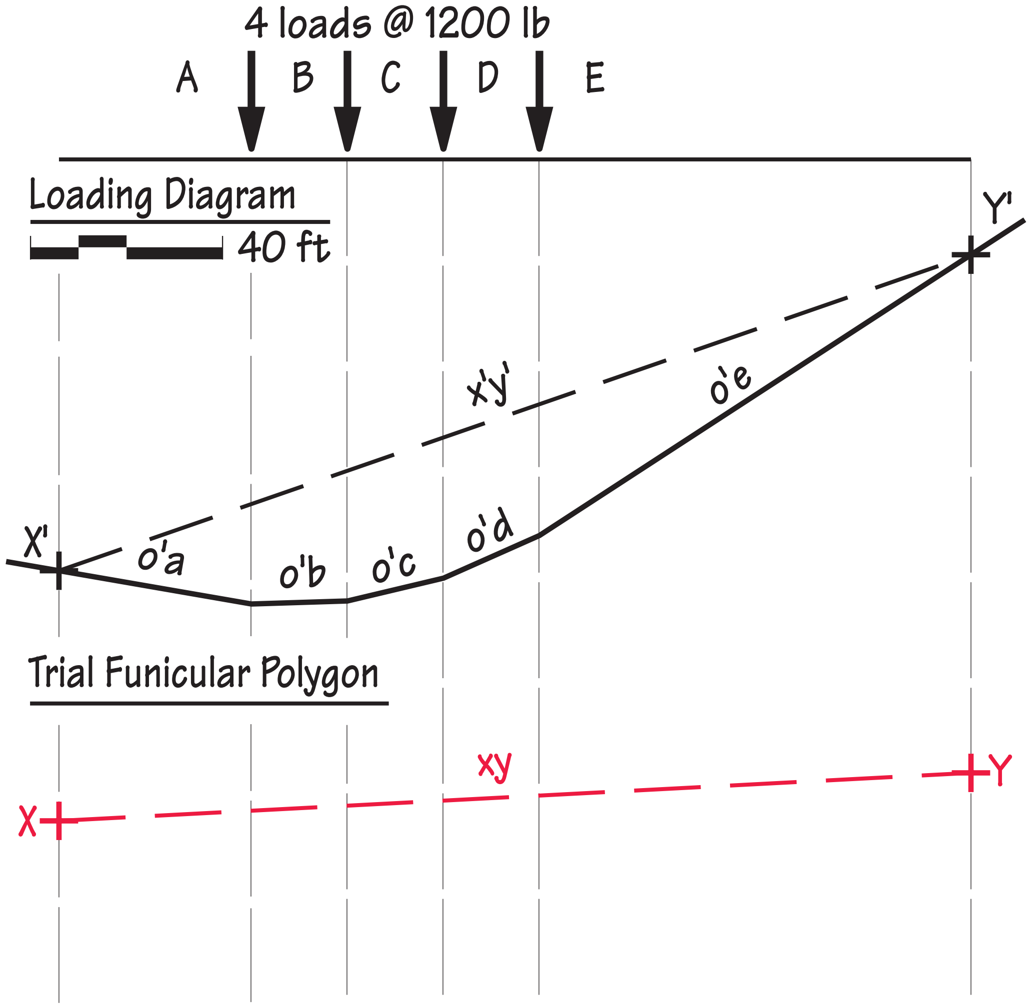
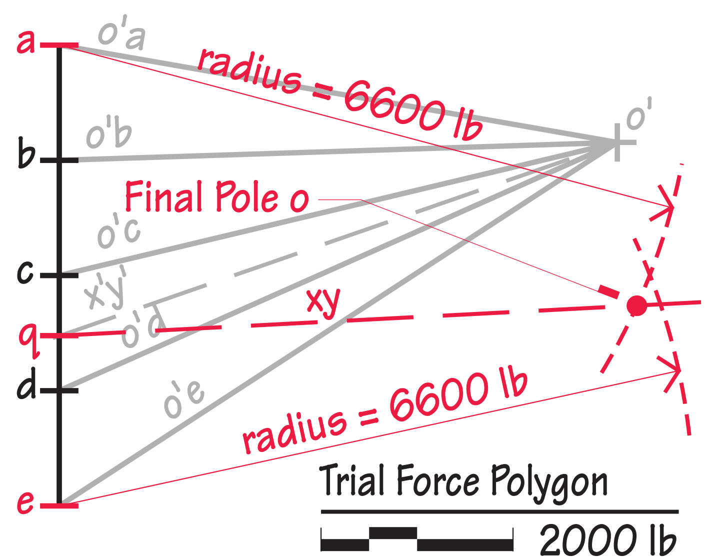



We strike an arc of radius 6600 lb from *a* to intersect ray *xy*, and another arc of 6600 lb radius from *e*.

We adopt the closer of these two intersections to the Load Line as the final pole, *o*.



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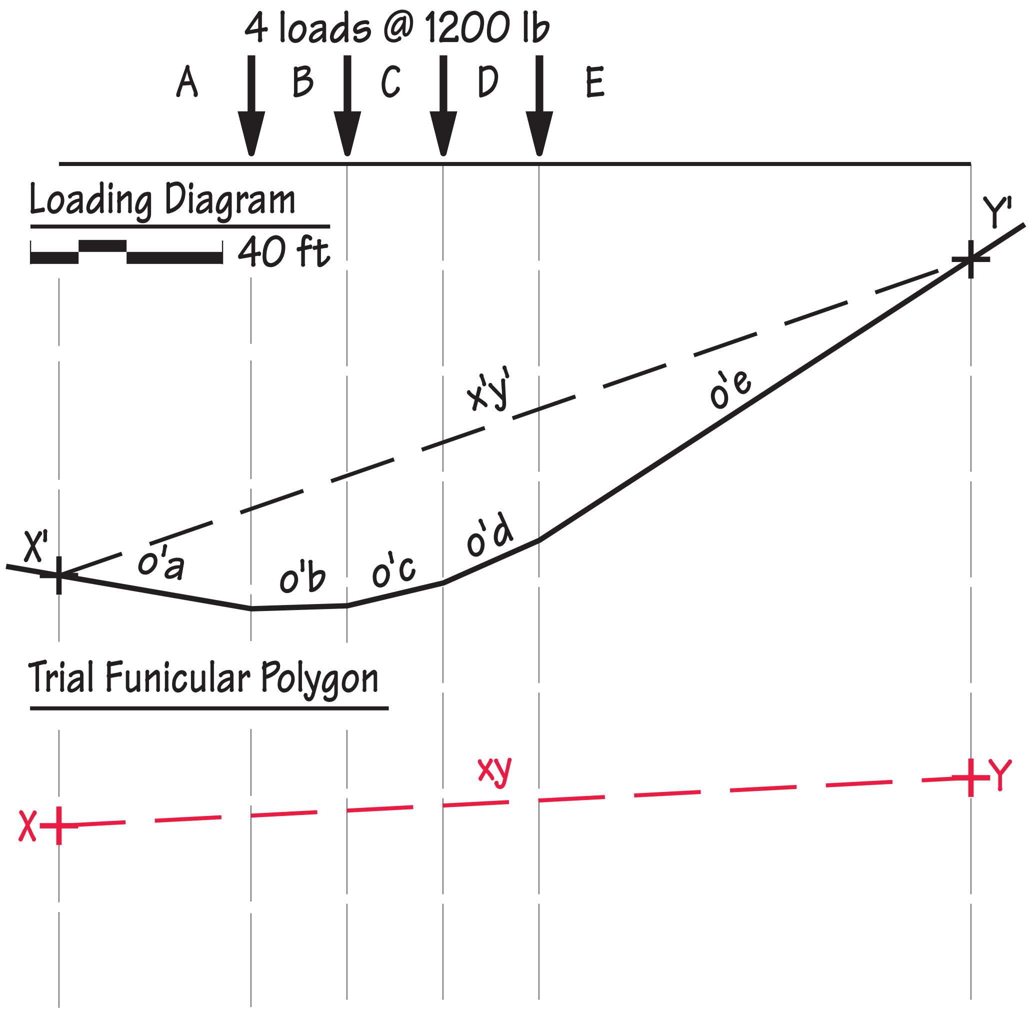
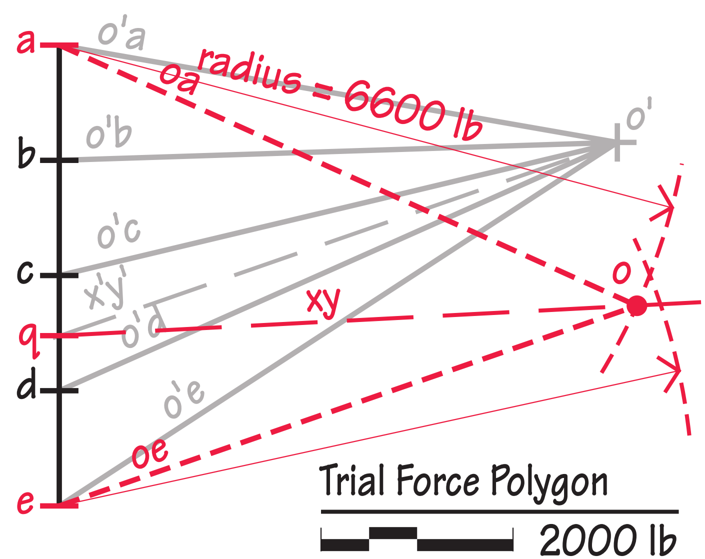



In this case, *o* is located at the intersection with ray *xy* of an arc about point *a* whose radius is 6600 lb.

