Passivity Based Grid-Connectivity Criterion for Ensuring Stability of a Network With Controlled Power Injection Devices

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***Abstract*—In this paper, we present a passivity-based sufficient condition for the small-signal stability of an electrical network which has controlled real and reactive power injection devices connected to it. This condition can be easily applied to systems which can be made linear time-invariant in the D-Q variables, which include voltage source converter interfaced generation, storage and reactive power compensation systems. The equivalent admittance/impedance matrices of these devices, which are required for the checking of the stability criterion, can be obtained using analytical models or simulation-based frequency scanning techniques. Although the condition is conservative (sufficient but not necessary), an advantage of its use is that it can be applied individually to each device without the need for a system-wide study. Therefore the inclusion of this condition as one of the grid-connectivity criteria merits attention.**

***Index Terms*—Small-Signal Stability, Subsynchronous Control Interactions, Voltage Source Converters.**

I. Introduction

Future power systems are likely to have a large number of distributed power sources, interfaced to the grid using power electronic devices. This is in addition to conventional generation, and transmission controllers like HVDC and FACTS. Although such devices result in better utilization of energy resources and faster control, it is necessary to avoid adverse interaction with the network. Such interactions have been reported in the past [1], [2]. The unified stability analysis of distributed systems poses the following challenges: (a) a large number of possible operating conditions and network topologies have to be considered, (b) dynamic data/controller structure for individual sub-systems is difficult to obtain since these have not been standardized, (c) addition of new devices would require a fresh examination of the system, and (d) the stability analyses would have to be coordinated centrally. Therefore in this new scenario, it would be pragmatic and useful to specify the *decentralized* or *local* stability criteria for any device interconnected to the grid. Although this is expected be a conservative approach, it will ease the problem of stability analysis substantially. This paper presents such a criterion which can be easily checked using either the analytical model of the devices or simulation-based frequency scanning.

The criterion is based on the fact that a feedback system consisting of two or more passive sub-systems is stable. Since the electrical network (transmission lines, transformers) is passive, this means that the individual sub-systems connected to it also need to be passive. Passivity of the sub-systems can be checked by examining the equivalent admittance or impedance matrices (assuming that the sub-systems are time-invariant). Fortunately, the dynamical equations of most three-phase systems connected to the electrical network are normally balanced, and are linear time-invariant when formulated in the D-Q domain, for example a STATCOM [3]. The criterion of stability can thus be easily checked in the frequency domain. This is convenient because small-signal analytical models may not be easily extractable or available, but the frequency response can be numerically obtained by using simulation based frequency scanning techniques by using a black-box approach [4], [5].

We also explore the modification of control structure and parameters which will modify the frequency response in order to make the system passive. Attempts to make voltage source converter (VSC) based devices to behave as passive components near grid resonance frequencies using active damping strategies have been reported in [6]–[9]. Prior works have mostly concentrated on making these devices passive locally around the grid resonance frequencies, thereby requiring detailed information about the grid frequency response. We attempt to provide a simpler and robust criterion for the stability of these devices while restricting to minimal prior information about the grid frequency response. Moreover, unlike [9], the criterion is used without simplying assumptions about the structure of the transfer function matrices such as diagonal dominance. Using the example of a STATCOM, we attempt to achieve the passivity with minimal modifications in the controller architecture so that the additional consideration does not interfere much with the intended purpose of these devices. The updated control strategies have been tested on a detailed VSC based STATCOM and they are found to give the expected outcomes.

The paper has been organized as follows: Section II introduces the concepts of positive real systems and its applications in power systems. Section III discusses the practical applicability of this condition. The control strategies needed to ensure passivity are presented in Section IV. The simulation studies with a VSC based STATCOM are presented in Section V while Section VI presents future works and the conclusions.

II. Passivity Based Criterion

Modern power systems can be represented by a network consisting of passive components like transmission lines and transformers with series and shunt connected sub-systems connected to the network as shown in Figure 1.

The analysis of positive real (passive) systems is well defined

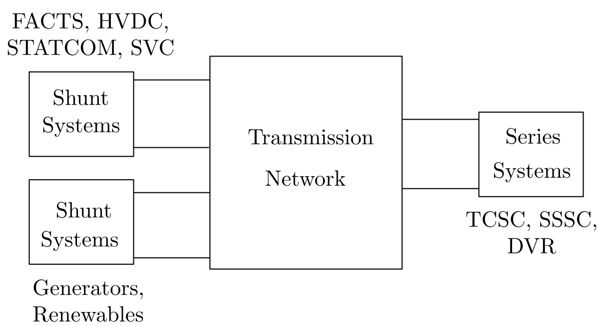

Figure 1. Representation of a power system

in terms of transfer functions, which are applicable for linear time-invariant (LTI) systems. It is therefore convenient to analyze the entire model in terms of transfer functions. It is assumed that all power system components that are balanced and three-phase can be converted to a multi-input multi-output (MIMO) LTI system using D-Q transformation, and can be represented by transfer function matrices.

