Designing A Concrete Arch Bridge

This is the famous Schwandbach bridge in Switzerland, designed by Robert Maillart in 1933. It spans 37.4 meters (122 feet) and was designed using the same graphical methods that will be demonstrated in this lesson.

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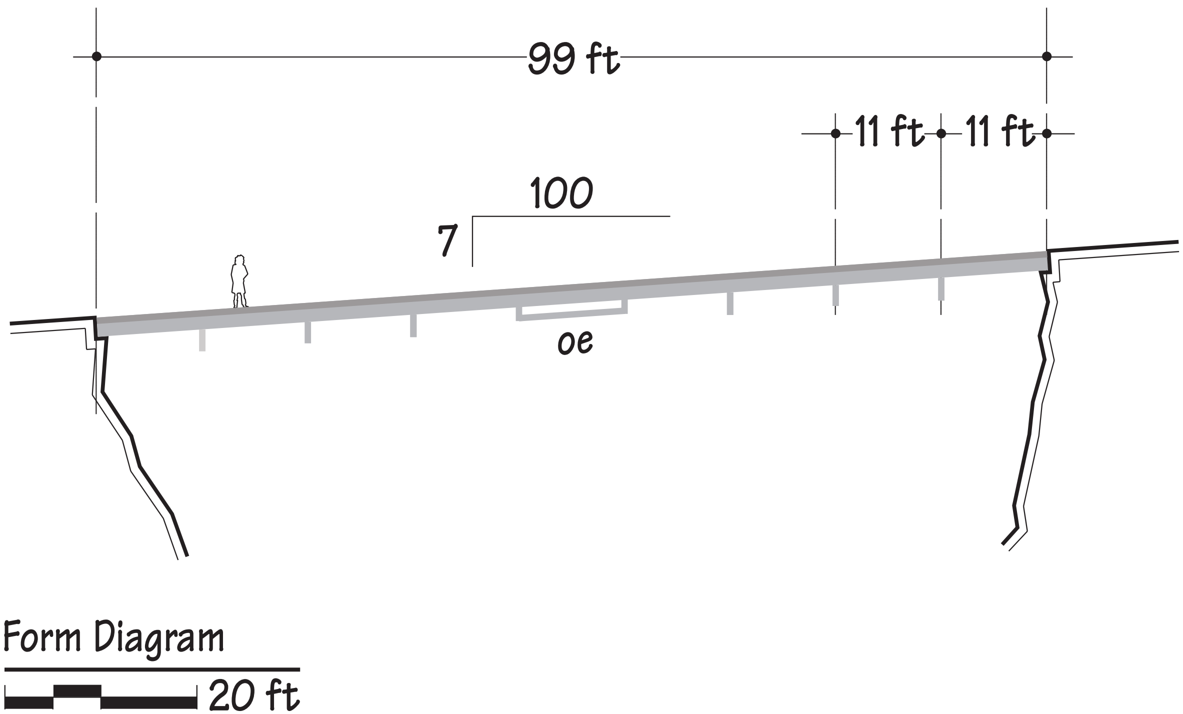

The Problem: This is a partially finished form diagram of a bridge that will span 99 feet. The roadway slopes at 7%. Eight walls spaced 11 feet apart bring total loads of 120 kips each from the stiff deck to a concrete arch below.

The arch is a concrete slab that is 8 inches thick and 16 feet wide throughout the span. The allowable axial stress in the arch is 800 lb/in^2.

Shape the arch in such a way that segment *oe* is parallel to the bridge deck as shown.

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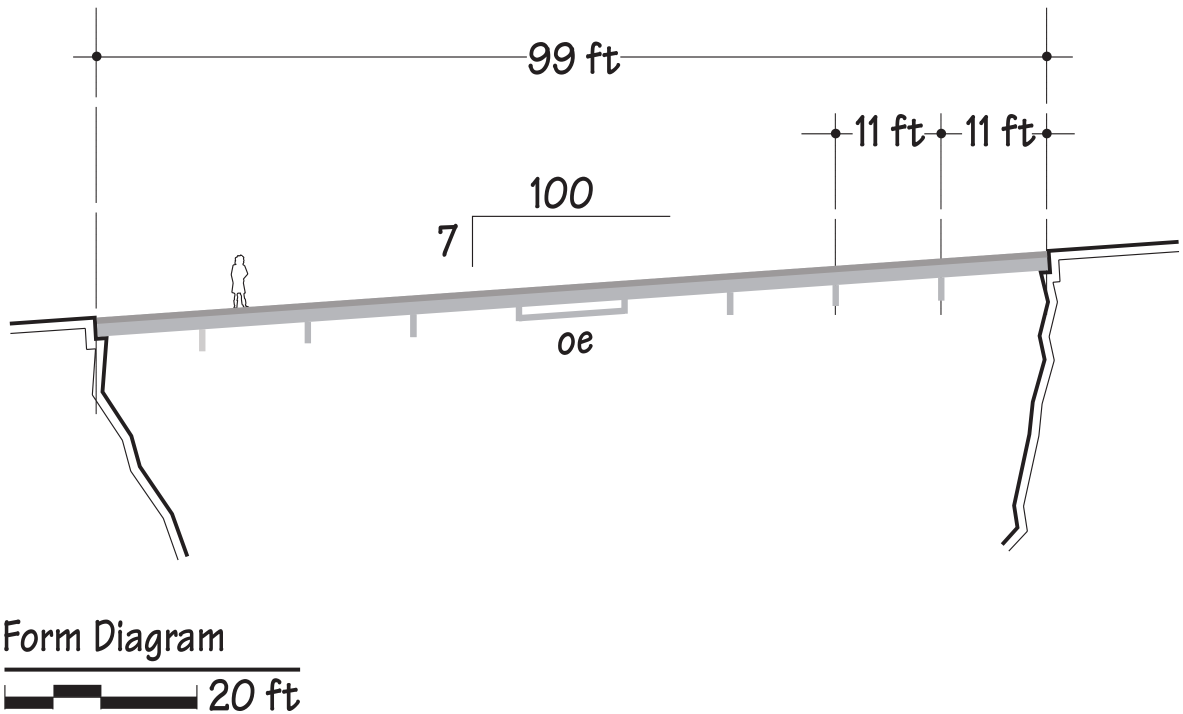

Step 1: What is the maximum allowable axial force in the arch?

The given parameters are an arch thickness of 8 inches, an arch width of 16 feet, and an allowable stress of 800 lb/in^2.

The cross sectional area of the arch is its width times its thickness:

$$\text{Area} = (\text{width})(\text{thickness}) = (16\text{ ft})(12\text{ in./ft})(8\text{ in.}) = 1536\text{ in}^2$$

The allowable axial force in the arch is the area of the arch times the allowable stress:

$$\text{Force} = (\text{Area})(\text{allowable stress}) = (1536\text{ in}^2)(800\text{ lb/in}^2) = 1{,}228{,}800\text{ lb} = 1229\text{ kips}$$

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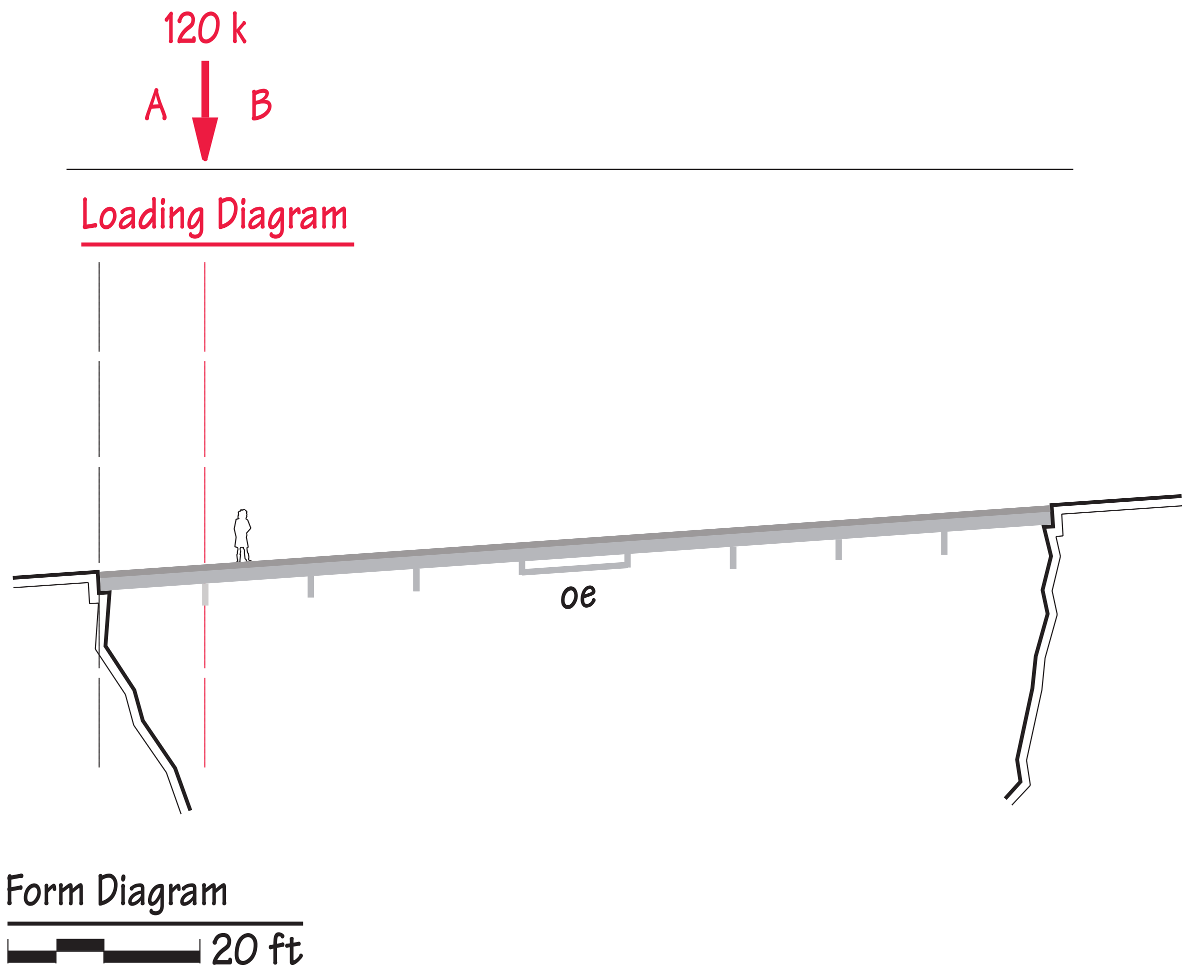

Step 2: Construct a loading diagram and apply interval notation.

Working clockwise from the upper left, place external gravity loads on the form diagram. The spaces or *intervals* between forces are labeled with the uppercase letters of the alphabet.

In this diagram, the first 120 kip load is placed over the leftmost arch wall. Letters *A* and *B* are assigned to the intervals on either side of the load. This is load *AB*.

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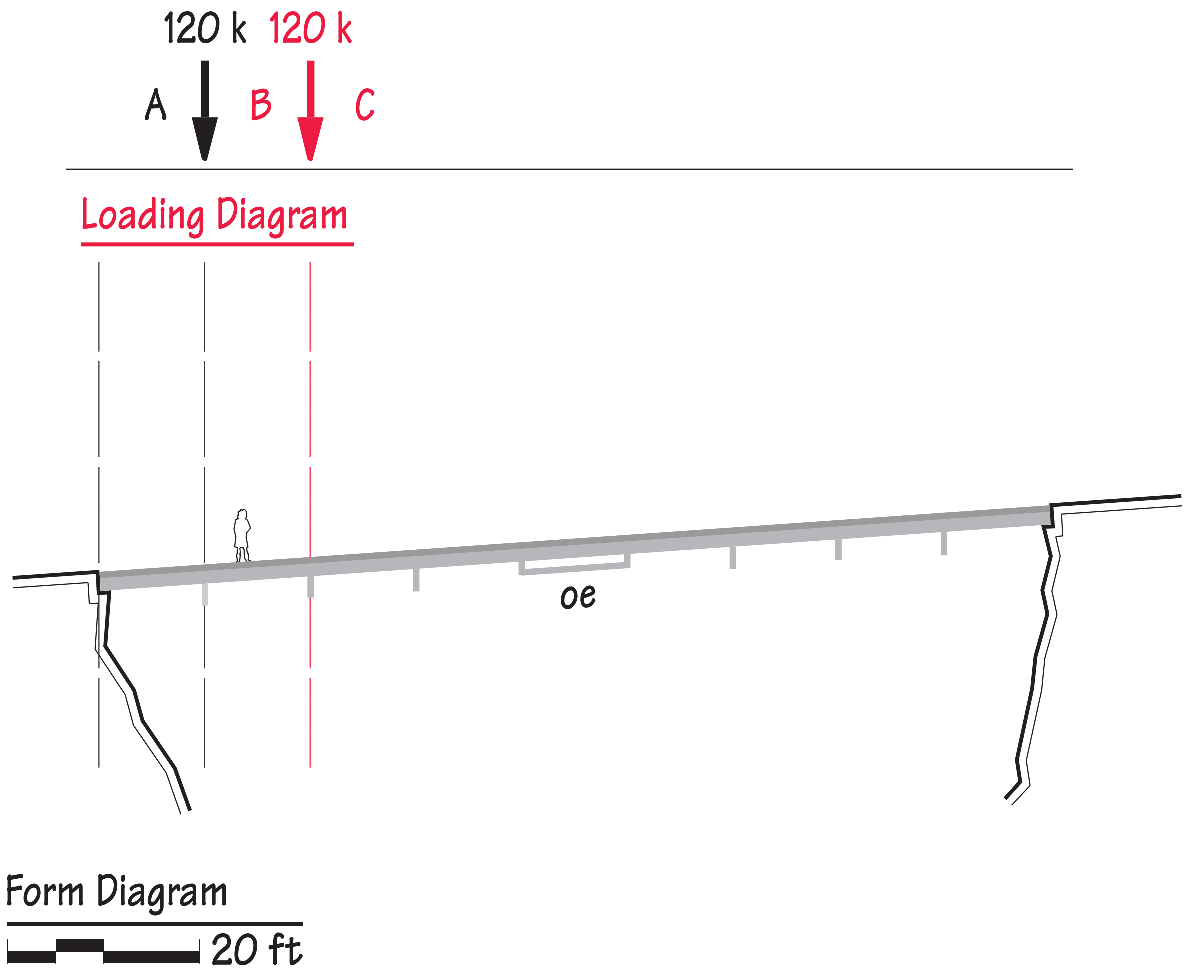

The second load is placed over the next arch wall. Letter *C* is added to the diagram, and this becomes load *BC*.

With each load, a vertical line of action is extended down through the area of the form diagram where the arch will be constructed.

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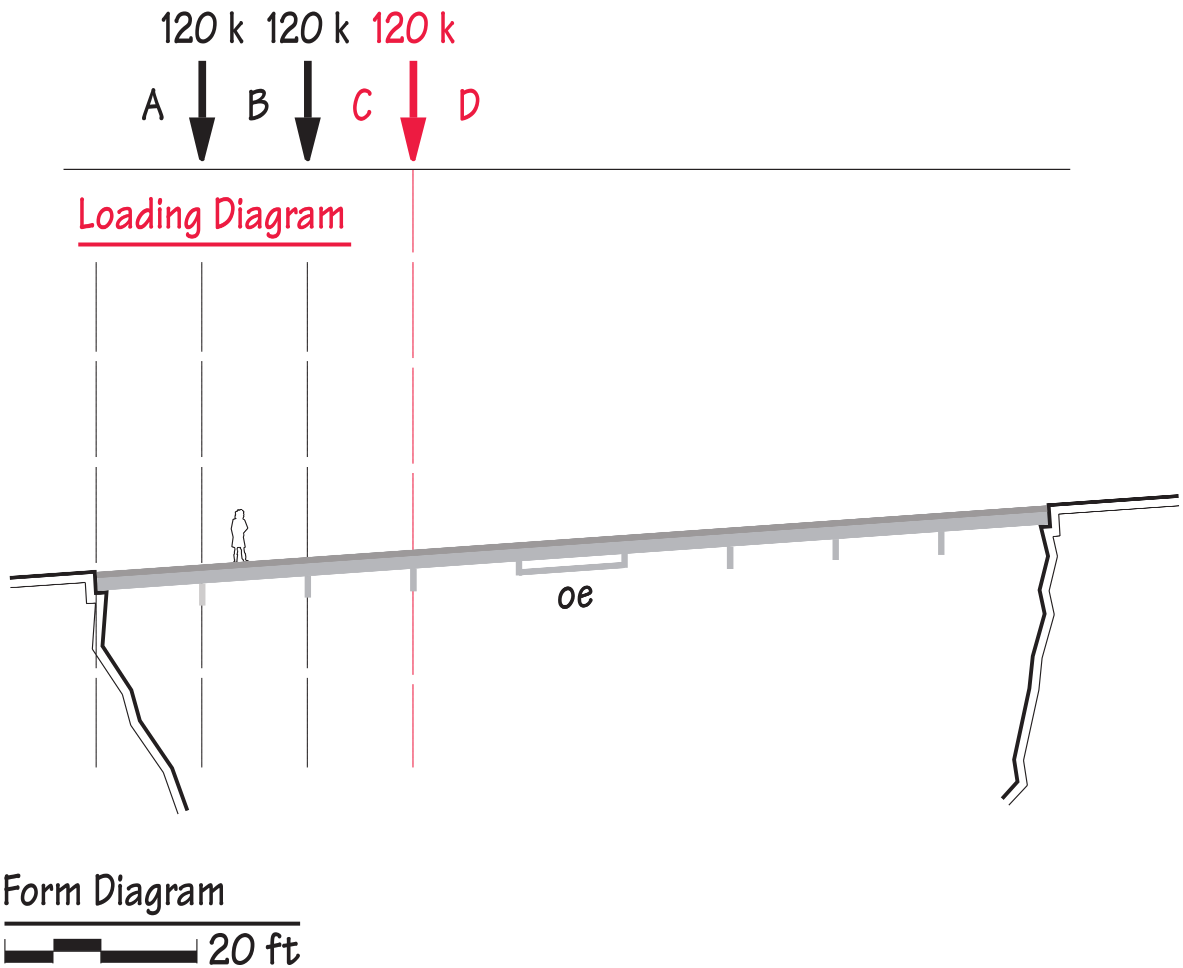

The process continues until all the loads on the arch have been constructed and labeled.