This means that ray *oa* will be 6600 lb long, and ray *oe* slightly less.



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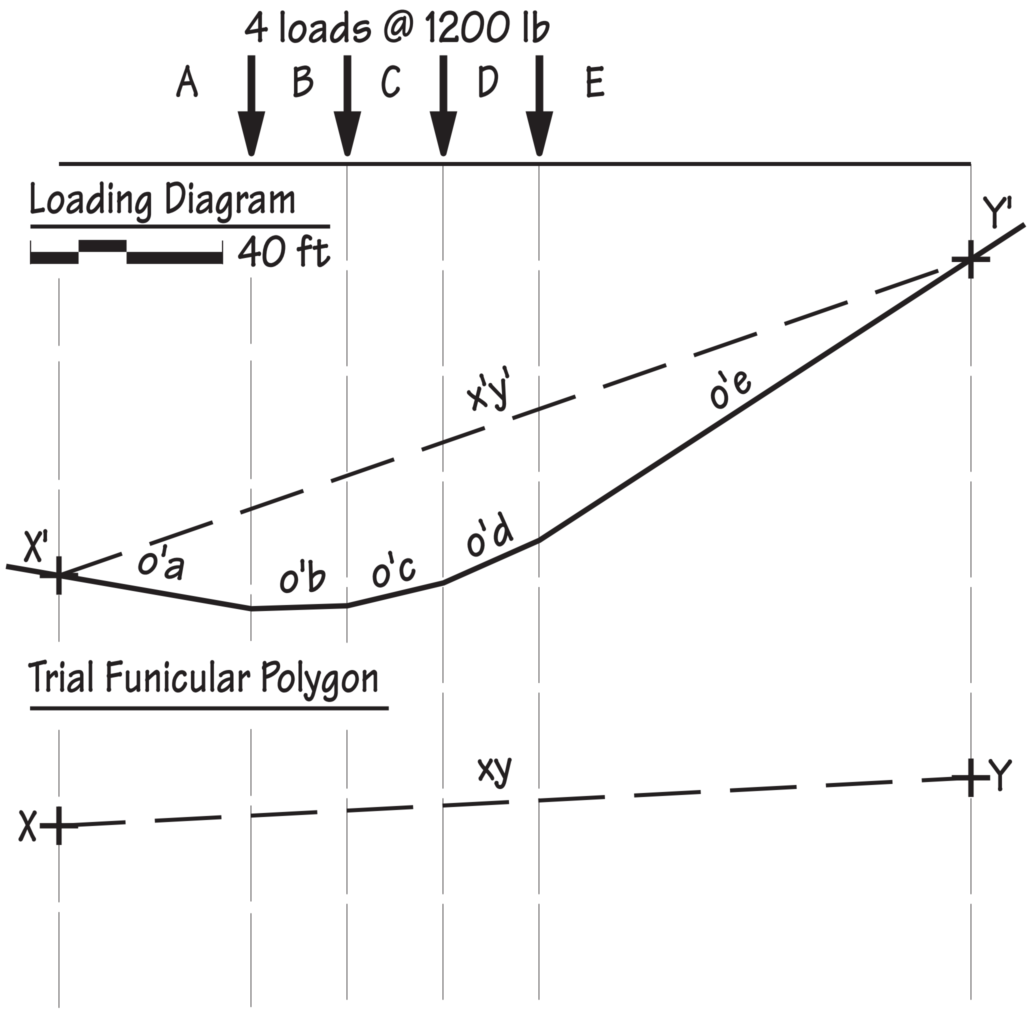
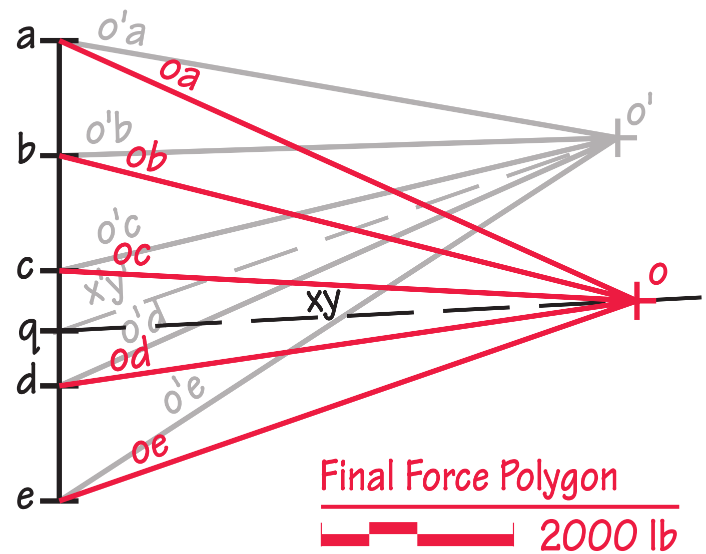



**Step 5: Construct the Final Funicular Polygon.**

Now we are able to complete the Final Force Polygon. From Final Pole *o* we construct a set of rays.



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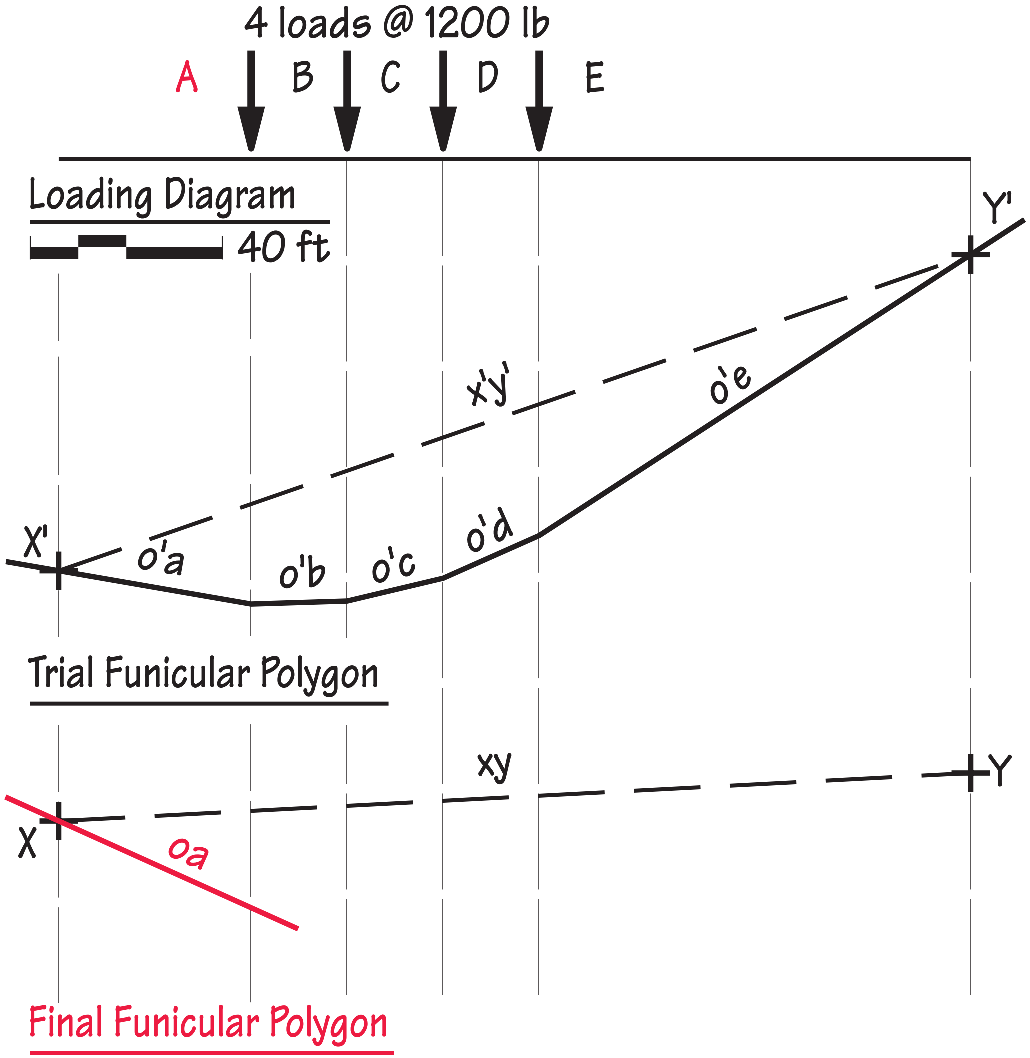
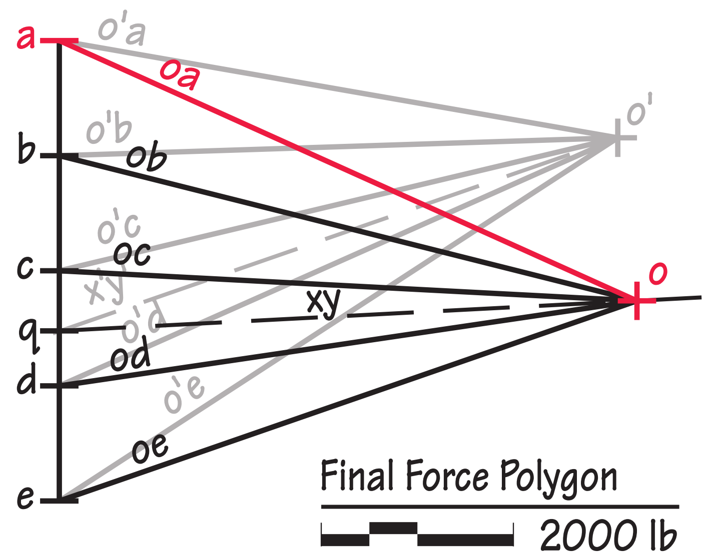