An $n \times n$ transfer function matrix $H(s)$ is positive real if [10]

- it is *Hurwitz* i.e. no poles are in $\Re(s) > 0$[1],
- $H(j\omega) + H^T(-j\omega) \geq 0$ for $\omega \in \mathbb{R}^+$. [2]

It should be noted that a positive real system will be always stable but not vice versa. The following conslusions can be inferred [10]:

- Positive real systems are always stable.
- Negative feedback connection of positive real systems are also positive real.
- Sum of two positive real functions is also positive real i.e. parallel connection of two positive real systems is also a positive real system.

The open loop system (electrical network) can be modeled as series and parallel combinations of R-L-C components. Since R, L, C components are positive real and series & parallel combination of positive real systems are also positive real, the open loop system is therefore positive real. However, these grid connected sub-systems like FACTS devices, HVDC links, synchronous generators with excitation systems, renewable based generation etc. need not be passive systems.

For the simplicity of analysis, we consider here only shunt

[1] $\Re(s)$ denotes the real part of the complex variable s.

[2] $\mathbb{R}^+$ denotes the set of positive real numbers.

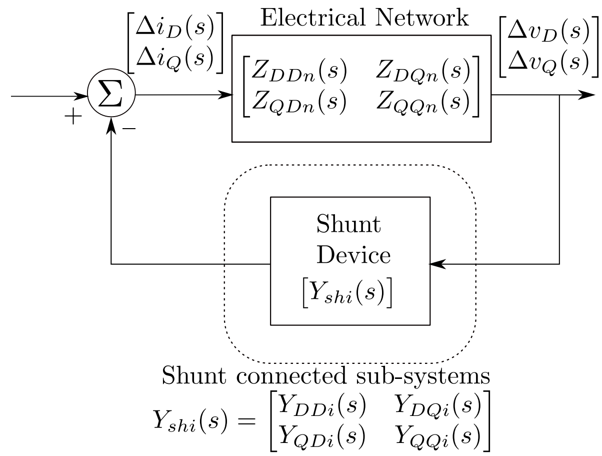

Figure 2. Schematic of feedback structure in power systems (shown here for only one shunt connected device)

connected sub-systems. The overall system can therefore be represented as shown in Figure 2. The open loop transfer function matrix corresponds to the impedance matrix of the network, $Z_{DQn}(s)$, while the transfer function of the feedback blocks correspond to the admittance matrices of the individual sub-systems, $Y_{shi}(s)$. It has been already stated that the negative feedback connection of two positive real systems is also positive real.

Since the open loop system (electrical network) is positive real, the closed loop system will be stable if the shunt connected sub-system is also positive real (passive).

Therefore, if all the shunt connected sub-systems have positive real admittance matrices, the overall system will be positive real and will therefore be stable. The closed loop transfer function can be evaluated as in (1).

$$\begin{aligned} G_{cl}(s) &= \left(Z_{DQn}^{-1}(s) + Y_{sh}(s)\right)^{-1} \\ &= \left(Y_{DQn}(s) + Y_{sh}(s)\right)^{-1} \end{aligned} \tag{1}$$

where $Y_{DQn(s)}$ is the admittance matrix of the electrical network. It can be seen from (1) that at any particular frequency $s = j\omega$, if the admittance of the shunt connected device is much less than the self-admittance of the network at that bus, the closed loop system can be reasonably approximated by the open loop transfer function at that frequency i.e. the shunt connected device behaves approximately as an open-circuit at that frequency.

III. Application to Practical Systems

The validation of positive real criterion requires examination of the admittance matrix of the shunt connected sub-systems. The transfer functions can be derived using an analytical model of these sub-systems. Due to the complexity and non-standardization of these systems, the small-signal models may not be easily extractable or available. A pragmatic alternative to obtain a LTI model of the same is to use simluation-based frequency scanning. A brief description of frequency scanning is presented in Appendix A.

The positive real criterion is defined on the entire frequency range (positive real numbers). This necessitates the discussion about the range upto which frequency scan needs to be done. Typically the controller bandwidth for the controllable power injection systems are upto 200 Hz. Systems like VSC based device can be modeled as constant modulation (magnitude and angle) index systems beyond this range, as the controllers are unresponsive above this frequency range. Depending on the controller architecture, the frequency range of analysis can be approximately divided into two distinct regions as shown in Figure 3.