This is load *CD.*

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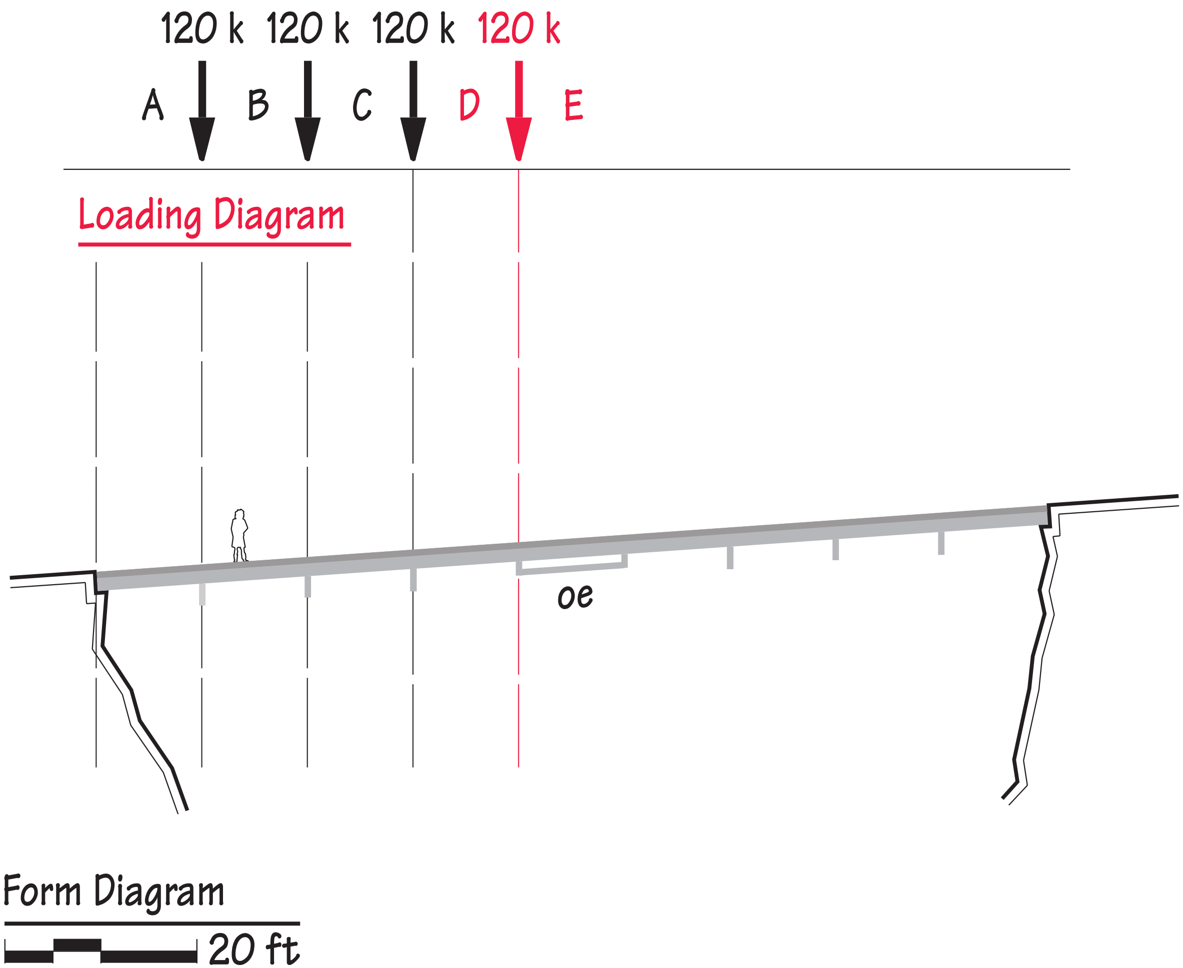

This is load *DE.*

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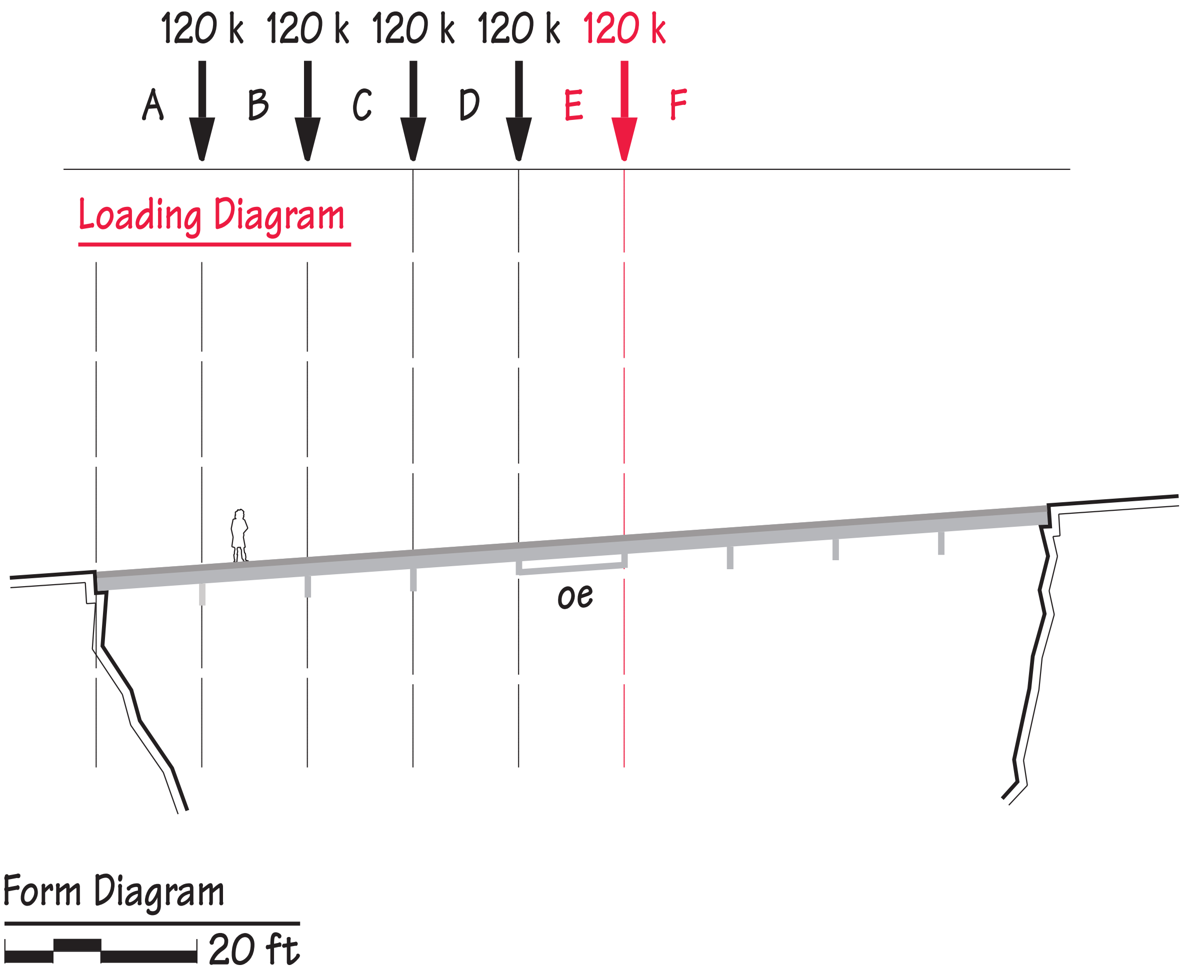

This is load *EF*.

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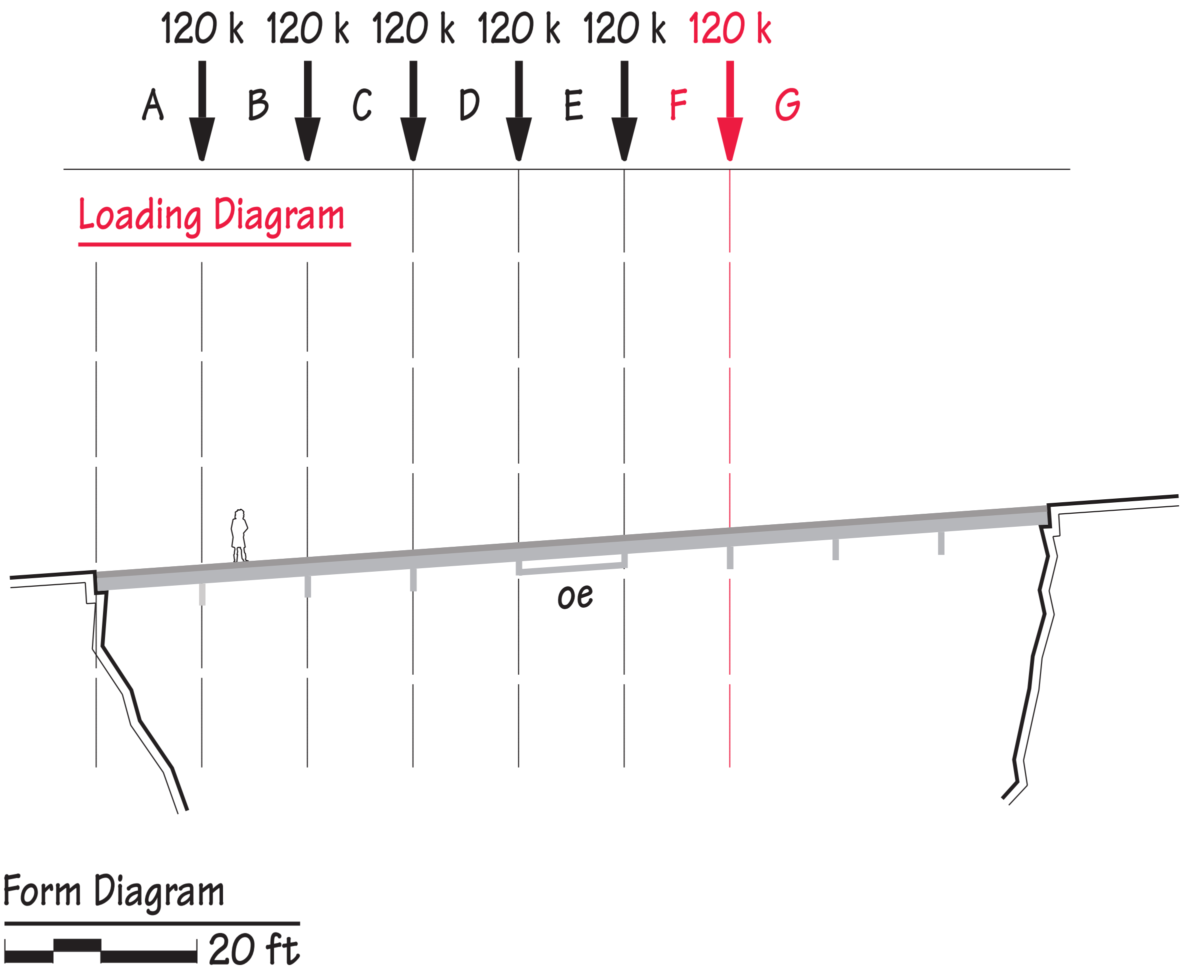
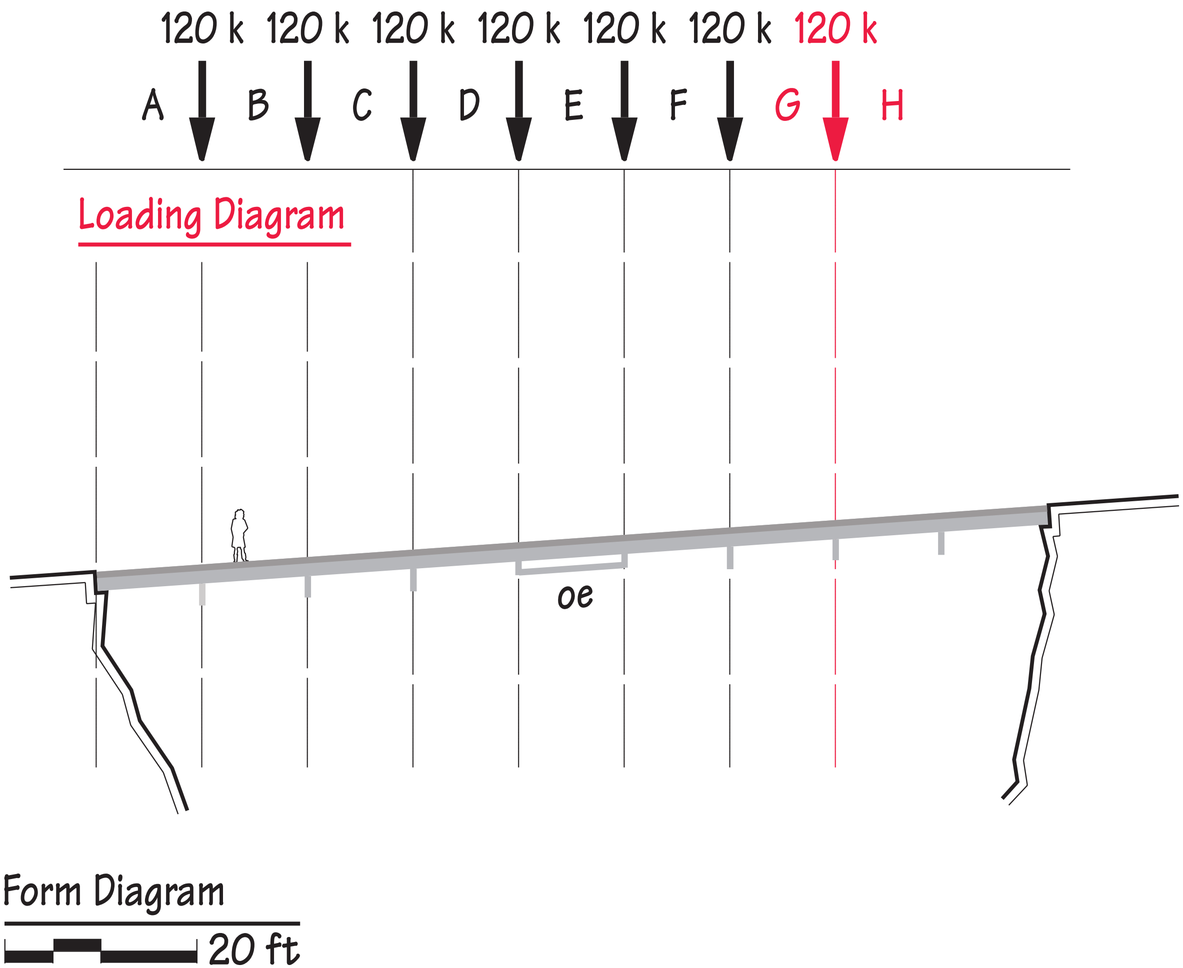


This is load *FG.*



This is load *GH.*

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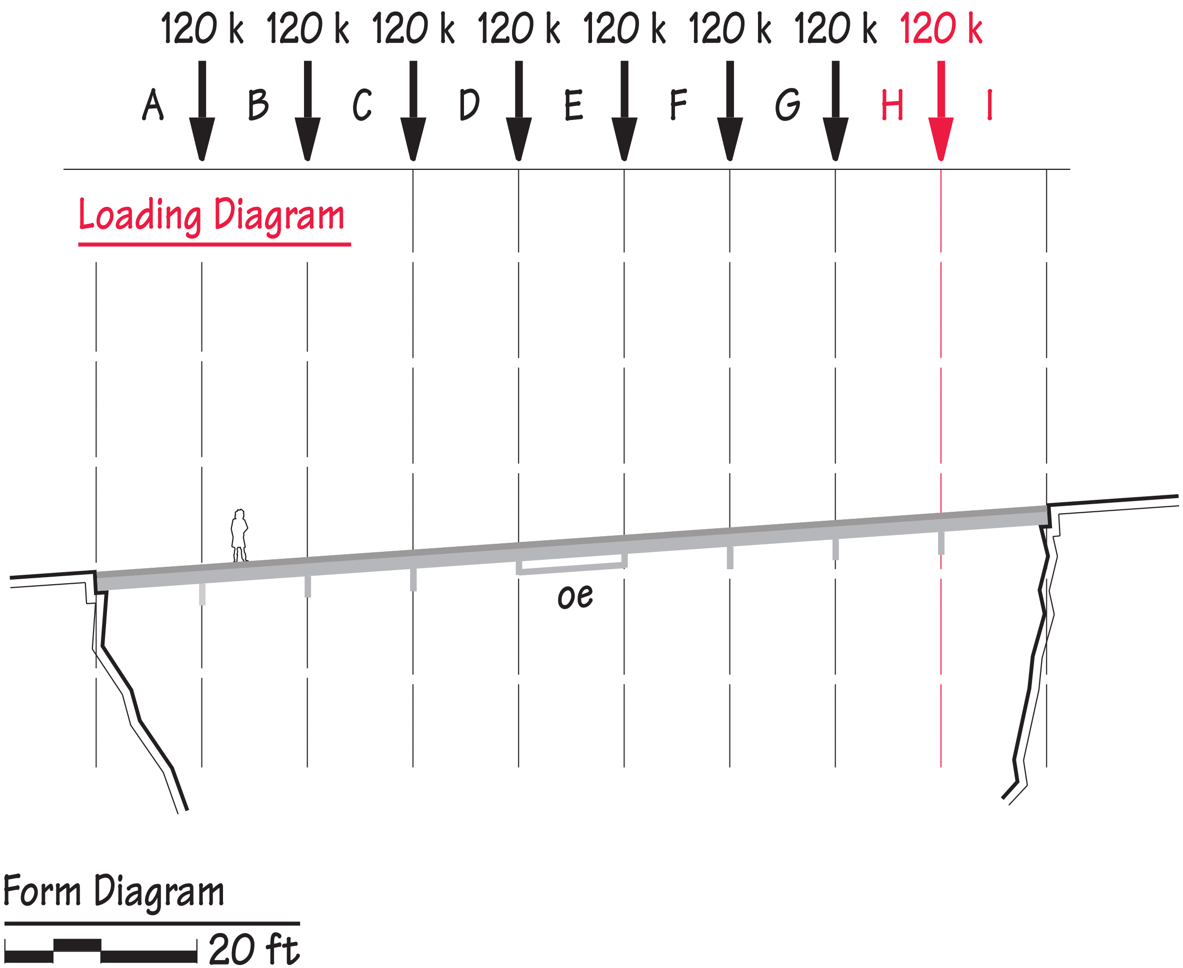

This is load *HI*.