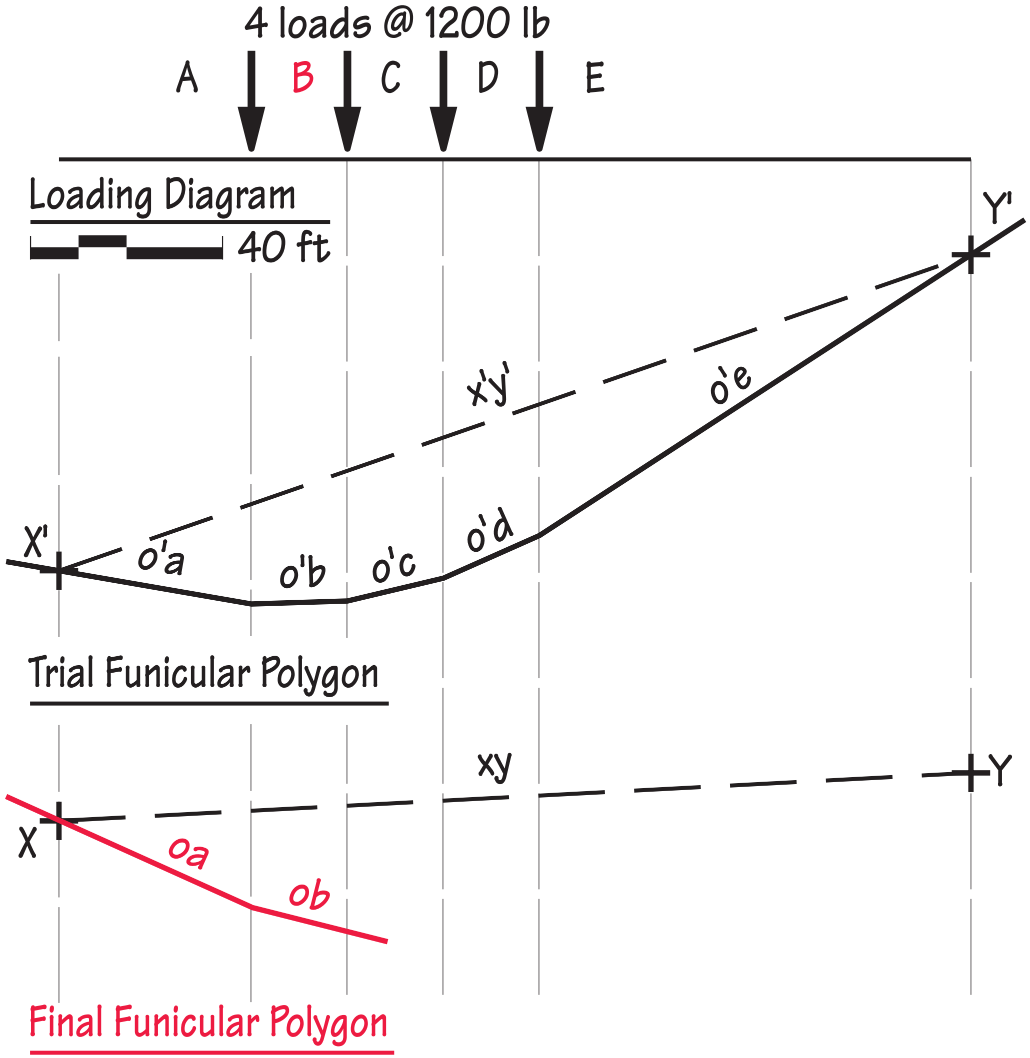

Parallel to each ray, we draw the corresponding segment of the Final Funicular Polygon.

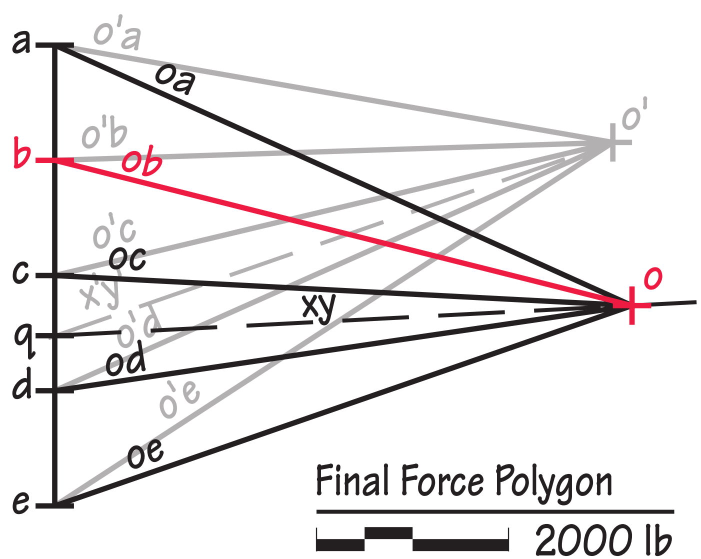
Segment *oa* falls in the interval below letter *A* on the Loading Diagram.



# Finding a Funicular Curve Through Two Points

Segment *ob* falls in the interval below letter *B* on the Loading Diagram.



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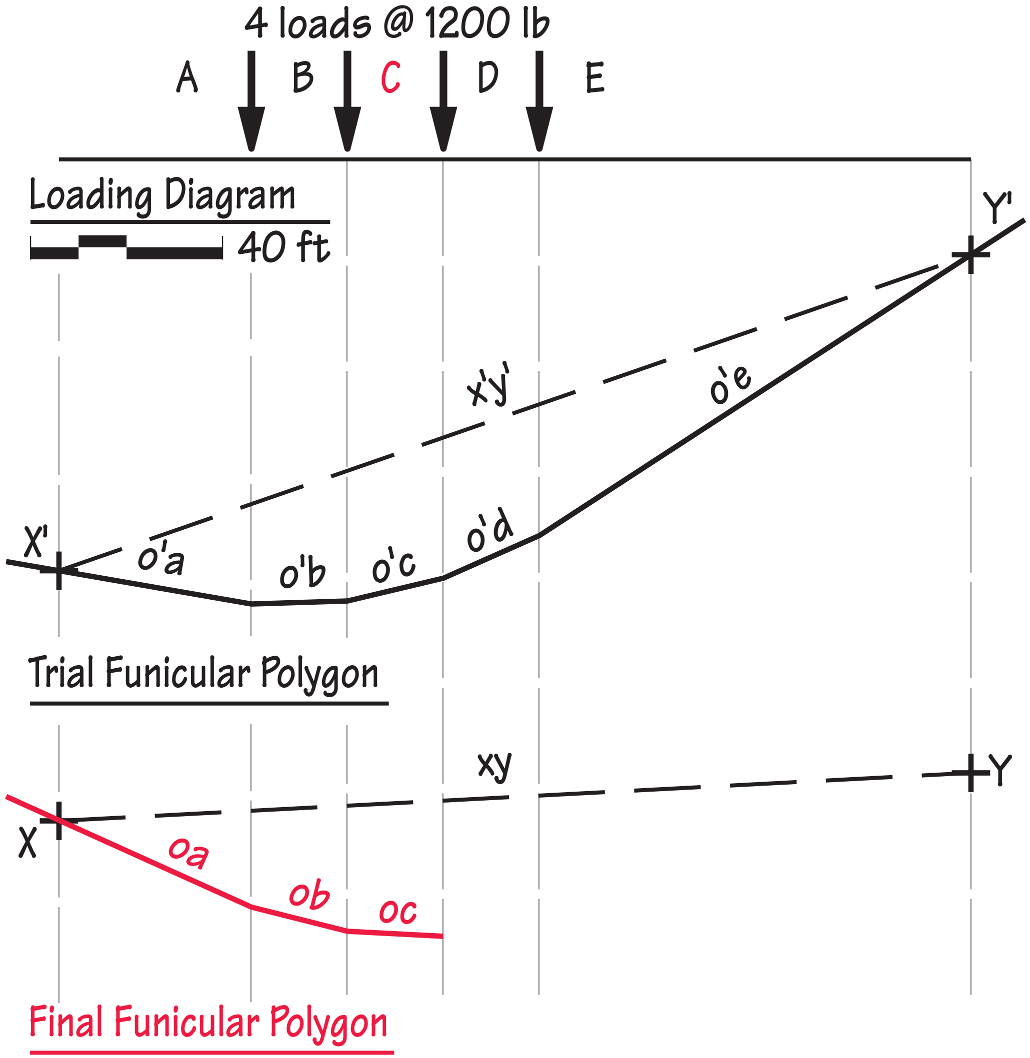
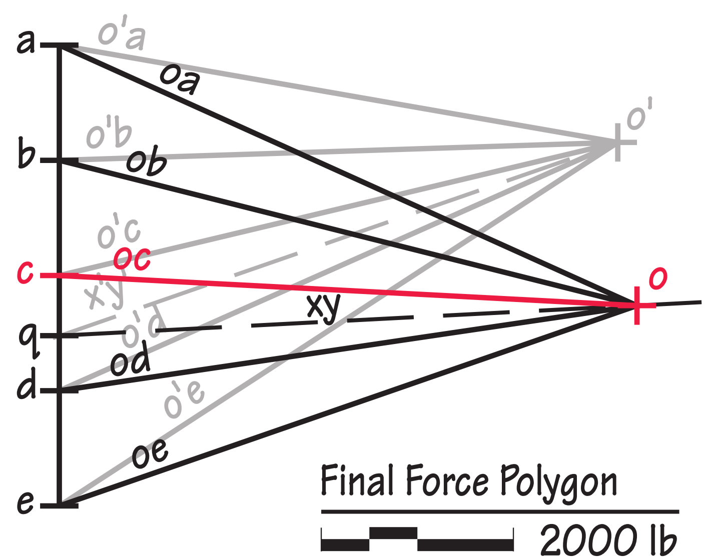



Continuing from left to right and top to bottom, we complete the Final Funicular Polygon.



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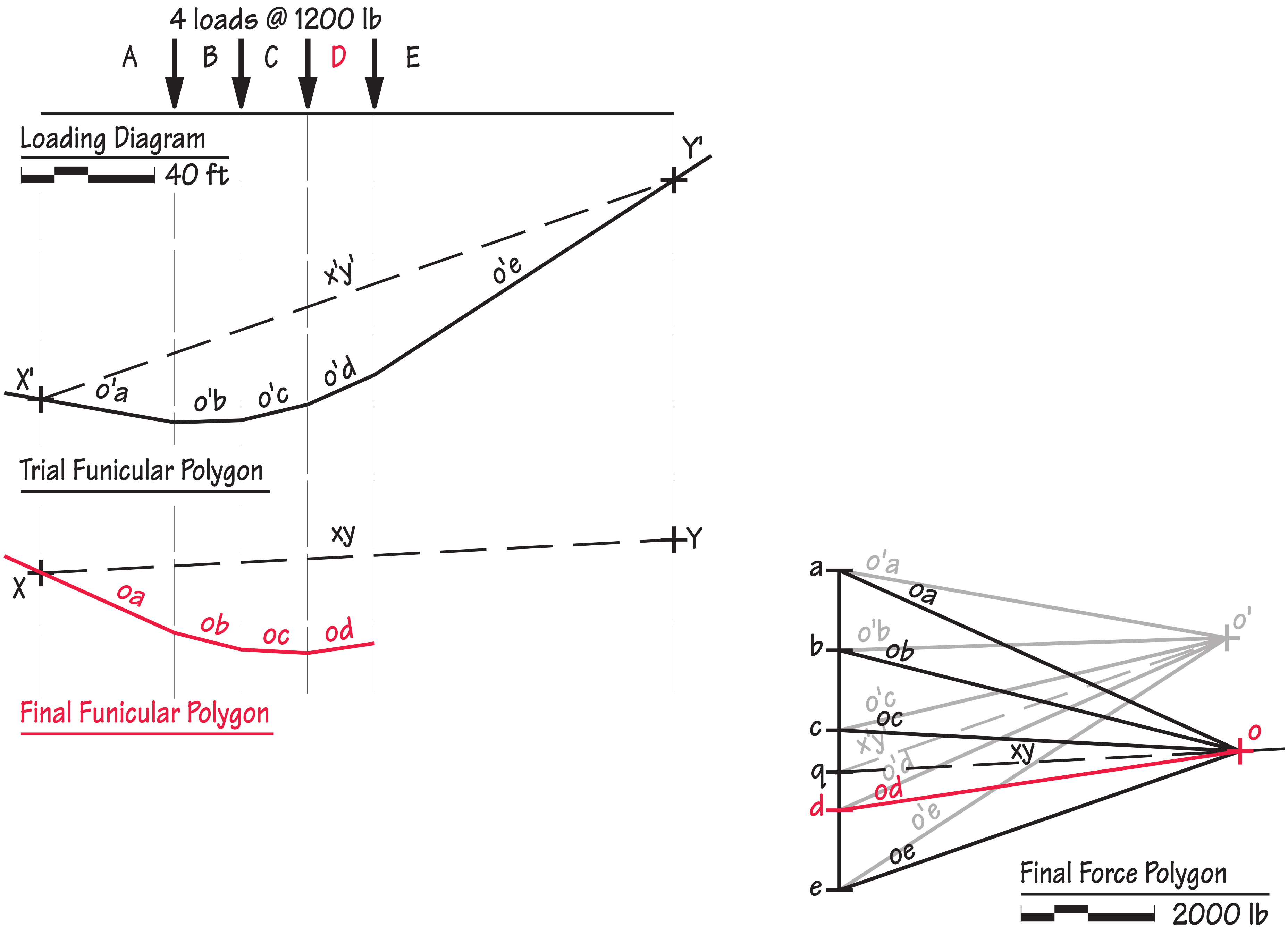
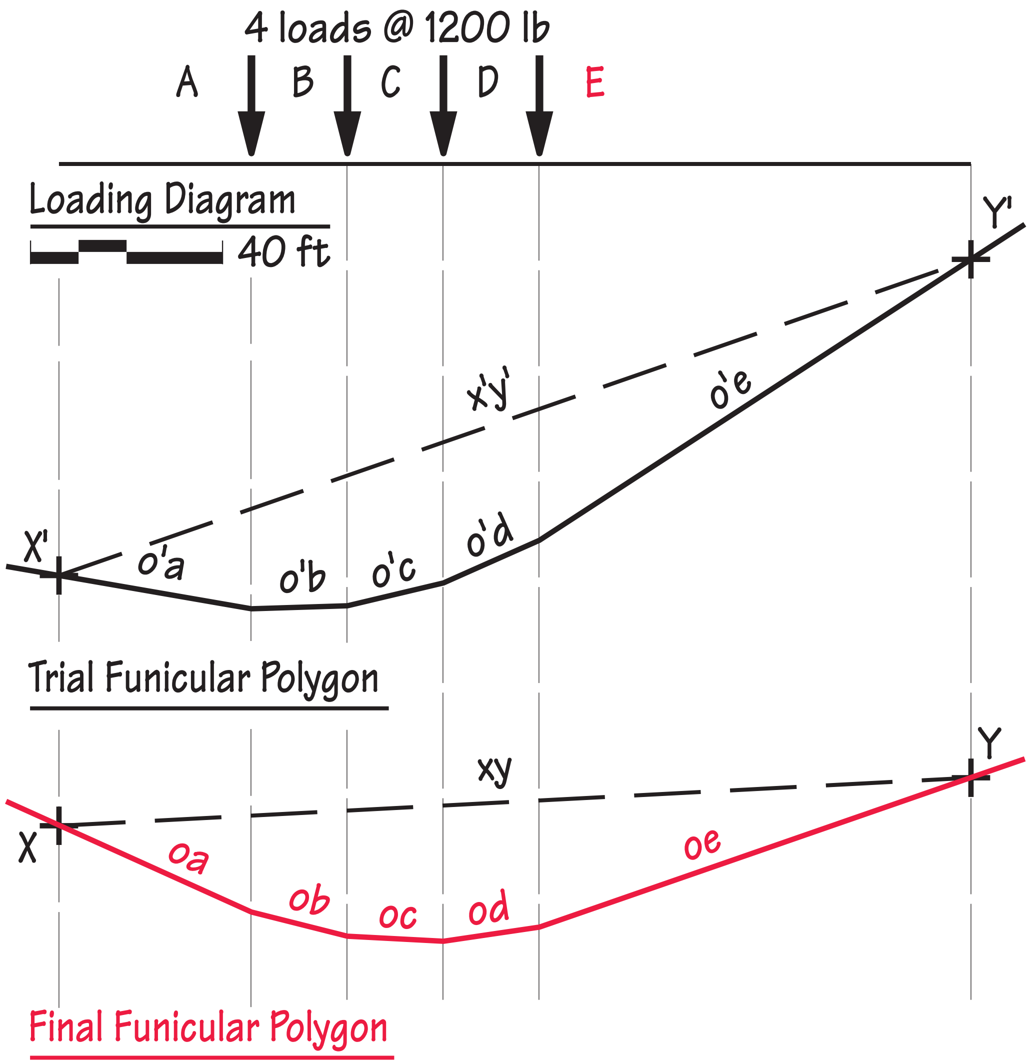
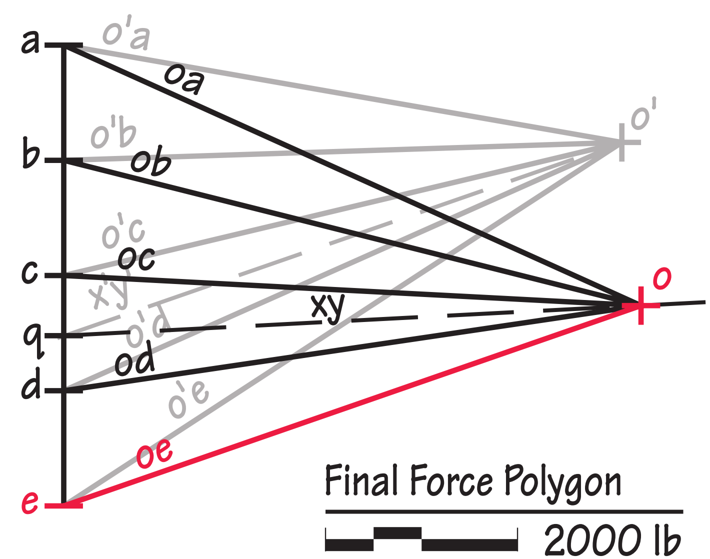


# Finding a Funicular Curve Through Two Points

Segment *oe* closes the Final Funicular Polygon at point *Y*, verifying the accuracy of our construction.