The loading diagram is now complete. All of the external gravity loads acting on the arch have been placed on the loading diagram.

Vertical lines of action have been drawn for each load, extending down through the area of the form diagram where the arch will be constructed.

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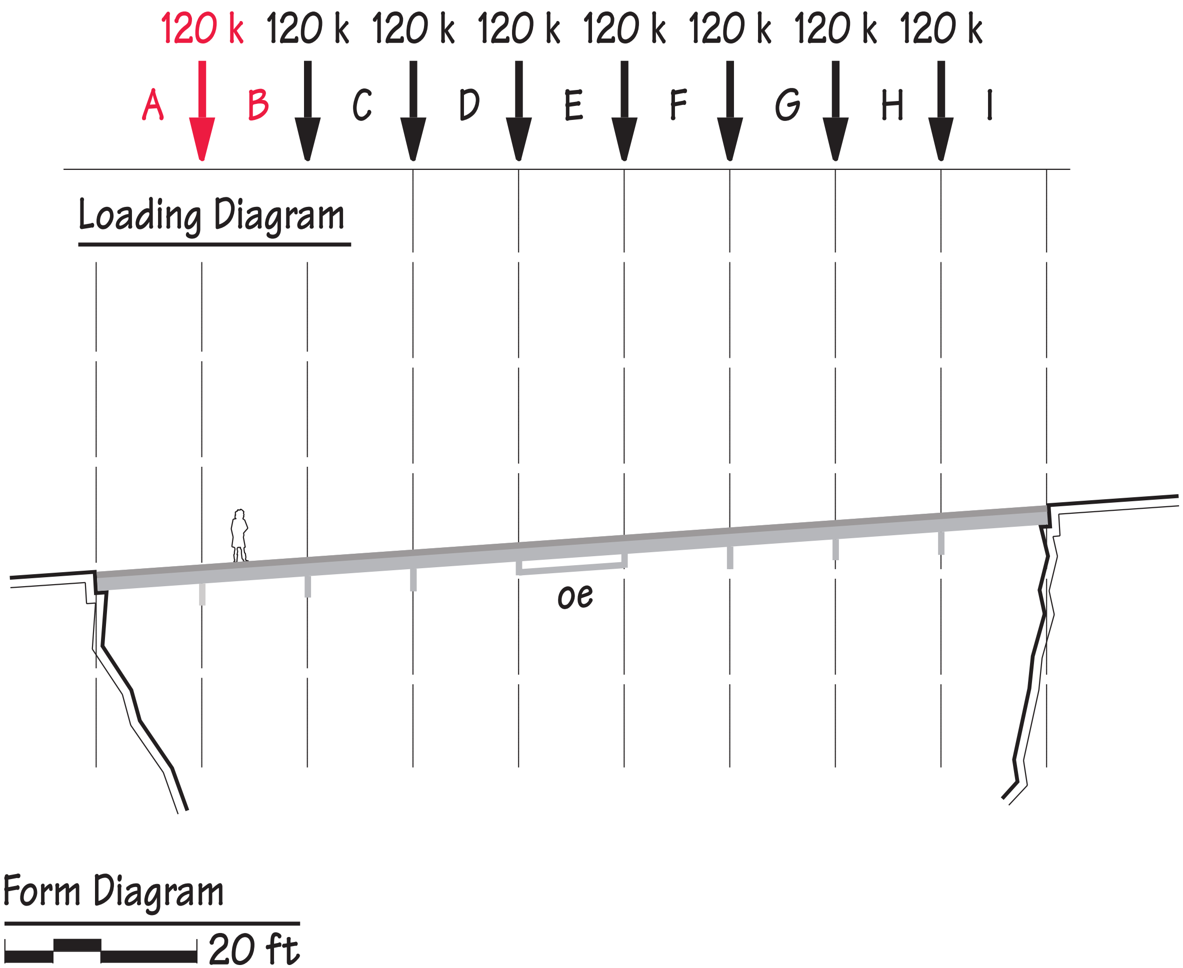
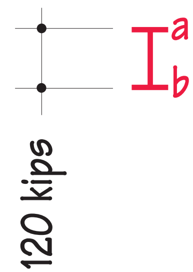
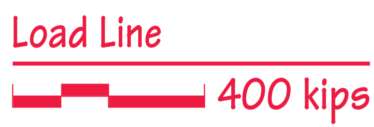


Step 3: Construct a load line to any convenient scale.

The Load Line is a graphical summation of the loads acting on the structure. On the Load Line, the magnitude of the forces from the loading diagram are drawn to scale.

Beginning with force *AB*, we plot a segment of the load line, parallel to *AB*, labeled *ab*. The length of *ab* scales to 120 kips, the magnitude of the force.





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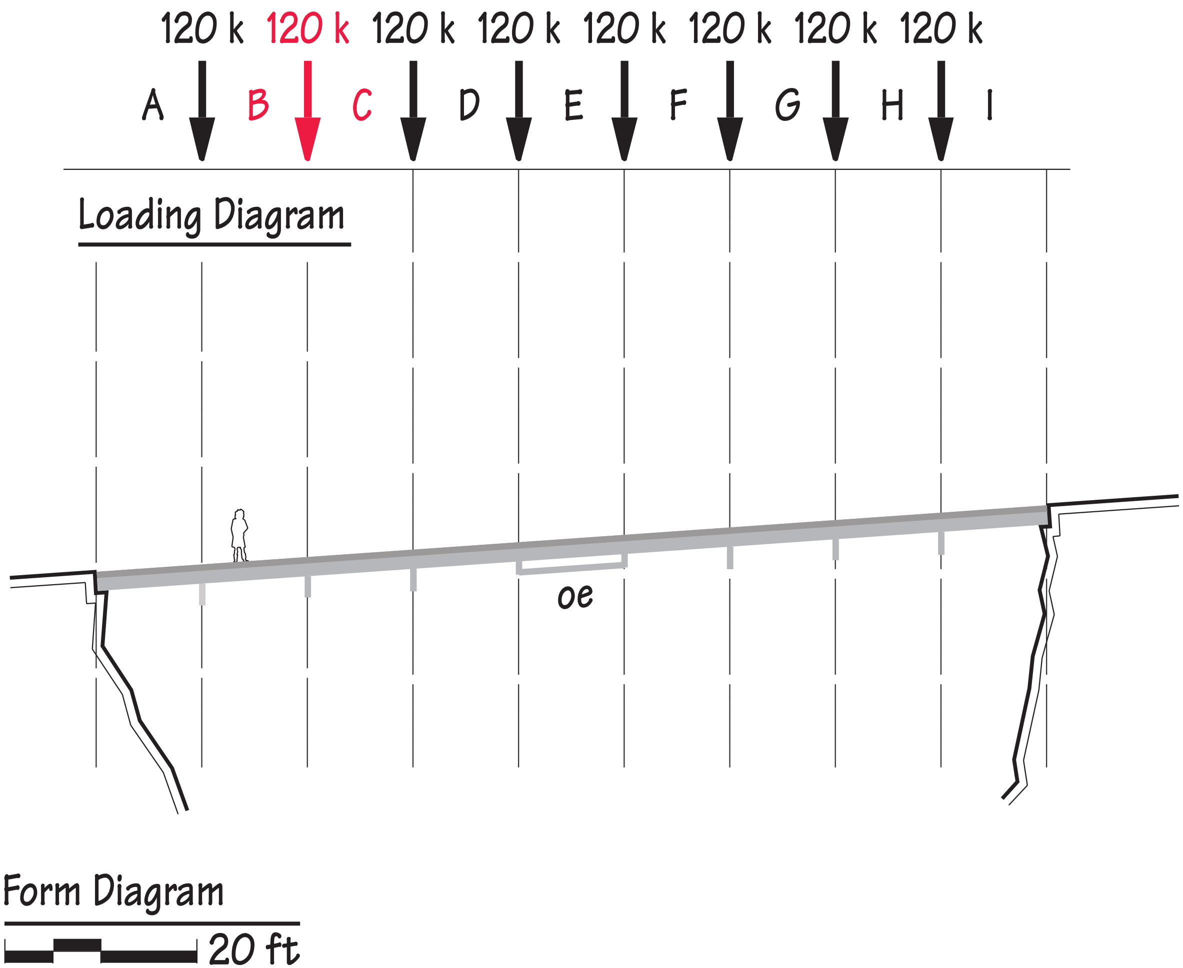
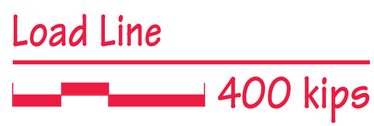
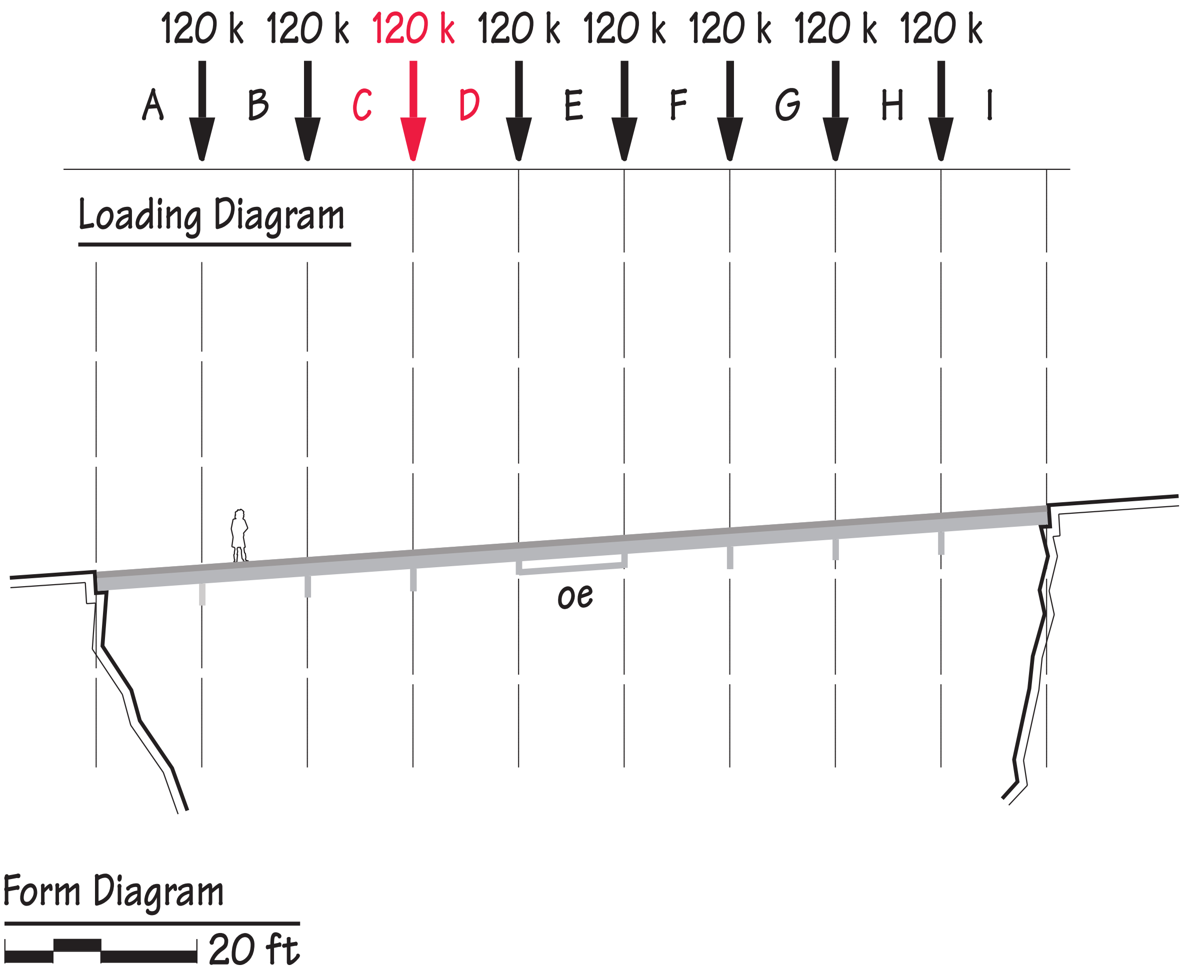


Once again, we work clockwise around the Form Diagram, plotting each load onto the Load Line in a tip-to-tail fashion.

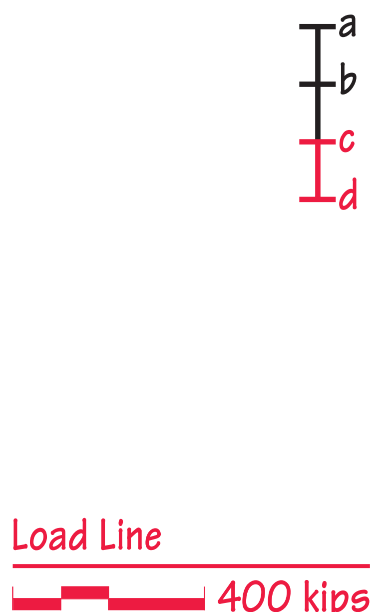
Although the Load Line is composed of vectors, the ends of the force segments are marked with horizontal tick marks rather than arrow heads. This helps to keep the diagram legible and accurate.





Since all of the loads on our arch are of equal magnitude, each segment of the Load Line will be equal in length.

When the loads on a structure vary in magnitude, or are not all strictly vertical in direction, the segments of the Load Line will vary in length and direction as well.



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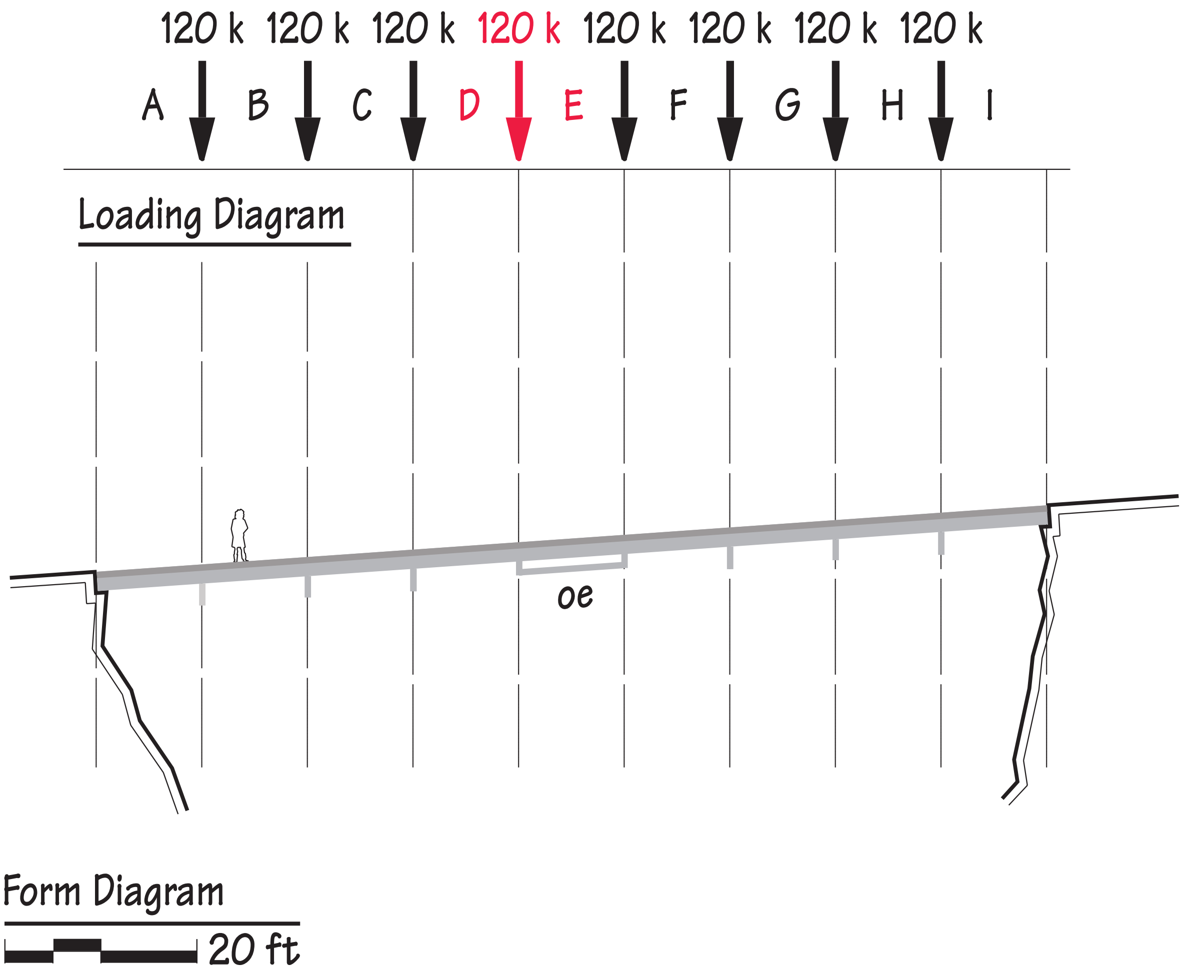
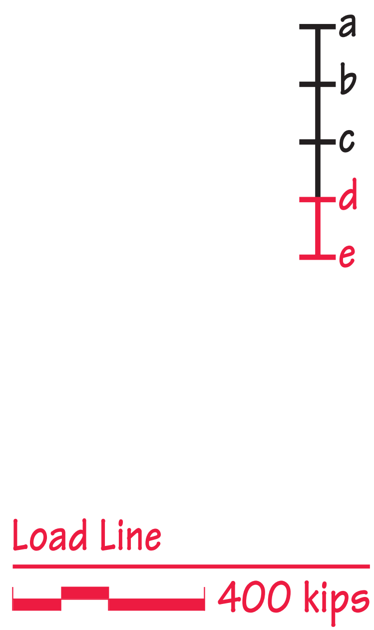


We continue across the Loading Diagram, plotting all the loads on the arch onto the Load Line.