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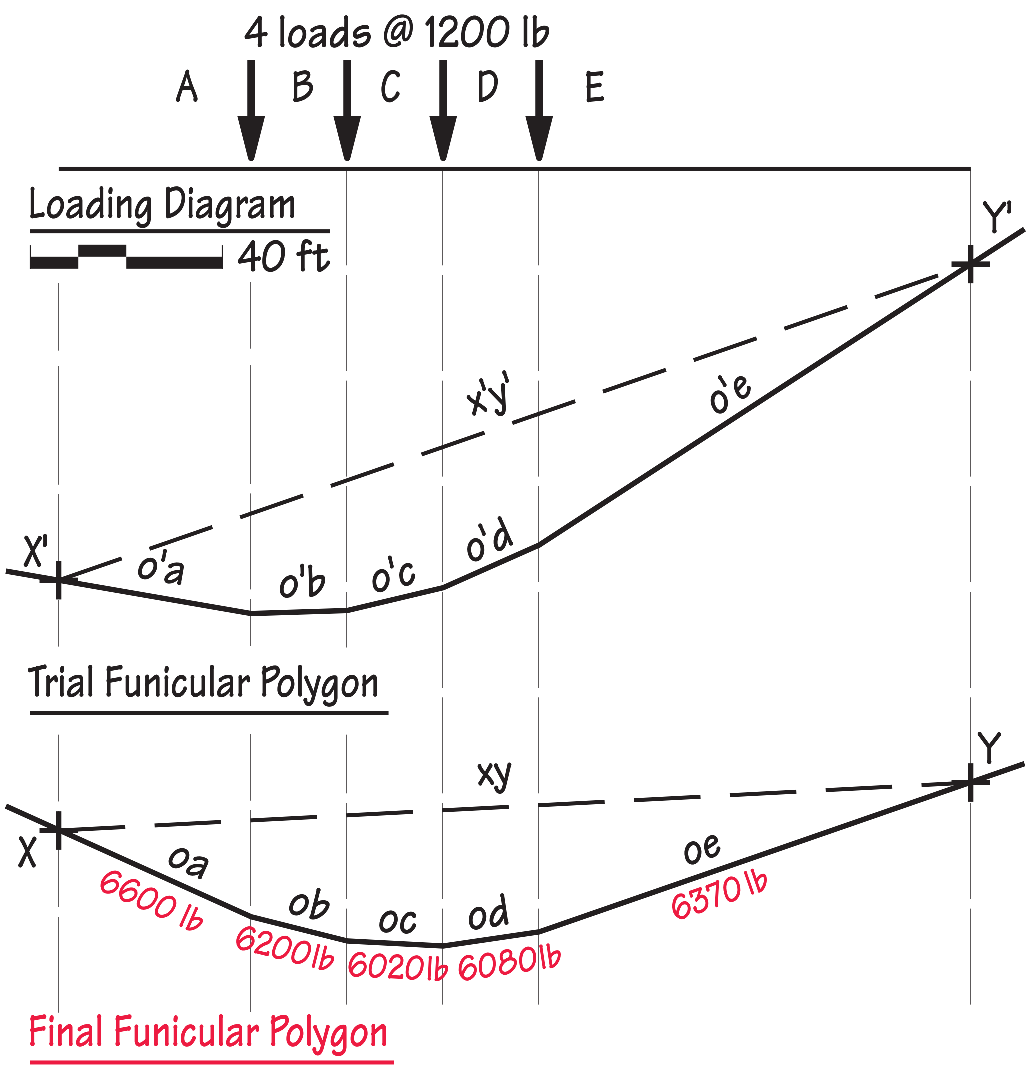
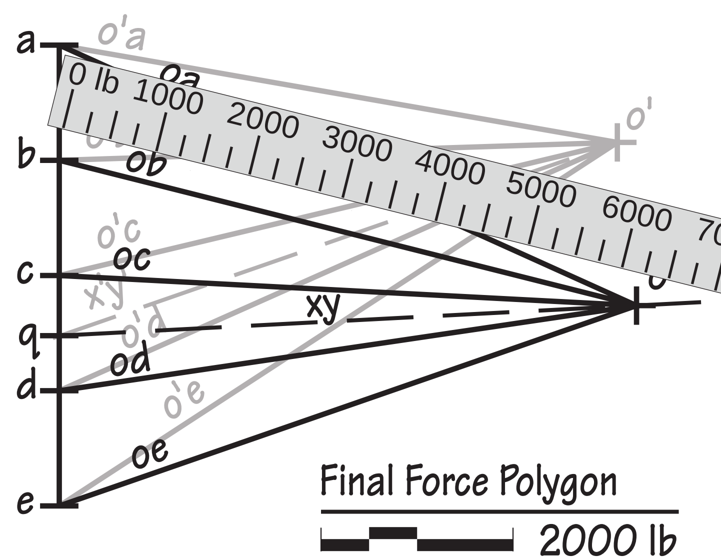



We may scale each ray on the Final Force Polygon to determine the force in the corresponding segments of the cable.

Notice that the highest force is indeed in segment *oa*, and that the maximum force in the cable is limited to 6600 lb as desired.



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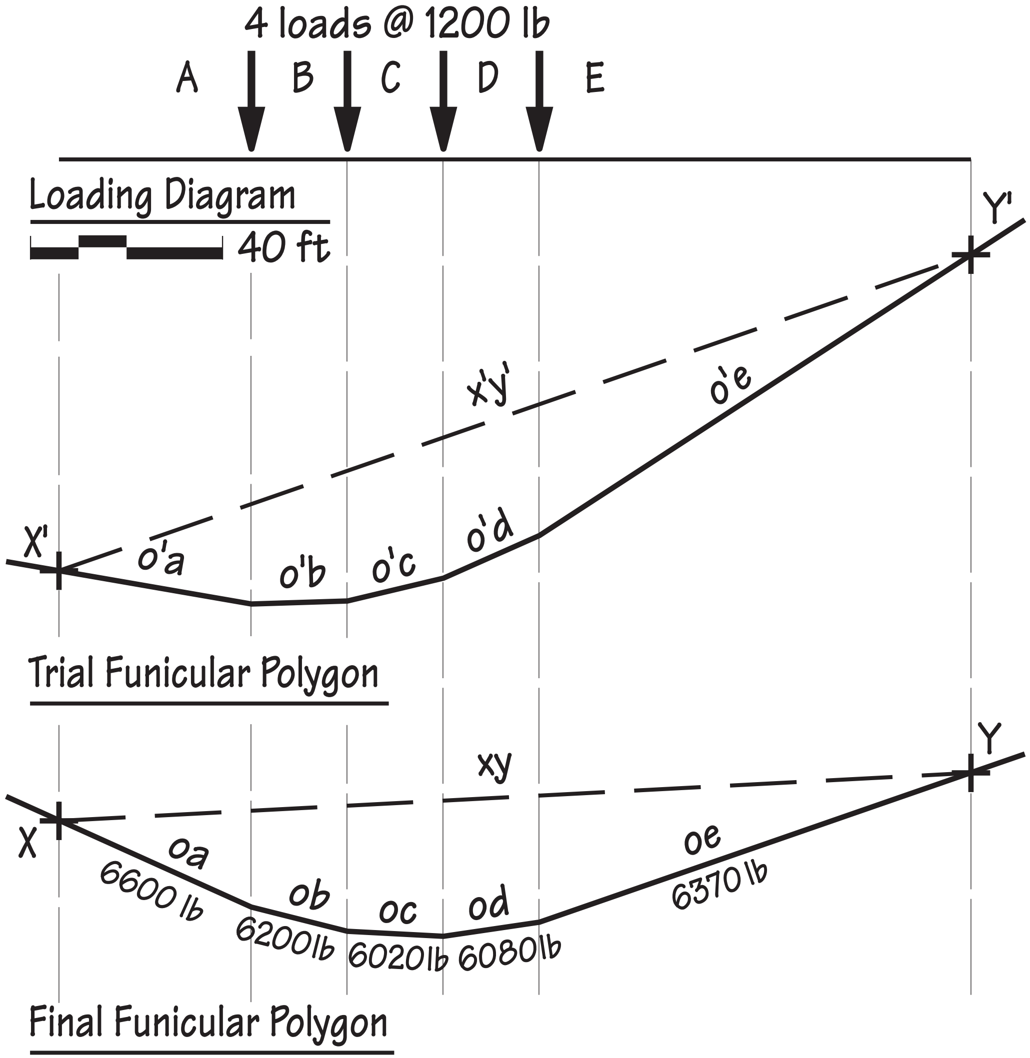
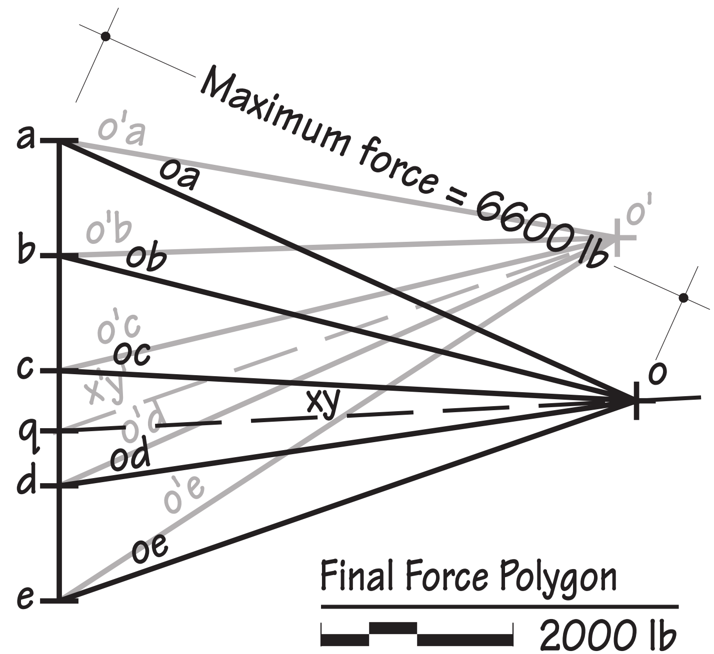



The Final Funicular Polygon is now complete.

It is the shape that the cable will take between support points *X* and *Y* while experiencing a maximum tension of 6600 lb.



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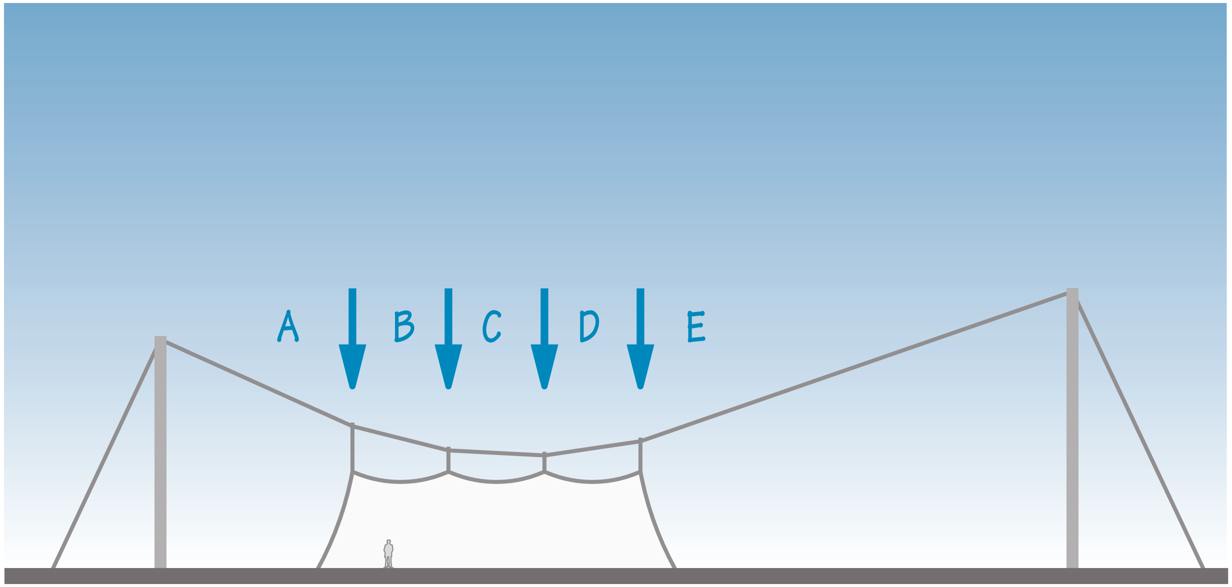
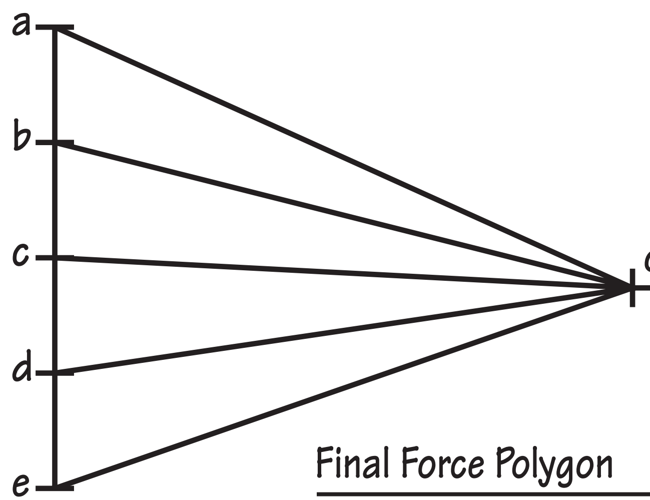



Form Diagram

40 ft

This is the complete structure with the form of the cable that we have determined.

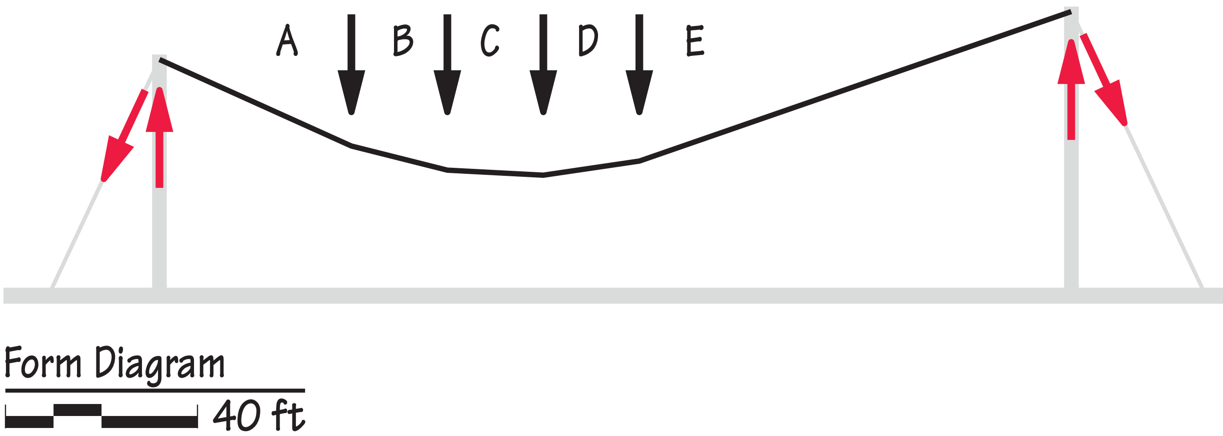
Now that we have determined the form and forces in the main cable, we will briefly investigate the forces in the cable towers and backstays.




Final Force Polygon

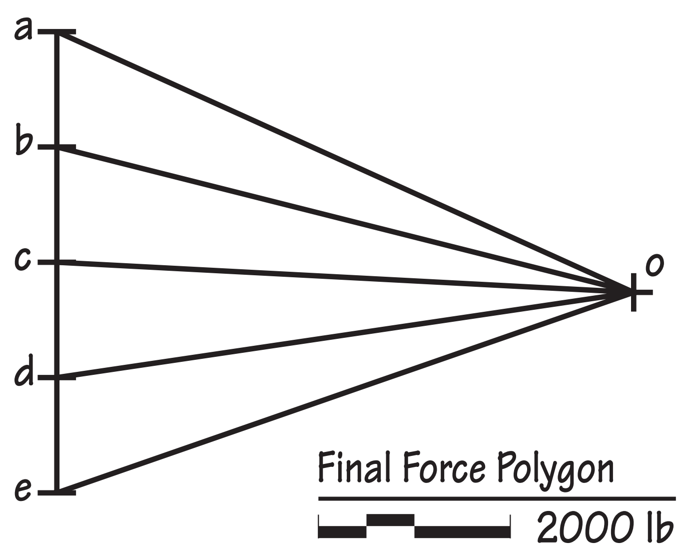
2000 lb

# Finding a Funicular Curve Through Two Points

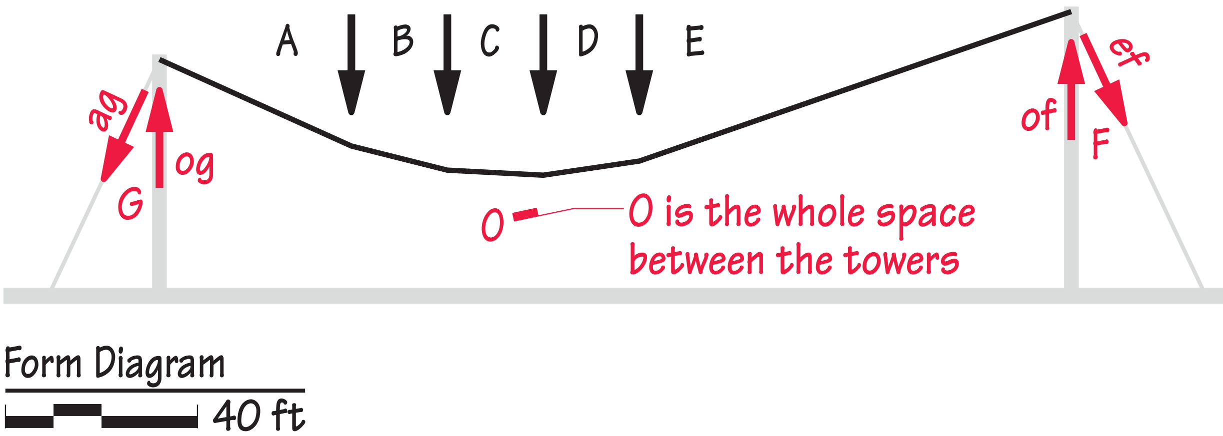



## Step 6: Investigate the tower and backstay forces.

We begin by constructing a free body diagram of the cable. The free body diagram shows all forces acting on the cable, including the forces exerted on the cable by the towers and backstays.