Notice how *intervals* on the Loading Diagram correspond to *points* on the Load Line.



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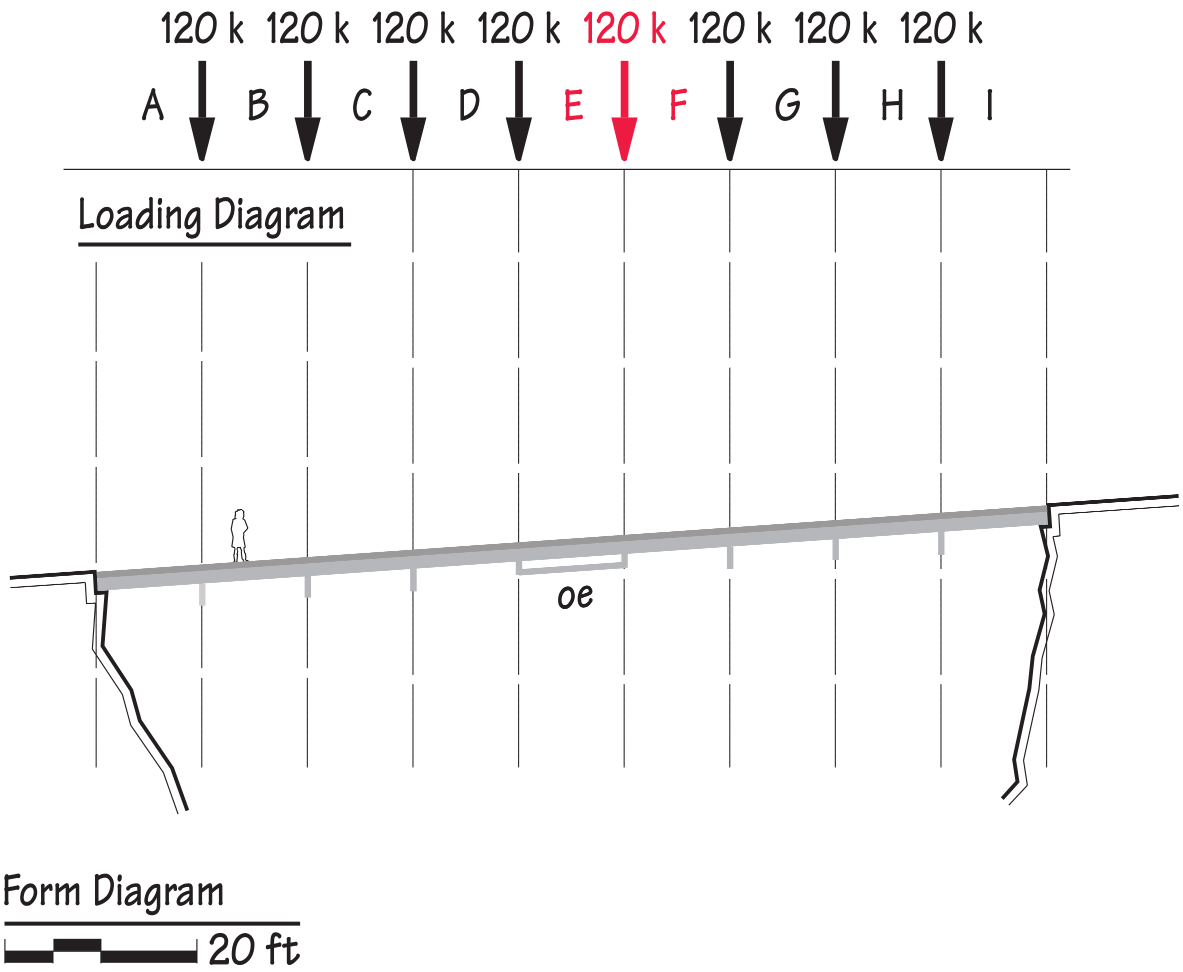
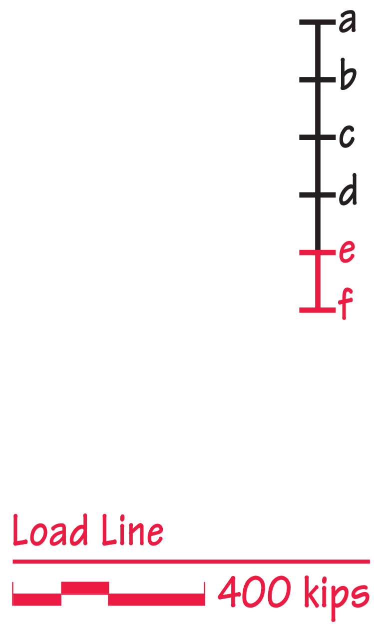
We continue across the Loading Diagram...





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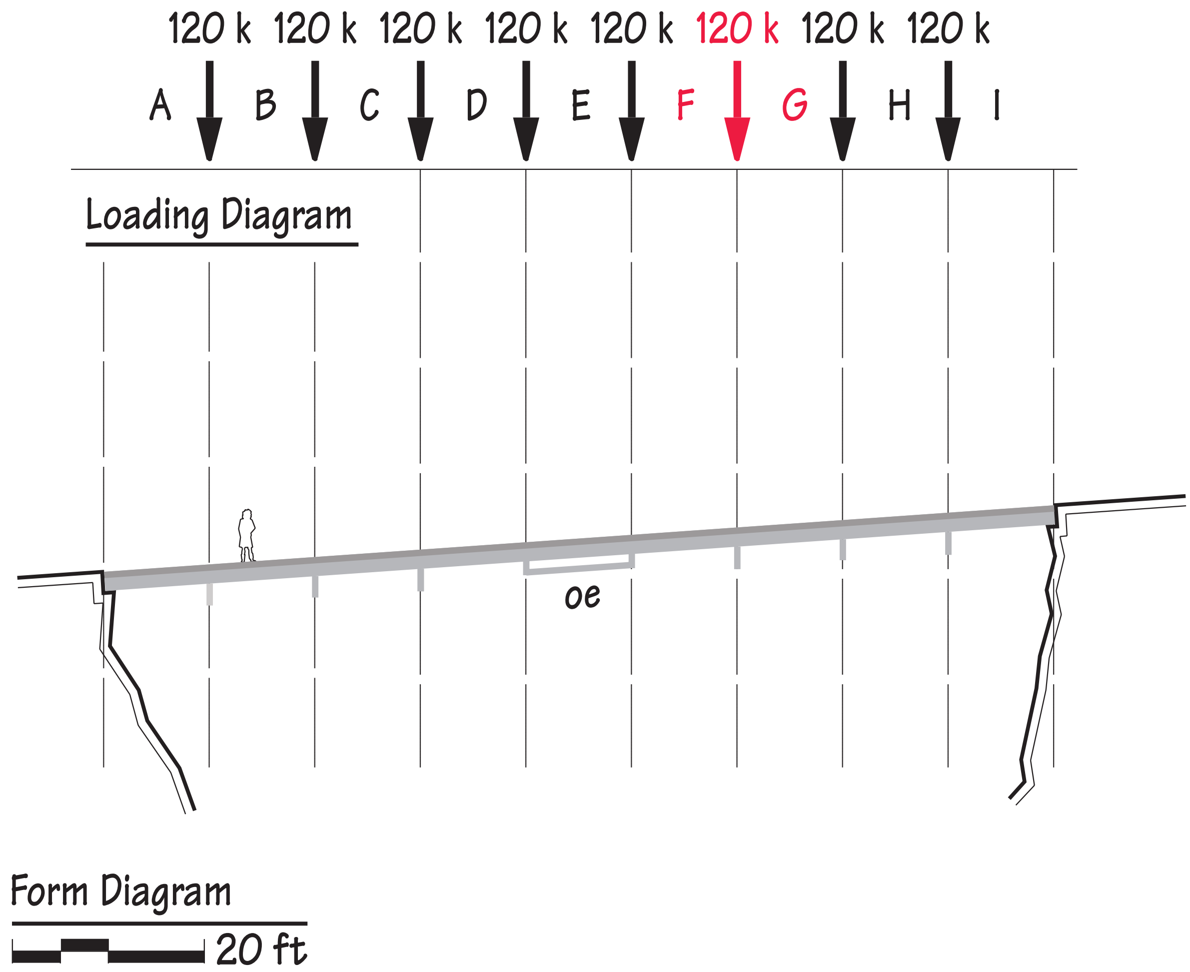
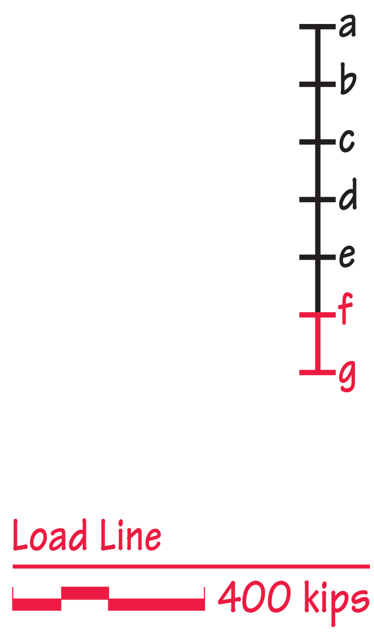


We continue across the Loading Diagram...



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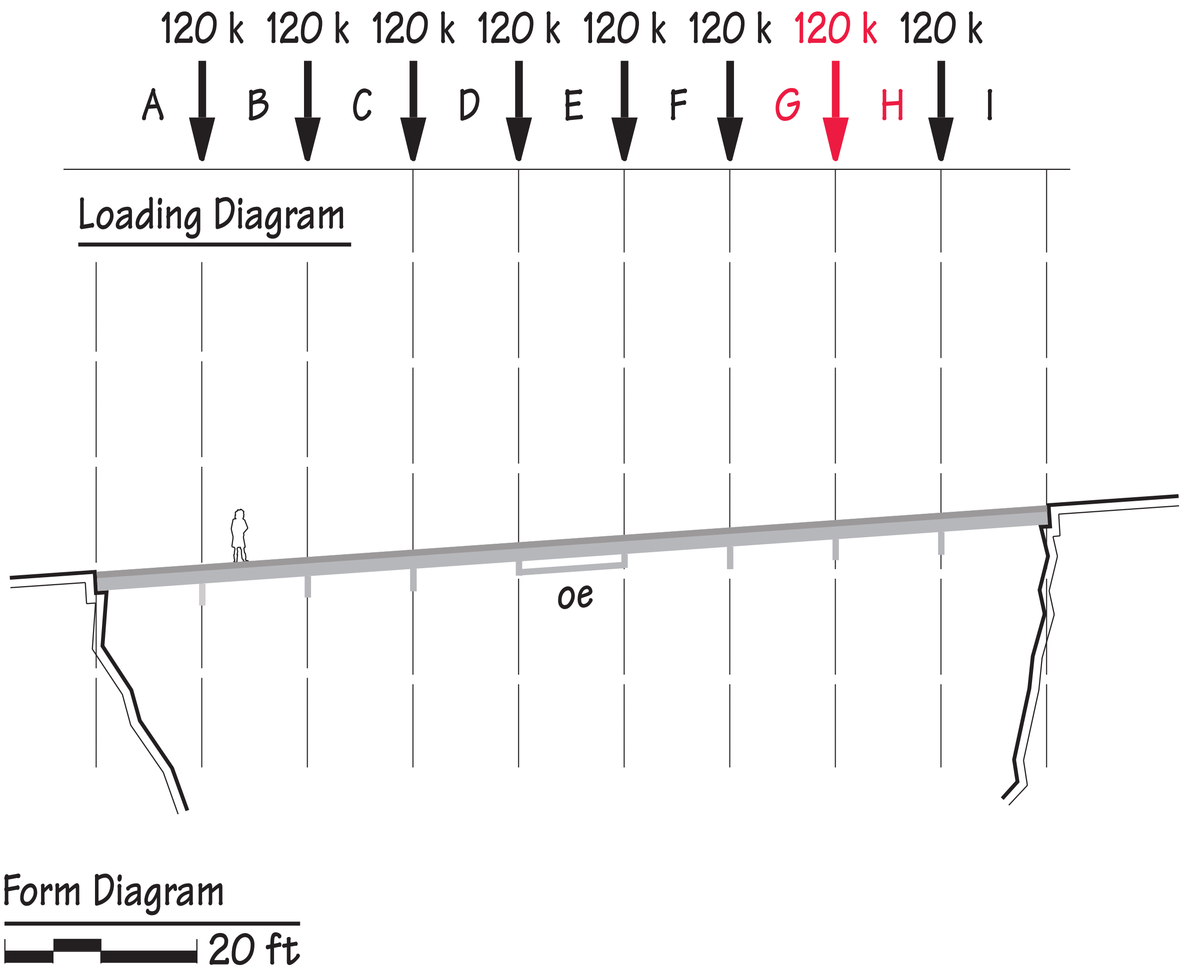
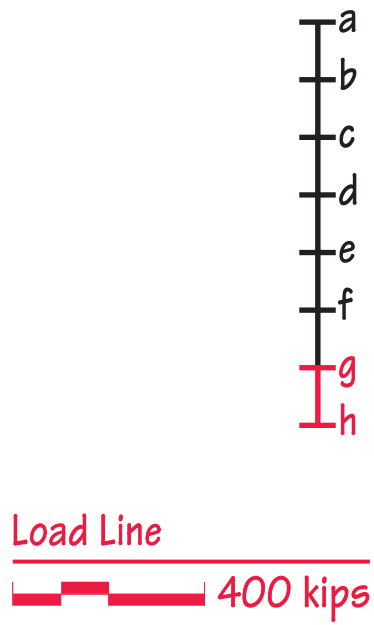
We continue across the Loading Diagram...





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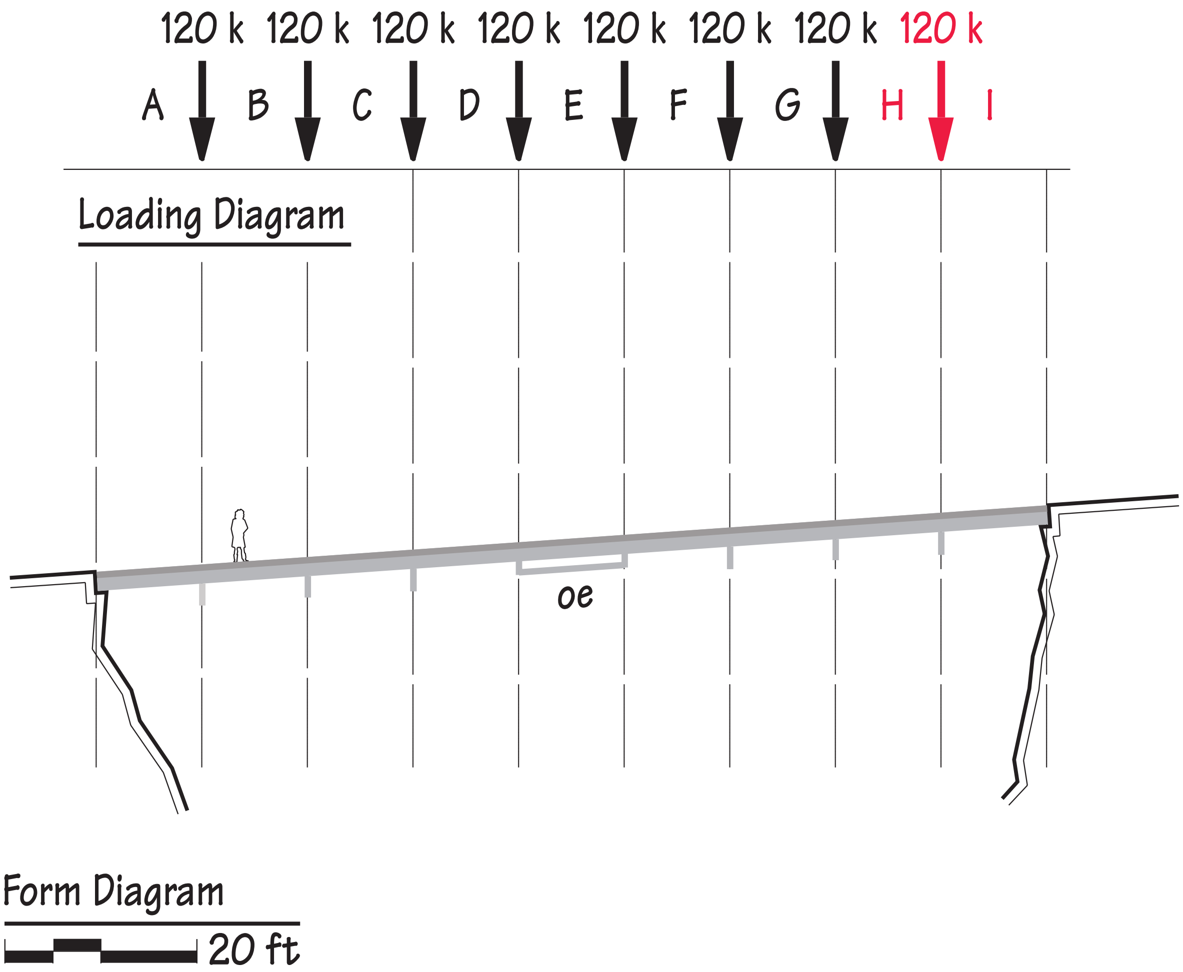
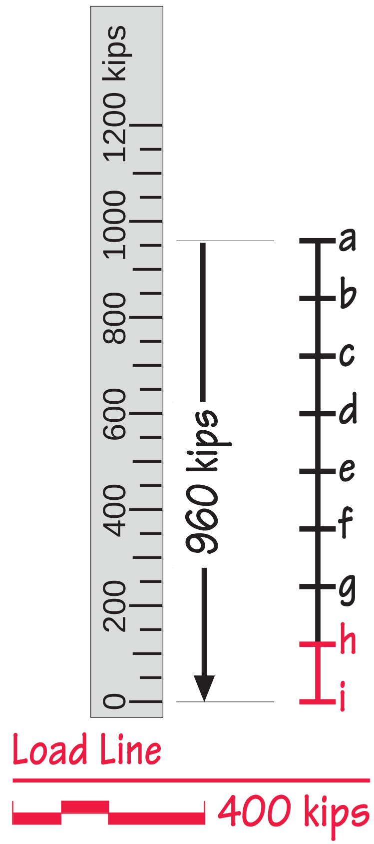
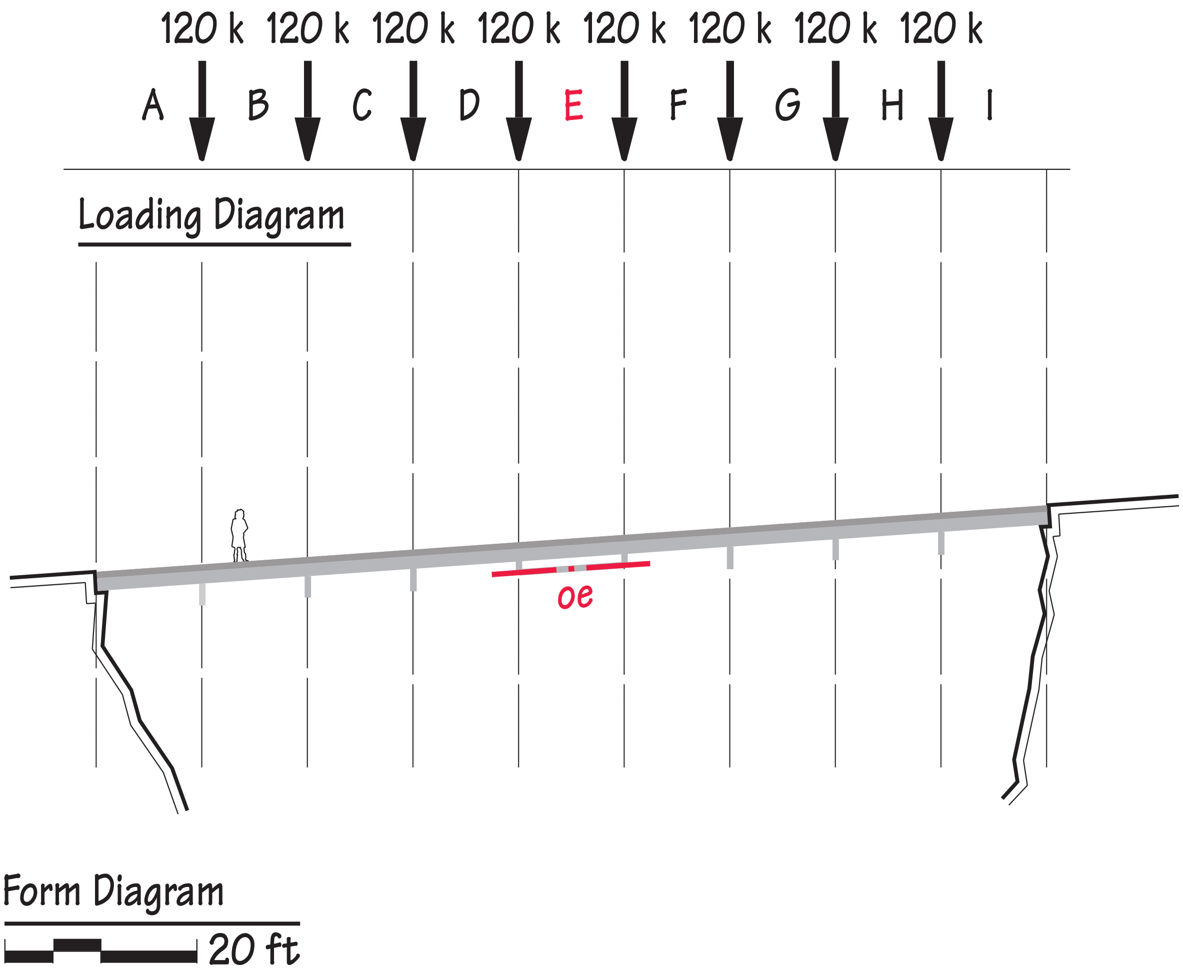