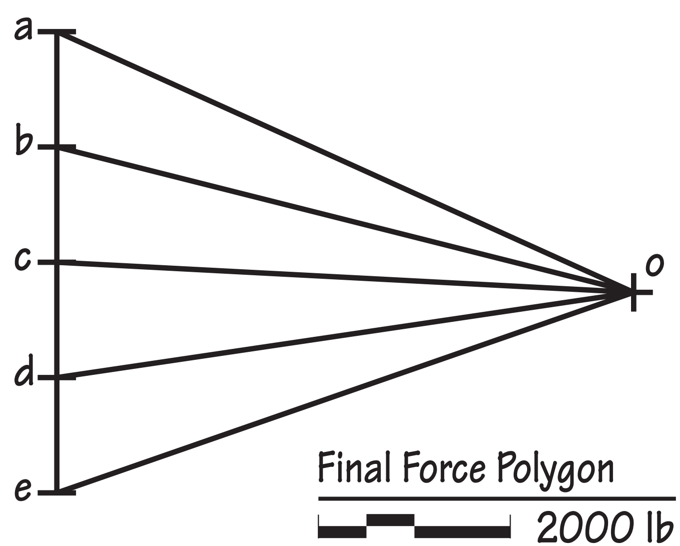
# Finding a Funicular Curve Through Two Points



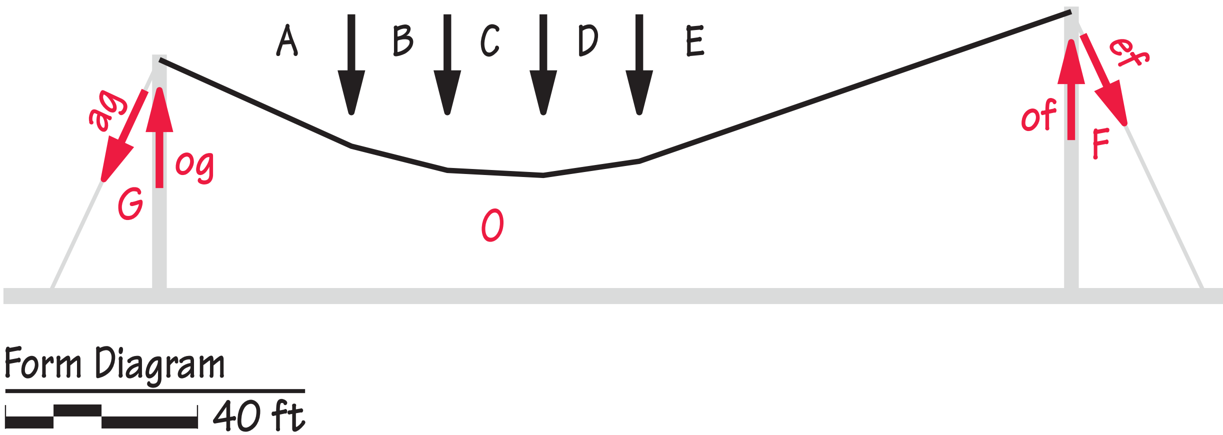

We use interval notation to keep track of the added forces.

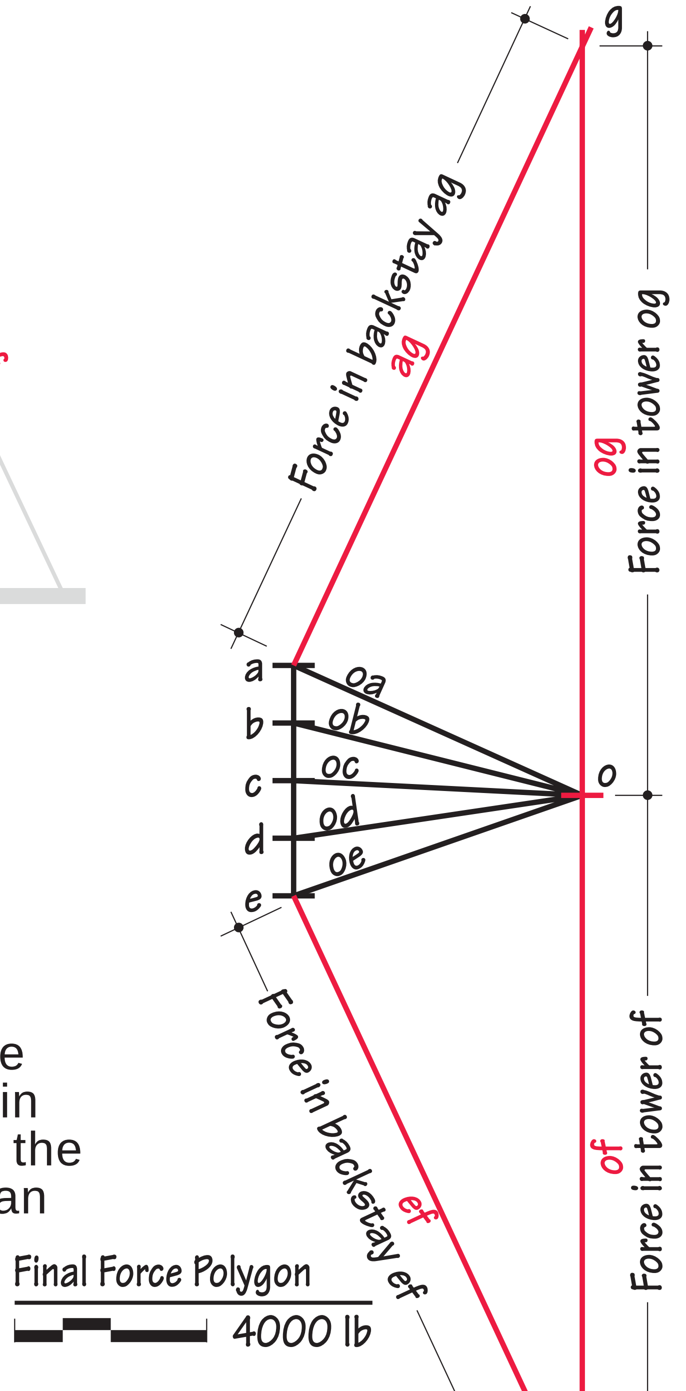
Notice that pole *o* on the Force Polygon corresponds to the interval below the main cable on the Form Diagram.

We will now add the tower and backstay forces to the Force Polygon.



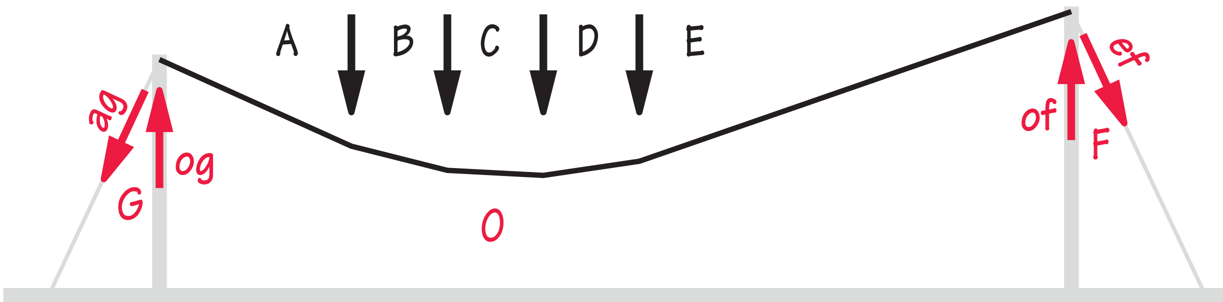
# Finding a Funicular Curve Through Two Points




Completing the Force Polygon, we discover that the forces in the towers and backstays are quite large in comparison to those in the main cable. Even when the scale of the force polygon is reduced by half, we can no longer fit the entire diagram on the page!