The Load Line is now complete.

The total length of the Load Line is equal to the total load on the arch, in this case 960 kips.

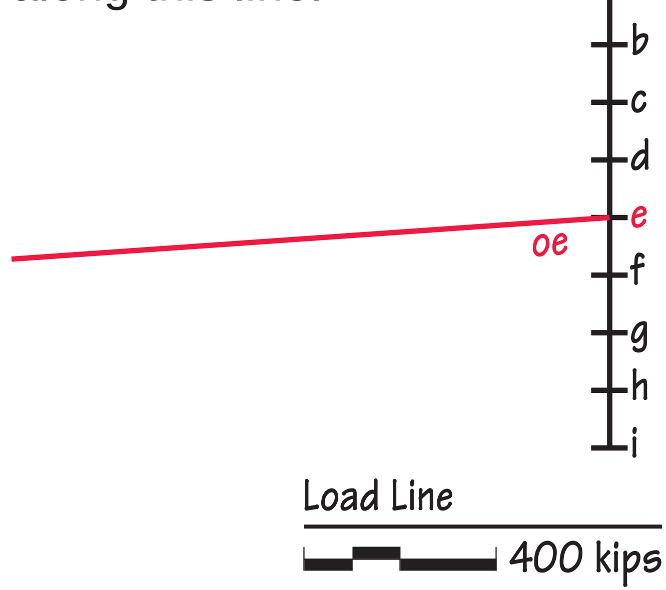


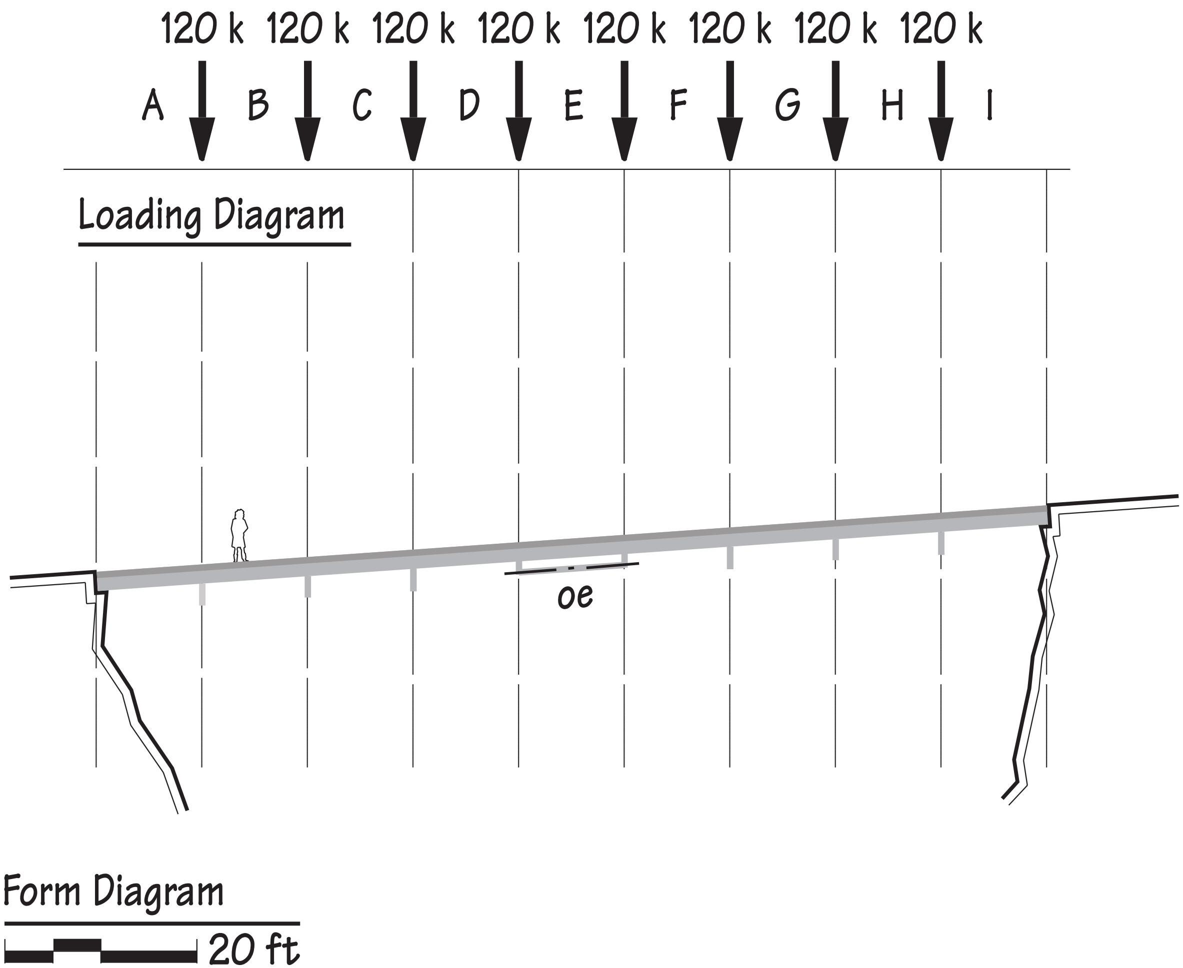


Step 4: Construct ray *oe* of known direction but unknown length.

Ray *oe* is a vector whose length is equal to the magnitude of the force in arch segment *oe*, and whose direction is parallel to that force.

Construct ray *oe* through point *e* on the load line, parallel to arch segment *oe*. Although we don't yet know the location of point *o*, the left end of ray *oe*, it must occur somewhere along this line.



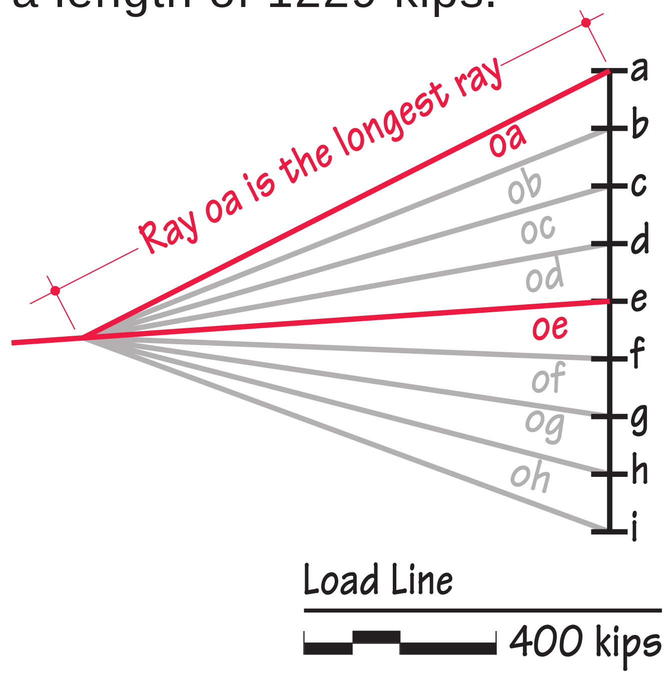


Step 5: Construct the longest ray and find point *o*.

Even before we know the location of point *o*, we can sketch the general layout of the rays of the Force Polygon.

We can see from this sketch that, for any point *o* that lies on ray *oe*, ray *oa* will be the longest ray, corresponding to the greatest force in the arch.

Thus we must limit ray *oa* to a length of 1229 kips.



Here is the equation we used in Step 1 to find the maximum allowable force in the arch:

$$\text{Force} = (\text{Area})(\text{allowable stress}) = (1536\ \text{in}^2)(800\ \text{lb/in}^2)$$
$$= 1{,}228{,}800\ \text{lb} = 1229\ \text{kips}$$

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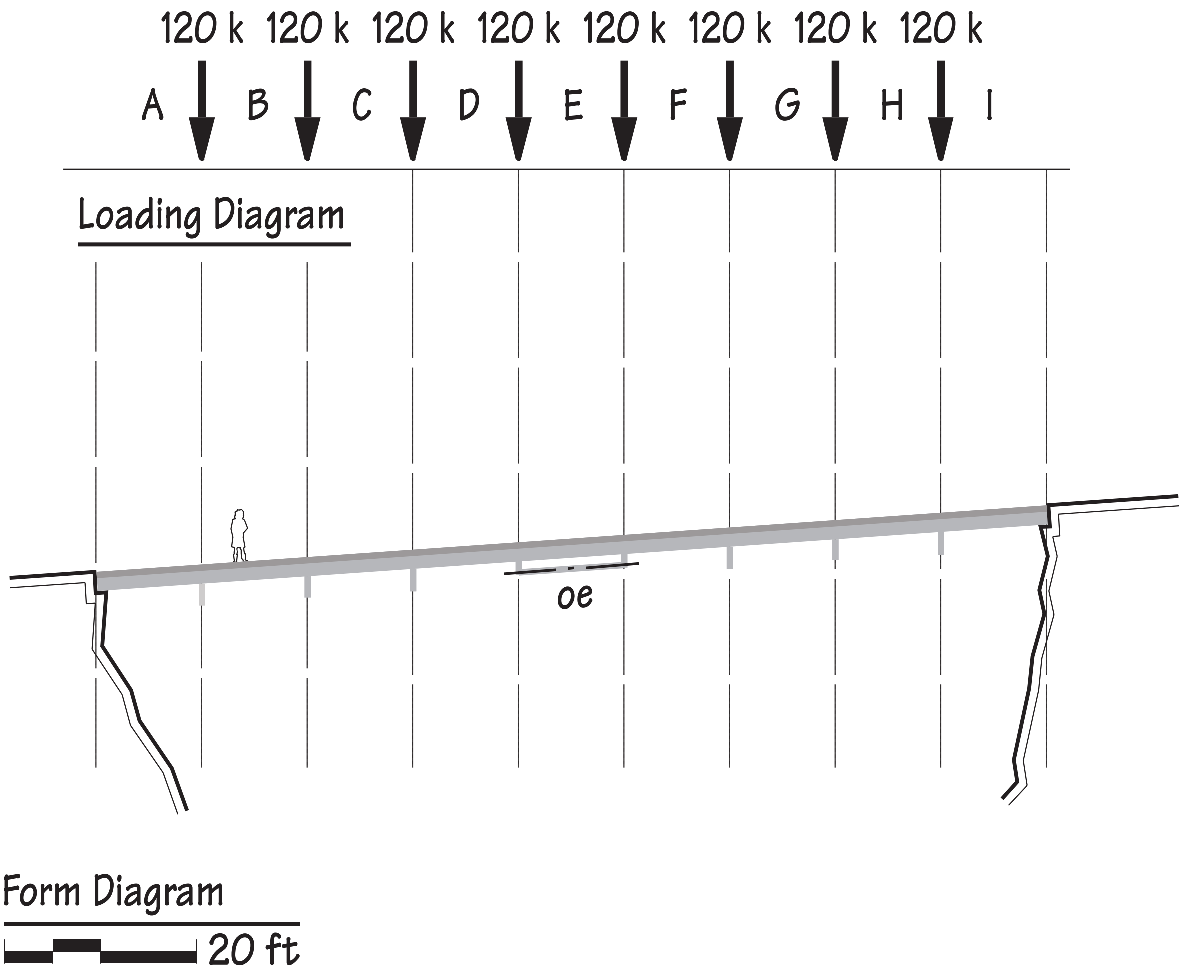
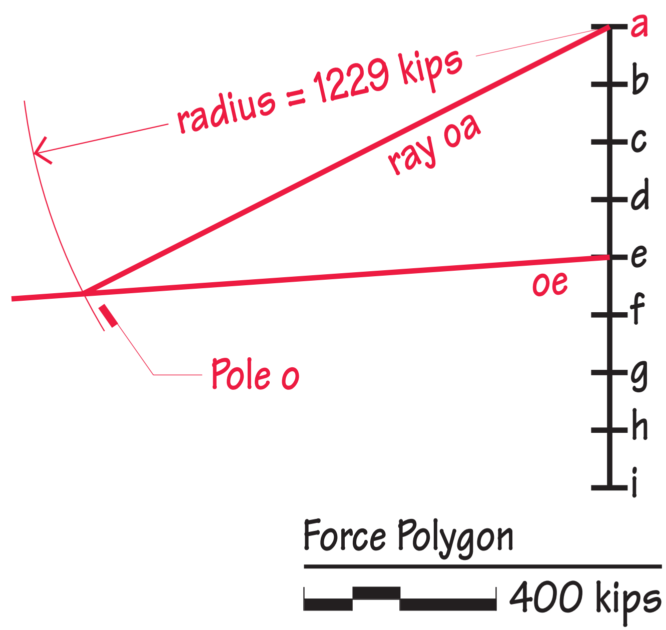


From point *a*, we use a compass to strike an arc of radius 1229 kips, the maximum force allowable in the arch. Where this arc intersects with ray *oe* is point *o*.

The Load Line has now become part of a Force Polygon. Point *o* is the Pole of the diagram.



Here is the equation we used in Step 1 to find the maximum allowable force in the arch:

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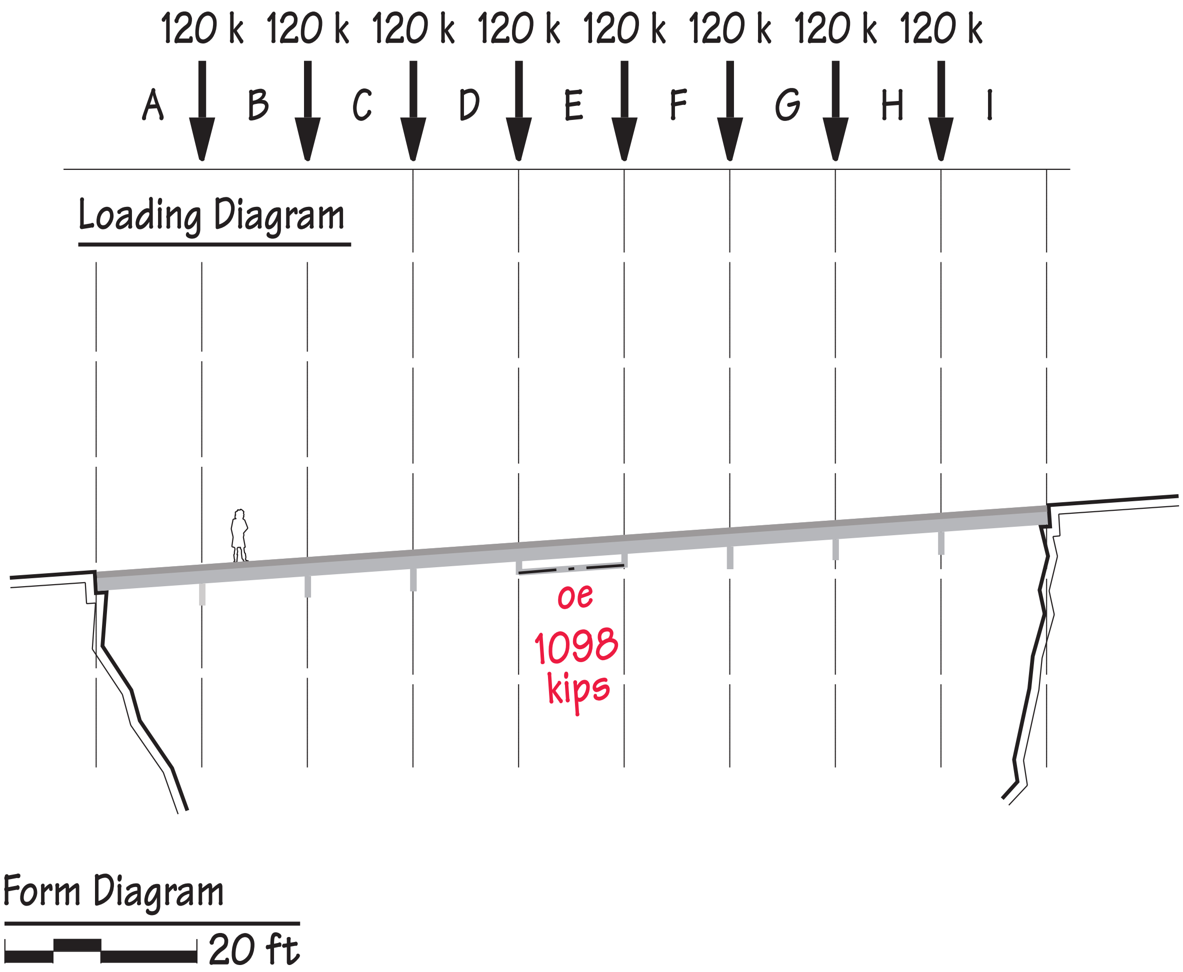
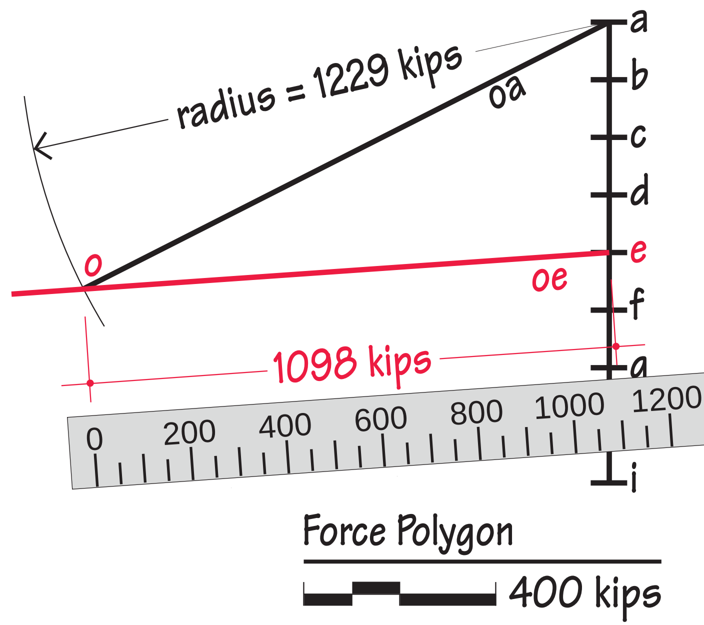
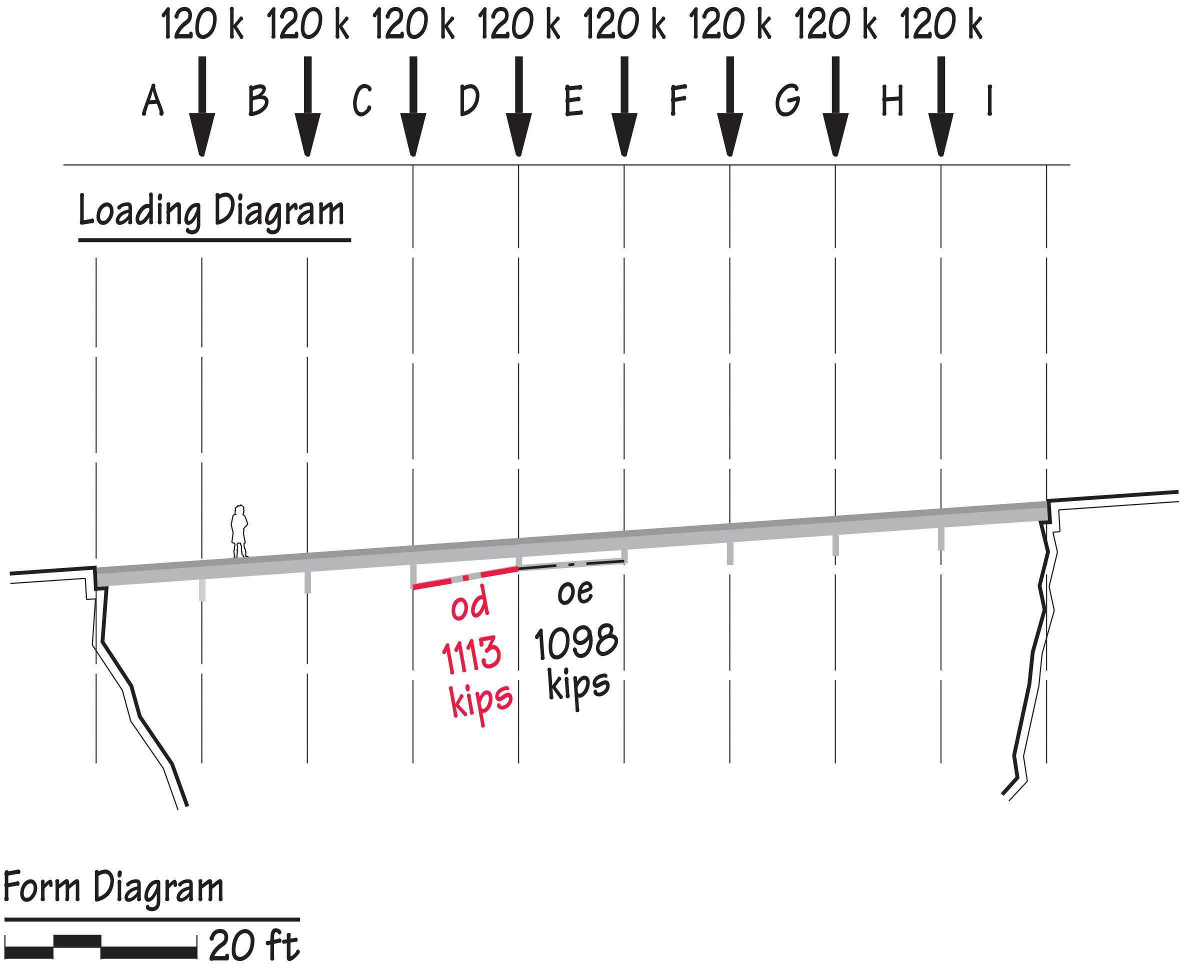


By scaling the length of ray *oe* on the Force Polygon, we can now determine the magnitude of the force in segment *oe* of the arch.

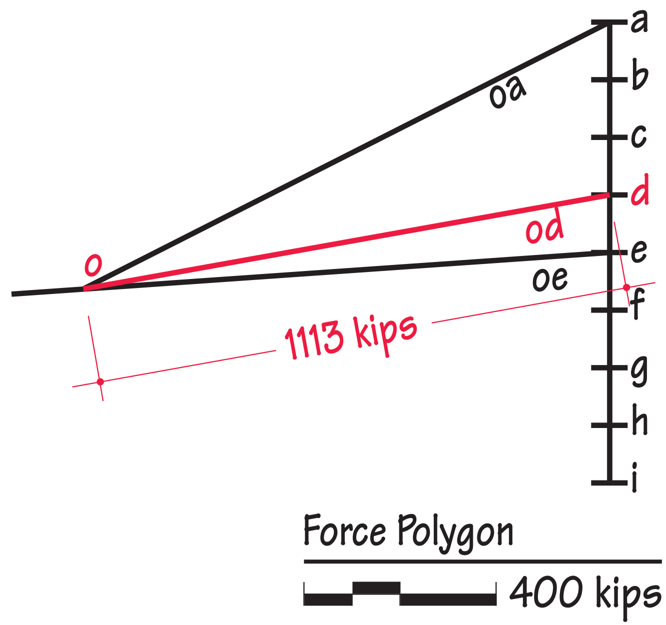




Step 6: Through the Pole *o*, construct the remaining rays of the Force Polygon, beginning at the center and working toward the ends.

As each ray is drawn, construct parallel to it the corresponding segment of the concrete arch on the Form Diagram.

Scale the length of each ray to find the magnitude of the force in the corresponding segment of the arch.



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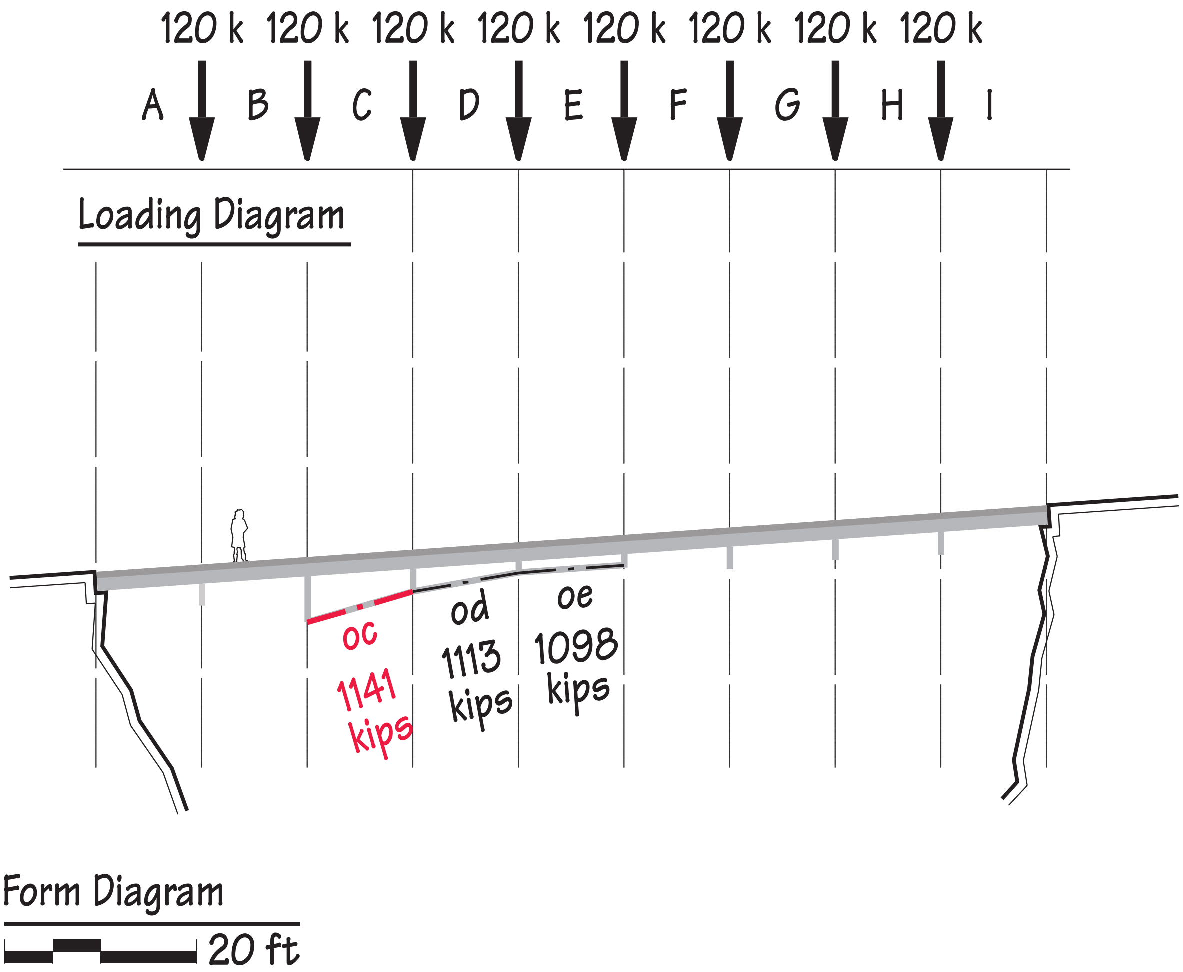
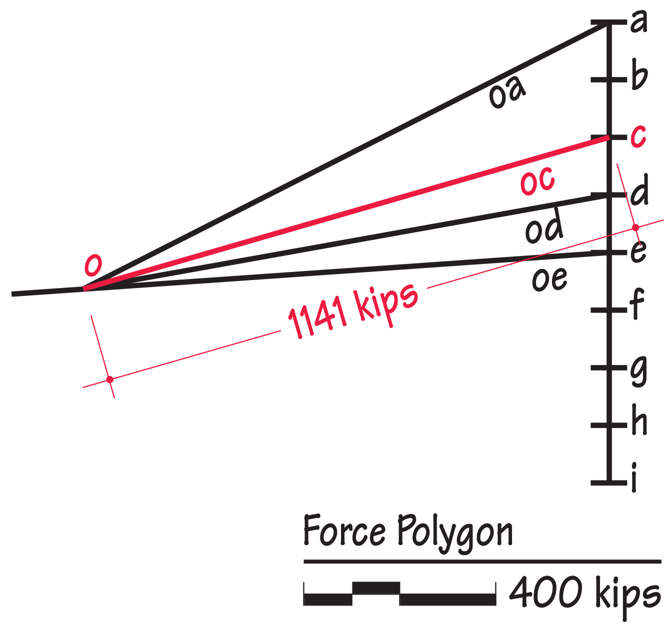
This is ray *oc*.





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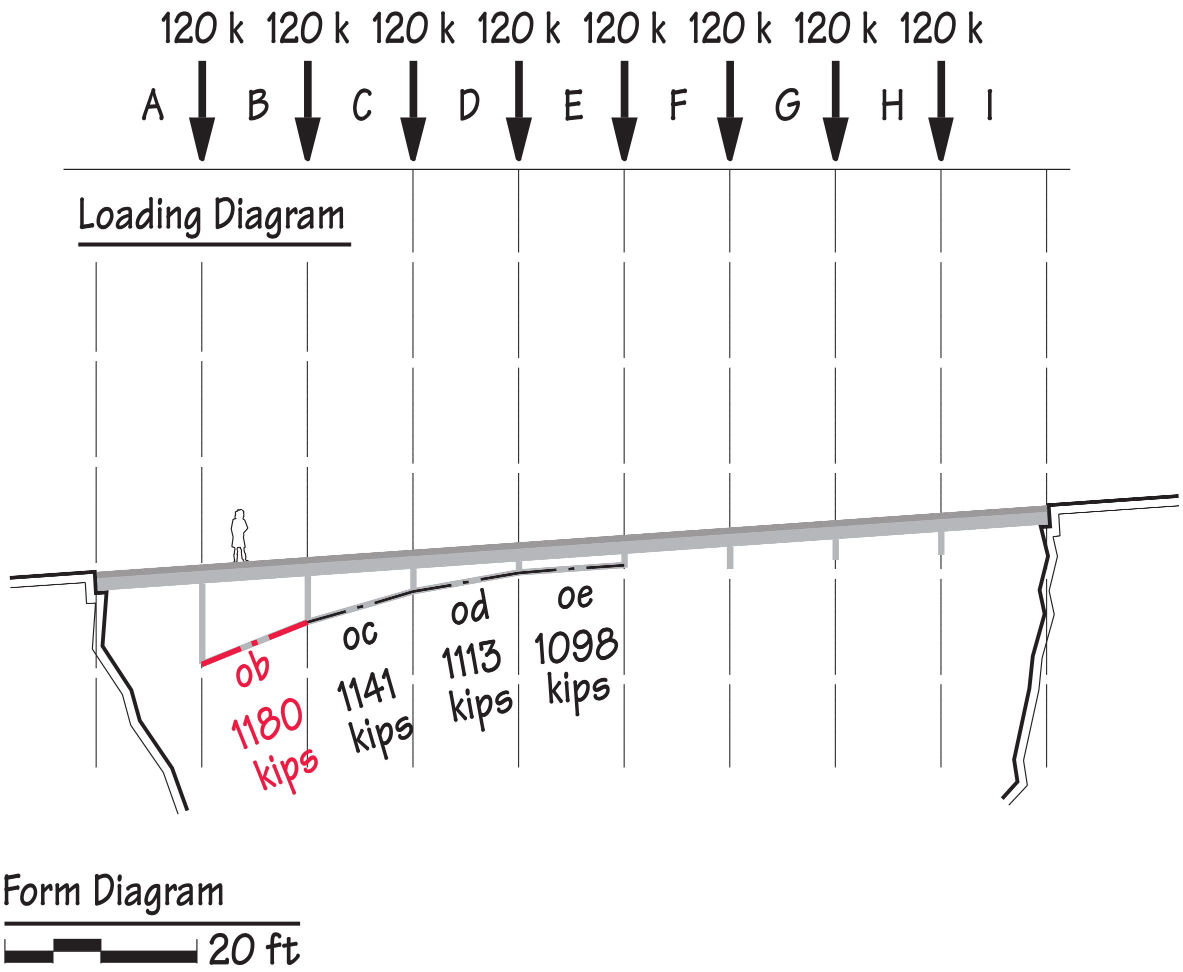
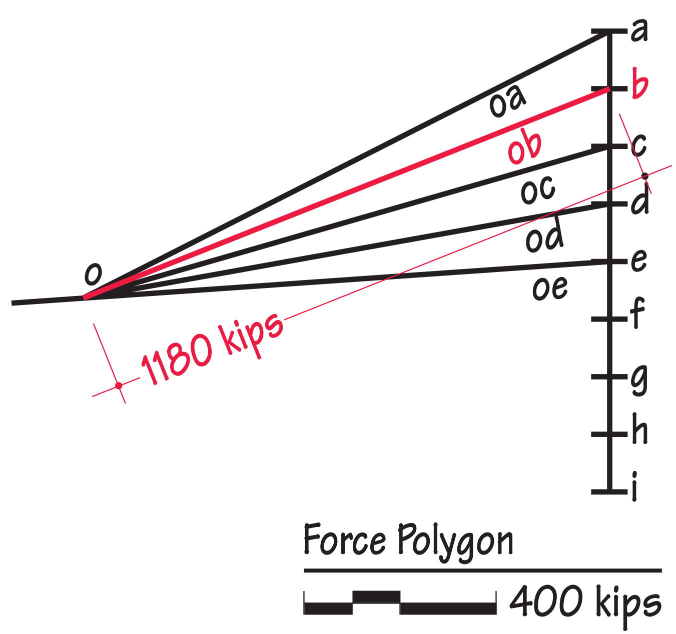
This is ray *ob*.