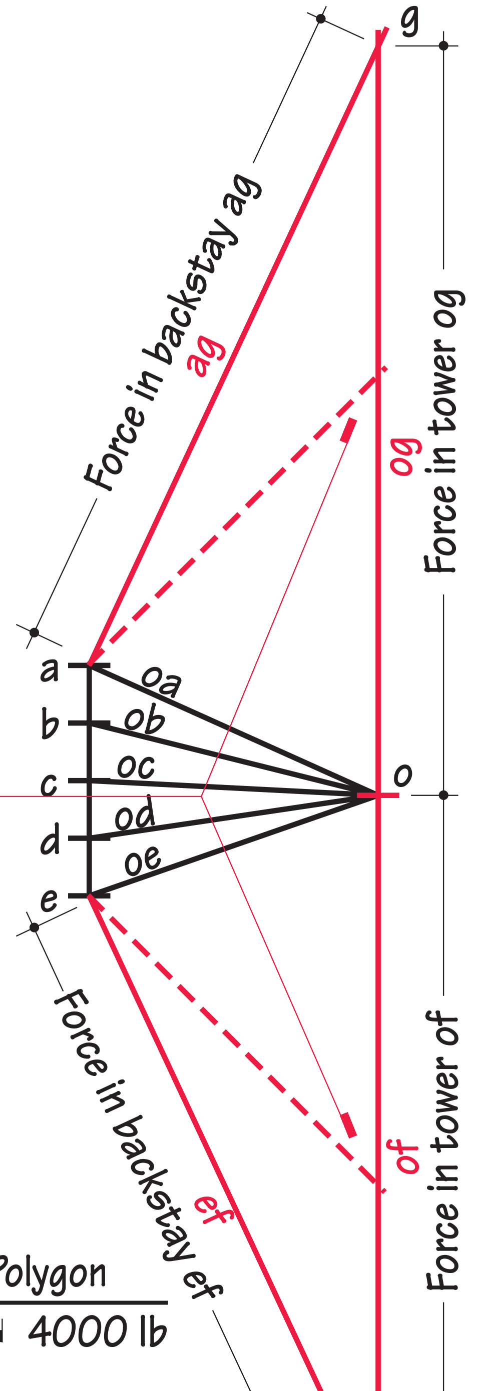
# Finding a Funicular Curve Through Two Points




Form Diagram

40 ft

Changing the slope of ag and ef moves their intersection with og and of so that all of these vectors become shorter.




Final Force Polygon

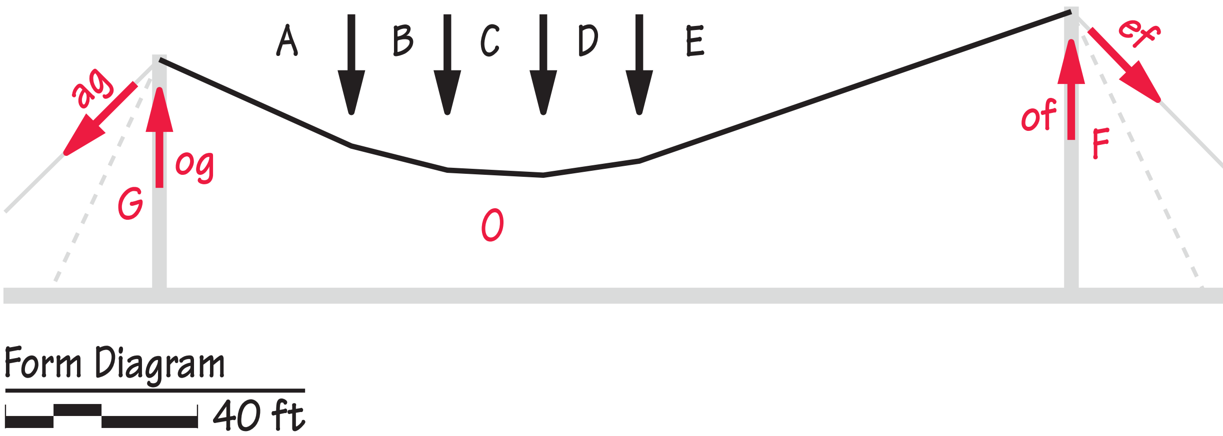
4000 lb

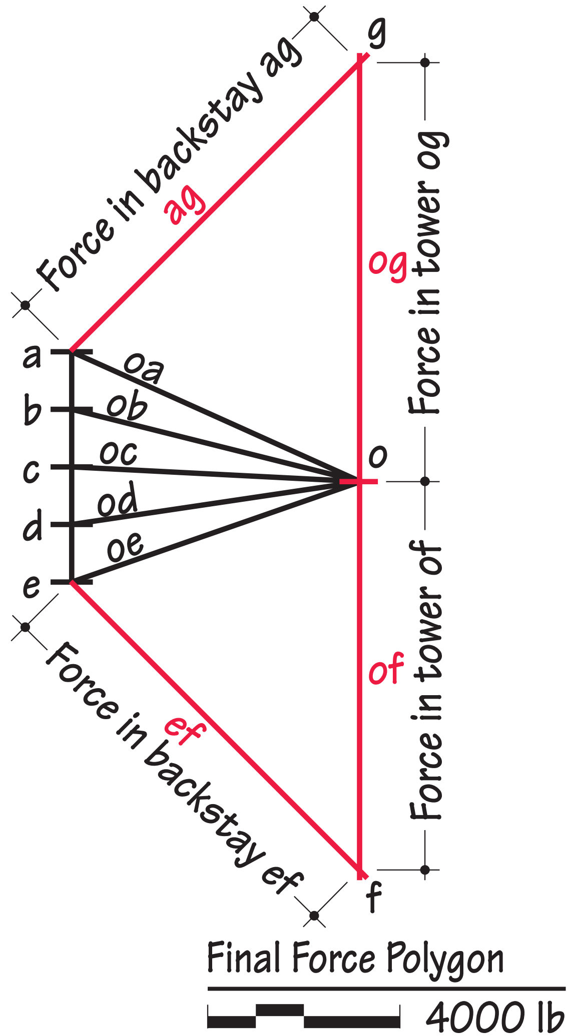
The red lines on the Force Polygon tell us that we could reduce the forces in the columns and backstays by reducing the slope of the backstays.

# Finding a Funicular Curve Through Two Points

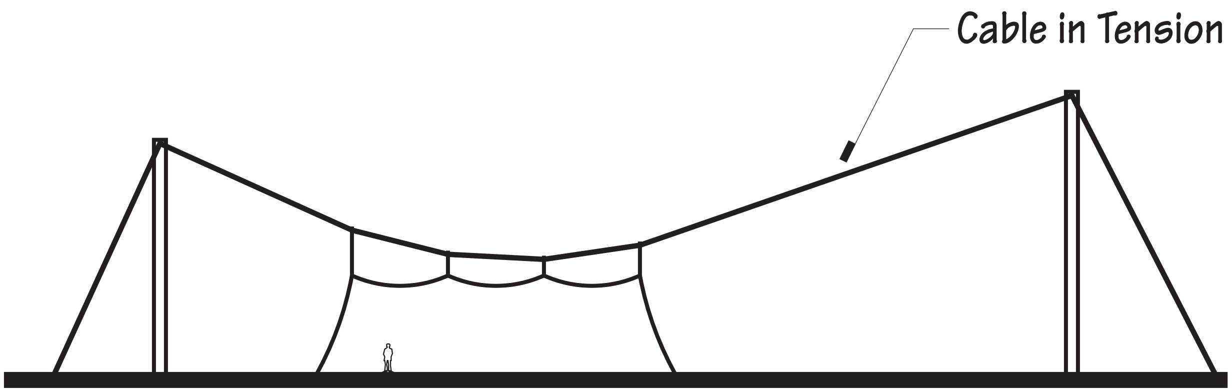
This Force Polygon shows the results of this change.

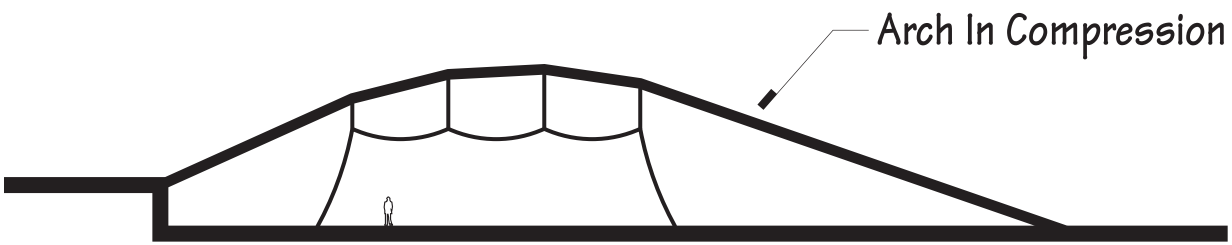
Does it suggest to you another way of reducing the forces in the backstays and towers?



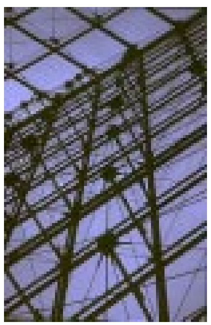



# Finding a Funicular Curve Through Two Points




How would you go about designing an arched concrete roof shell between two given support points so that it would experience a given maximum axial internal force?



*Click on the **Contents** button to begin a new lesson.*

*Click on the image of the Louvre Pyramid to return to the beginning of this lesson.*