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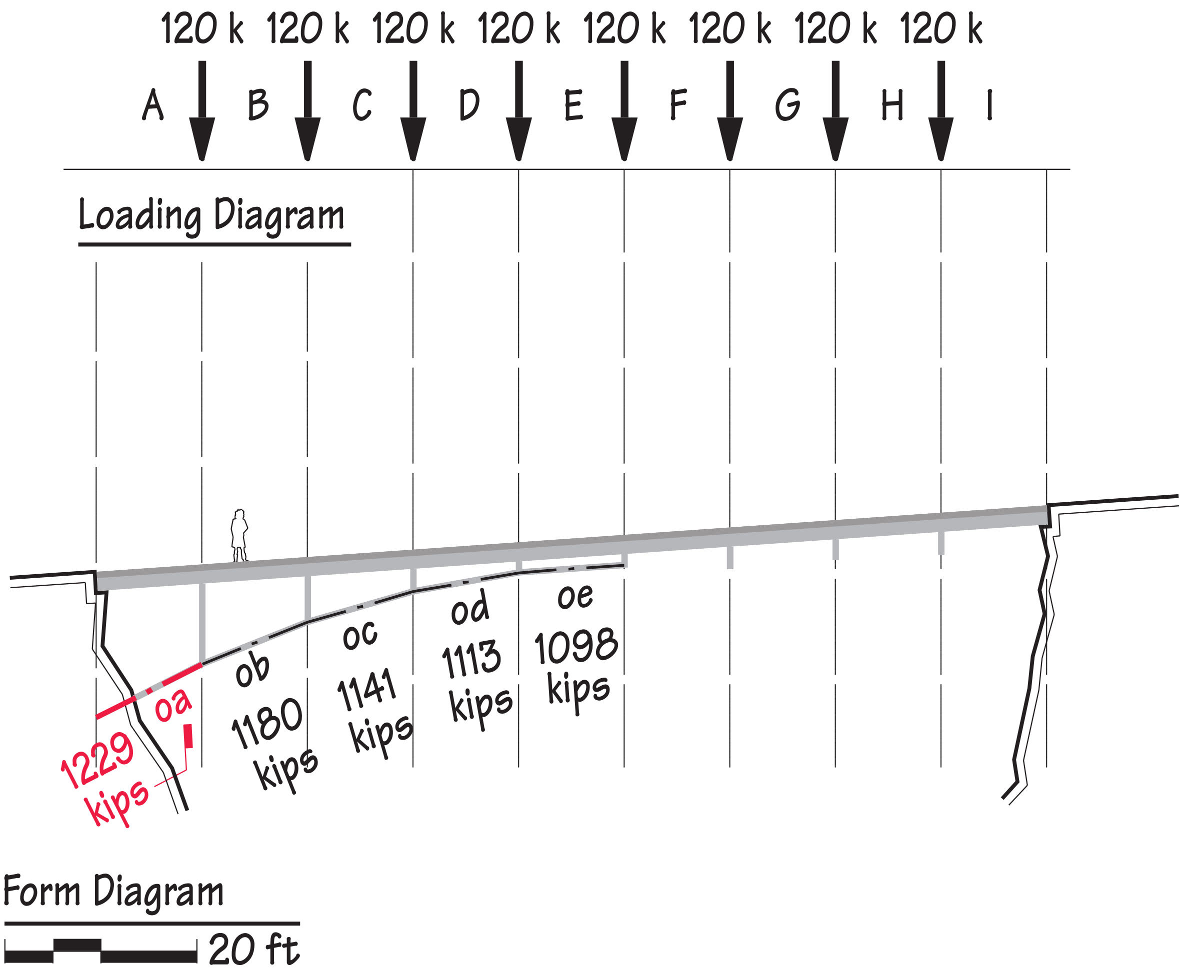
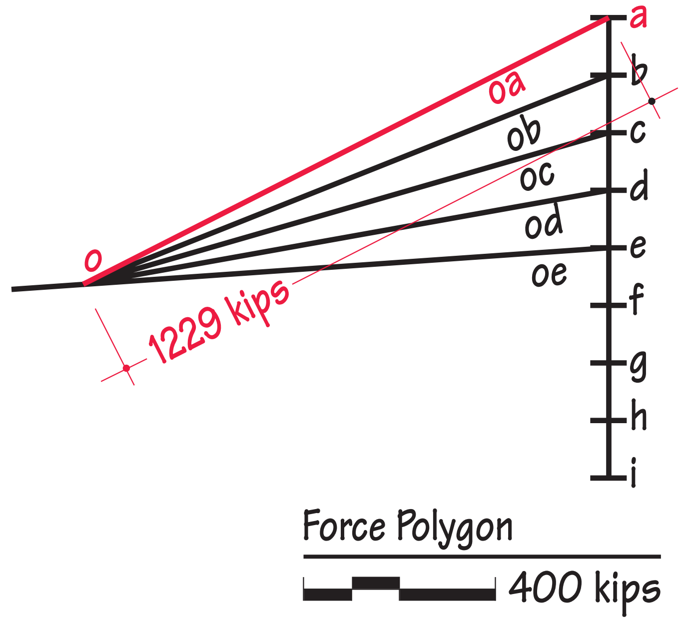
This is ray *oa*.





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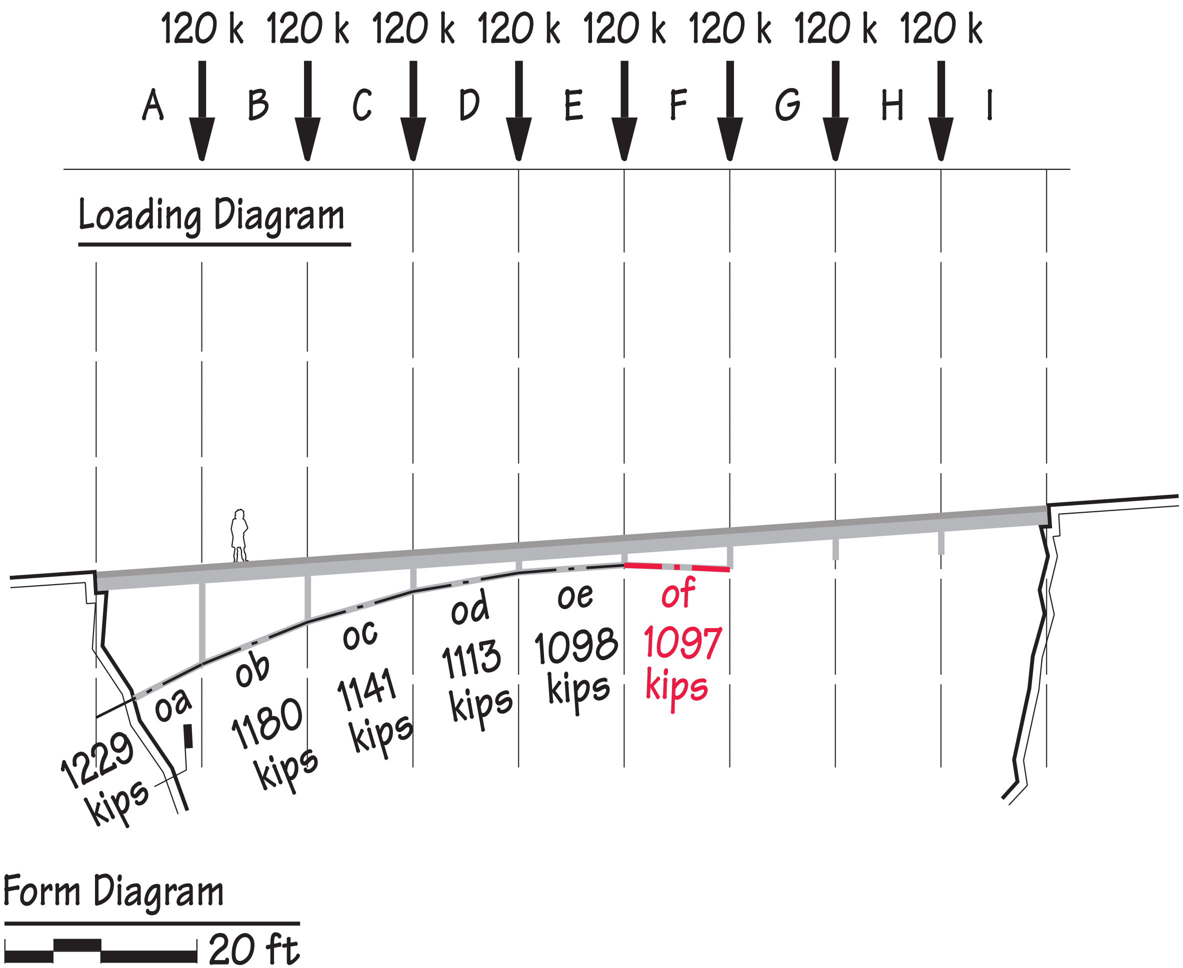
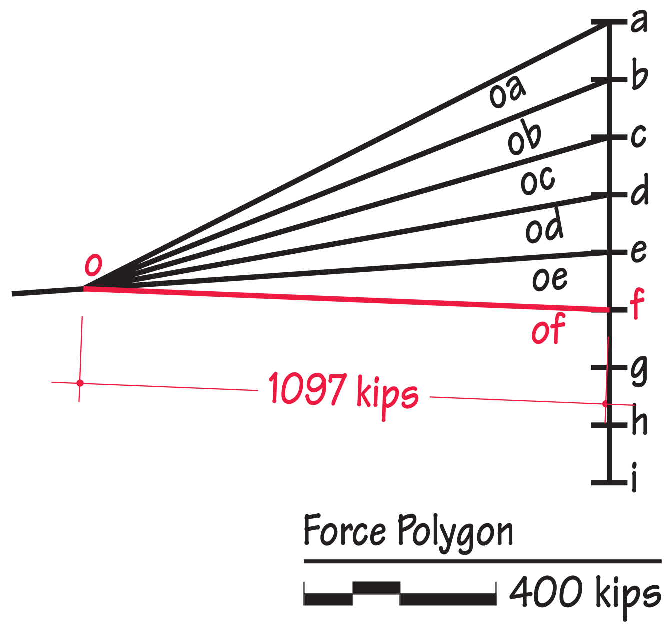
This is ray *of*.





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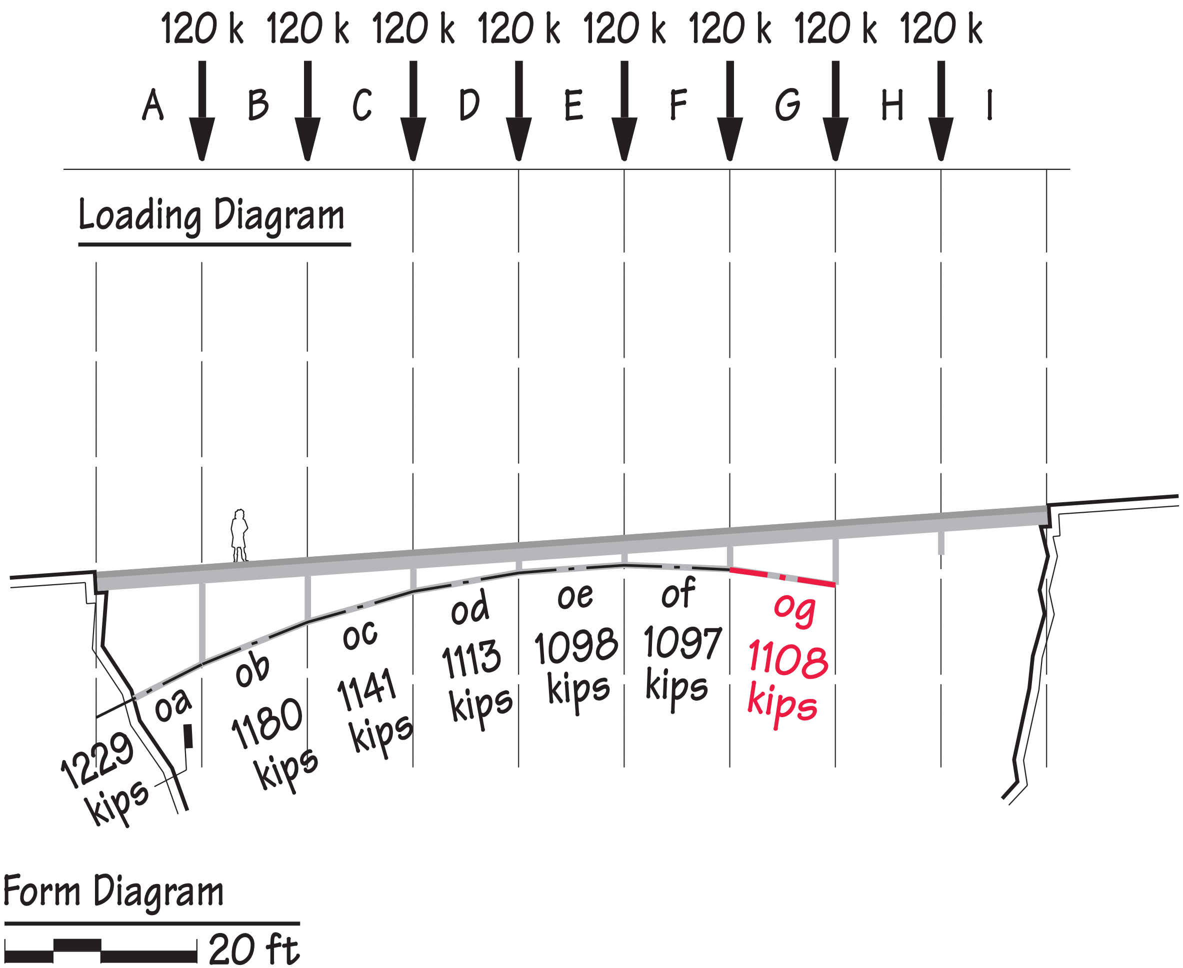
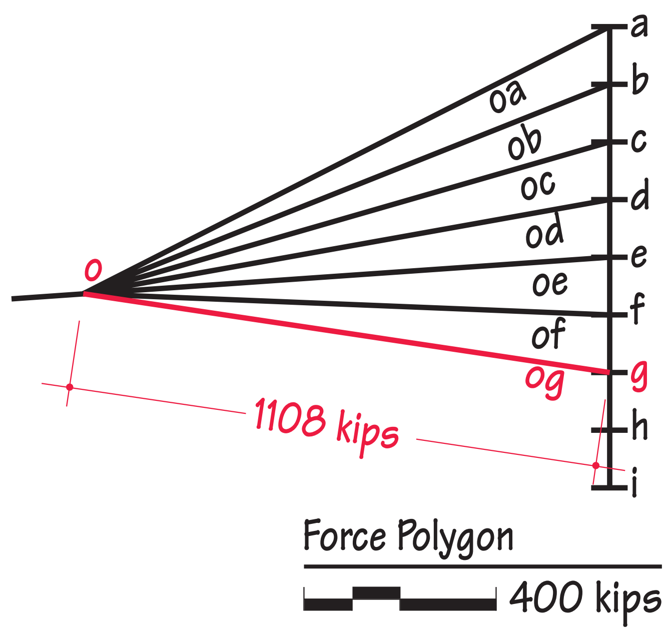


This is ray *og*.



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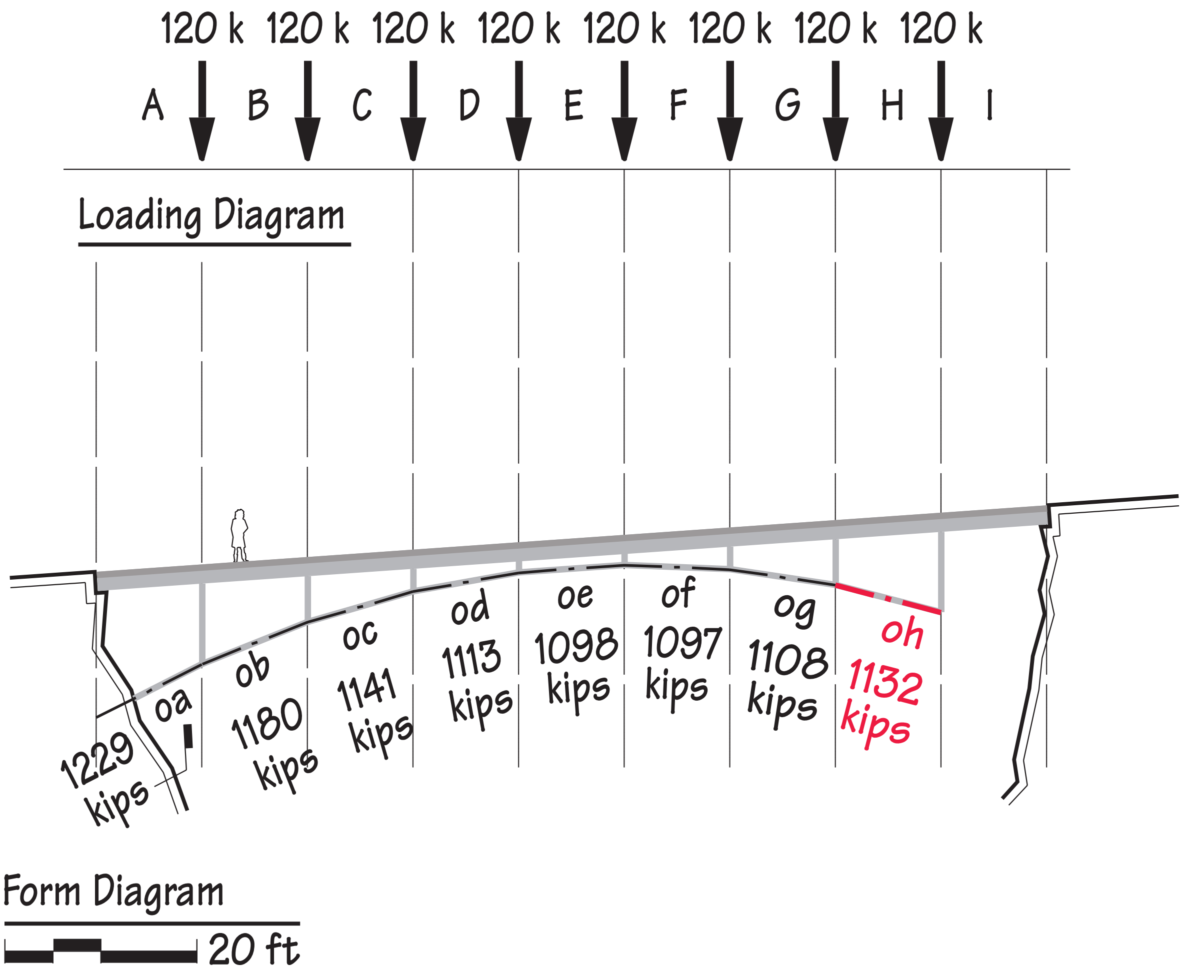
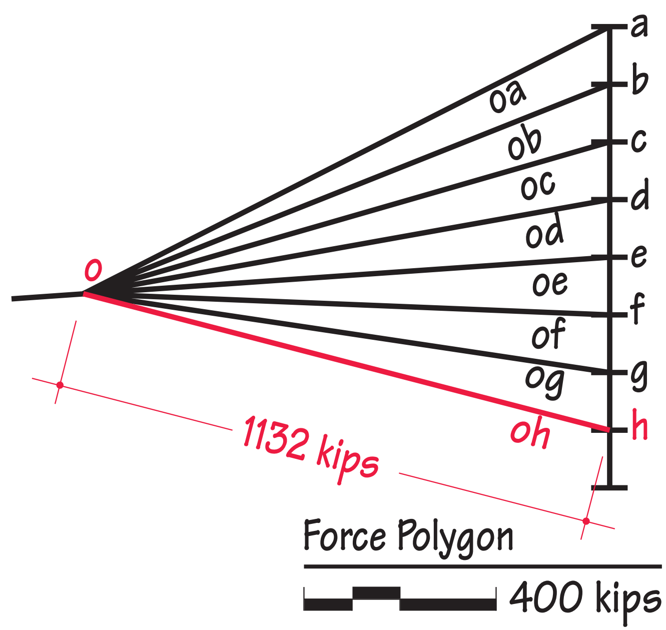
This is ray *oh*.





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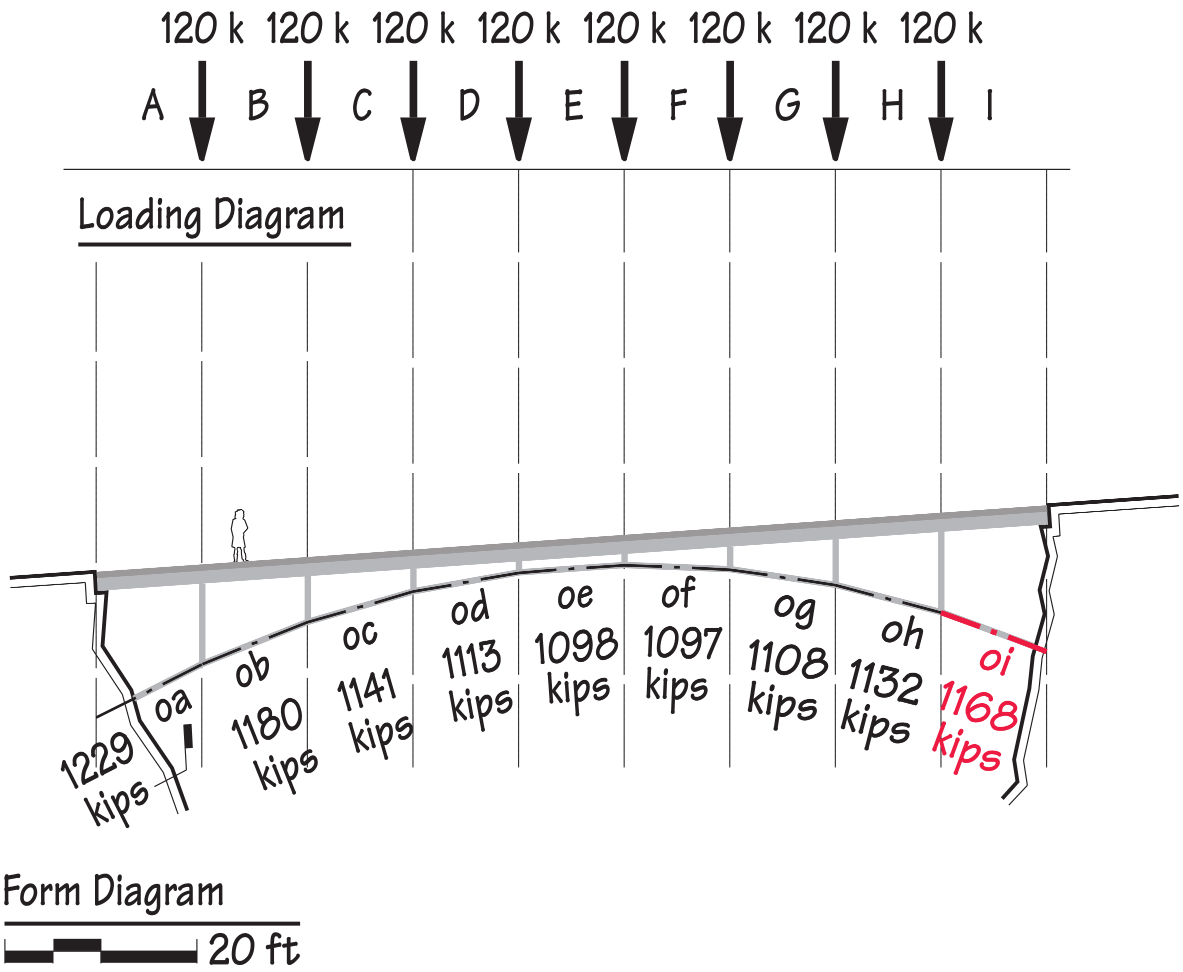
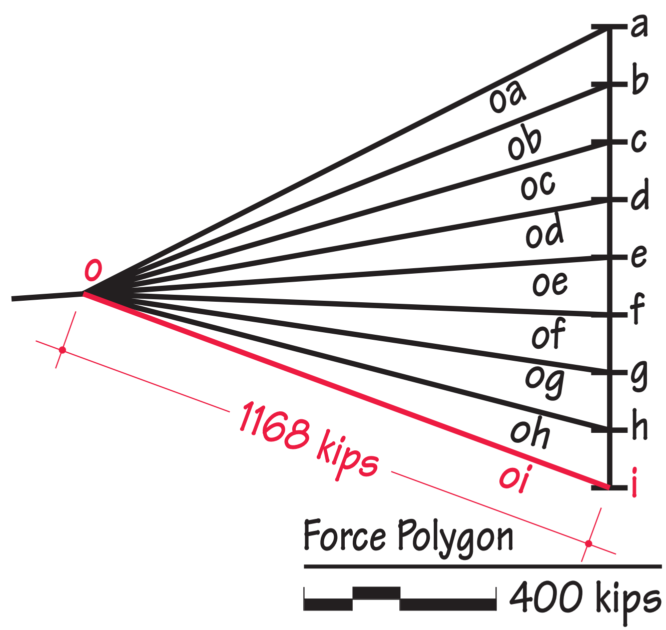
This is ray *oi*.





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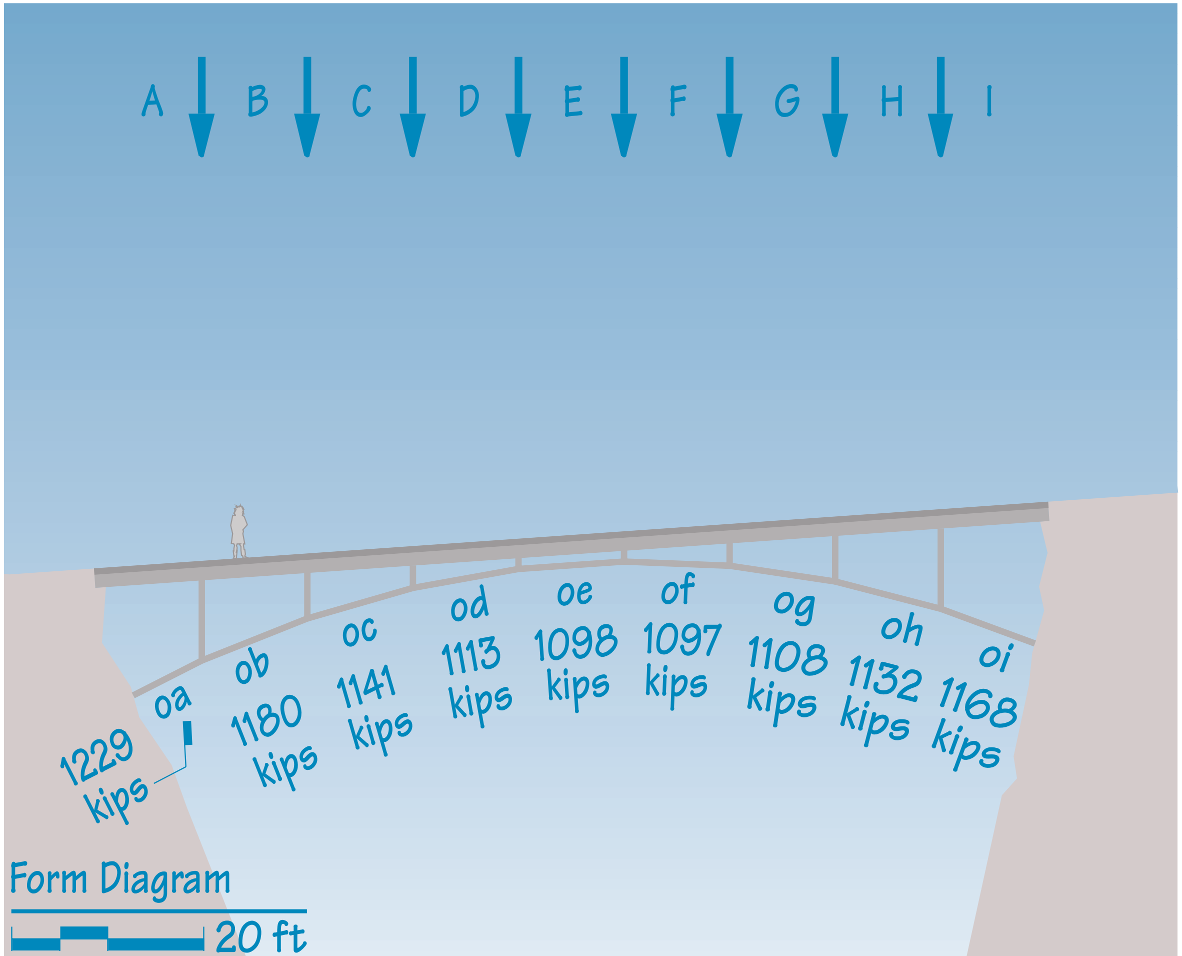
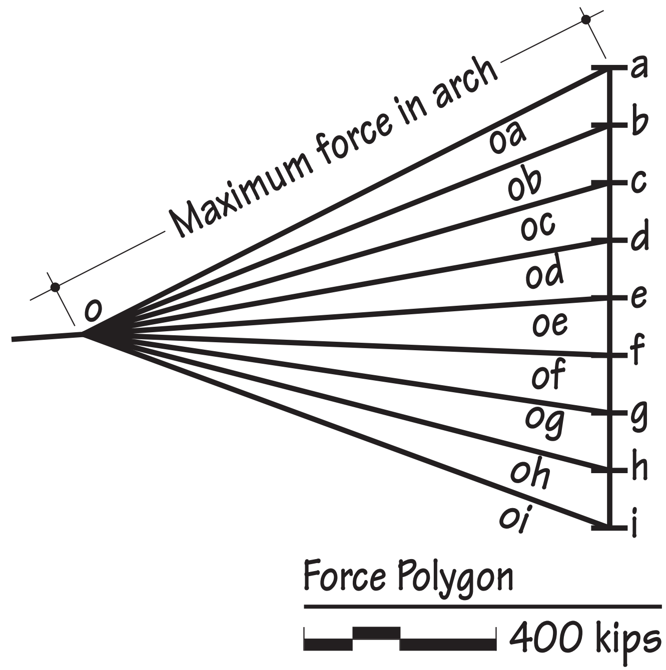


This is the bridge that you have designed.

The force polygon is a graphical tool that allows you to find the forces in a structure, and to find simultaneously the appropriate *form* for the structure.



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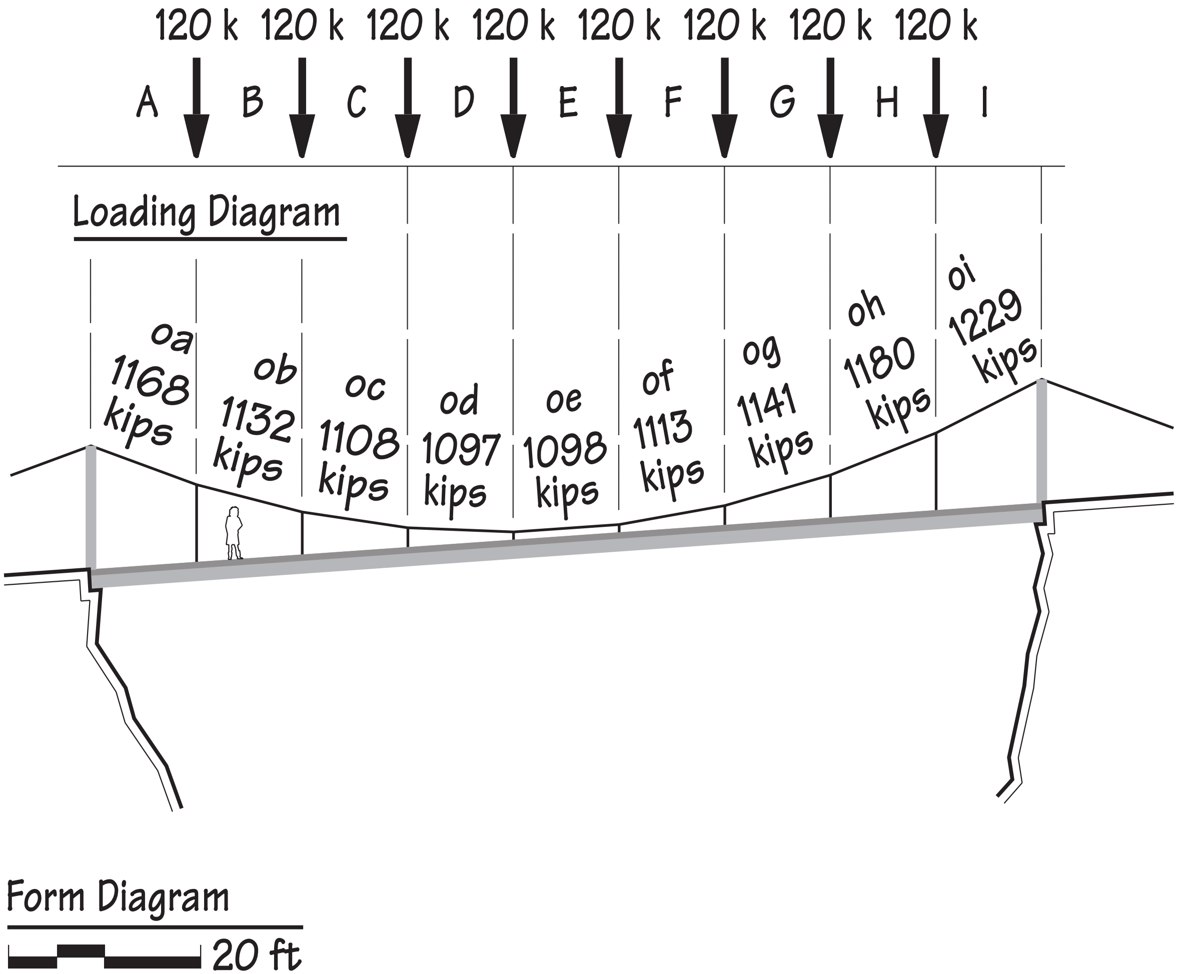
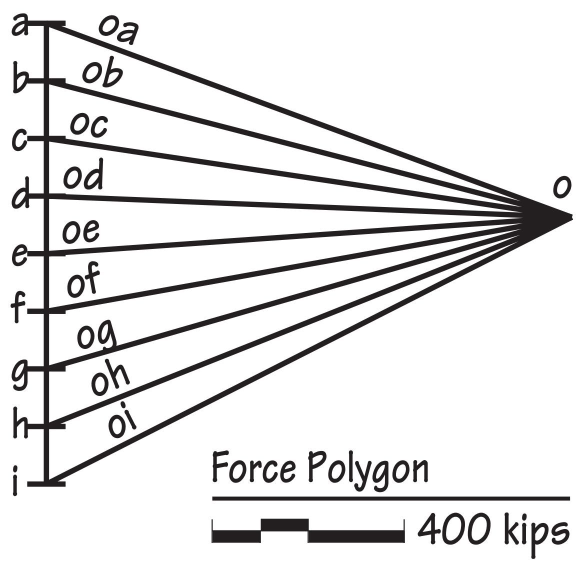


Using the same method that we used to find the form and forces for the arch, we can find the form of a suspension bridge whose cable can safely carry a given maximum force.

The pole of the Force Polygon lies to the right of the load line, rather than to the left. The forces in the cable are tensile, rather than compressive.



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