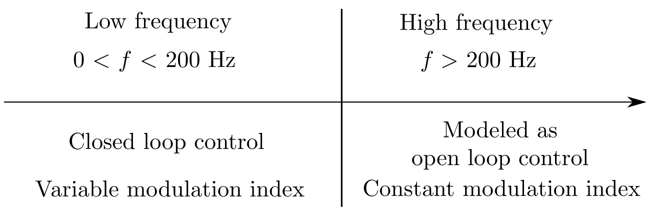

Figure 3. Frequency Spectra for passivity of shunt connected systems

The admittance matrix of a VSC device such as a STATCOM with constant modulation index is given in [11]. It can be seen that the system can be shown to be positive real under this condition. This concludes that in the high frequency range, the VSC based devices are usually passive in nature. The closed loop transfer function will also be positive real in this range since the network is also passive. We therefore need to focus only on the frequency range where the VSC based devices may not behave like a passive system, due to the presence of 'active' elements like closed loop controllers. The frequency scan therefore needs to be performed only in the medium and low frequency range (inside the controller bandwidth). As stated previously, the overall system will be positive real if the admittance matrix of the shunt connected systems satisfies the passivity criterion in this frequency range i.e.

$$Y_{vsc}^{\mathcal{R}}(j\omega) = Y_{vsc}(j\omega) + Y_{vsc}^{T}(-j\omega)$$

is positive semi-definite (eigenvalues ≥ 0) for $\omega \in \mathbb{R}^{+}$. We shall now focus on the modification of the control strategies of VSC based devices so that it can be made passive over the desired frequency range.

IV. Modifications in Control Strategies

The shunt connected systems may not be passive in the frequency range of their controller bandwidth. A case study of a non-passive VSC based STATCOM with P-I based voltage regulator is shown in [12]. Instabilities in reference to LCC based HVDC links are also presented in [13]. This is an indication that these systems in general, may not be passive within the controller bandwidth and their control strategies may need to be modified for the same. Since passivity is strongly related to the introduction of dissipative elements in the system like resistances, it is expected that the introduction of control strategies that mimic resistive elements can make these devices passive. It has been found that a modulating signal proportional to the voltage variation actually improves the passivity behaviour of these devices. The modulating signal is given in (2). This control strategy mimics the effect of a shunt conductance (resistance). This hints towards the use of active control strategies that can also mimic series resistance components in the network.

$$\begin{bmatrix} \Delta i_{RDm} \\ \Delta i_{RQm} \end{bmatrix} = \begin{bmatrix} G_{sh} & 0 \\ 0 & G_{sh} \end{bmatrix} \begin{bmatrix} \Delta v_{RD} \\ \Delta v_{RQ} \end{bmatrix} \quad (2)$$

The effect of the modulating signal will affect the transient response but it is not expected to alter the steady state response. If the bus voltage is converted to the Kron frame with the same phasor reference, then v_D will be zero and v_Q will be the rms value of the bus voltage. If the STATCOM has to inject a reactive power of Q pu, then the current injection is

$$i_Q + ji_D = \left(\frac{P + jQ}{v_Q + jv_D}\right)^{*} \quad (3)$$

It is apparent that if the losses in a reactive power compensation device such as STATCOM are neglected, then in sinusoidal steady state, $i_Q = 0$ and $i_D = -\frac{Q}{v_Q}$. The effect of the modulating controllers in steady-state response can be nullified by the use of a washout block but it will introduce attenuation in the low frequency range and can limit the range of frequency over which the modulating controller is effective. To avoid that, the feedback signals are synthesized by eliminating their steady state components as given in (4).

$$v_{Qm} = v_Q - V_{s(rms)}, \quad i_{Dm} = i_D + \frac{Q_{ref}}{v_Q} \quad (4)$$

The control strategy of a STATCOM Type-I controller alongwith the modulating controllers (to mimic the series and shunt resistances) is shown in Figure 4. The additional control blocks that are introduced to ensure the passivity of the system are enclosed in dotted boxes. The amount of series and shunt compensations are represented by the gains G_{sh} and R_s respectively. The outputs of the controller are applied appropriately as switching functions to the VSC switches.

V. Case Study: VSC Based STATCOM

The modified control strategies are tested on a detailed VSC based STATCOM with a Type-I controller [11]. The STATCOM is connected to a 500 kV bus and the dc capacitor voltage is regulated to 75 kV. The frequency scan of the STATCOM admittance with the basic Type-I controller (G_{sh} = R_s = 0) has been evaluated with the STATCOM injecting 100 MVAr reactive power. The simulation-based frequency scanning has been done in PSCAD [14] using multi-sine injection, as given in Appendix A. The variation of the eigenvalues of $Y_{vsc}^{\mathcal{R}}(j\omega)$ with frequency ω are shown in Figure 5. It can be seen that the device is not passive ($Y_{vsc}^{\mathcal{R}}(j\omega)$ not positive semi-definite) upto 104 Hz.

The modified controller, as shown in Section IV is now implemented and the frequency scan is performed again at the same operating point (100 MVAr reactive power injection), using multi-sine injections in PSCAD [14]. The variation

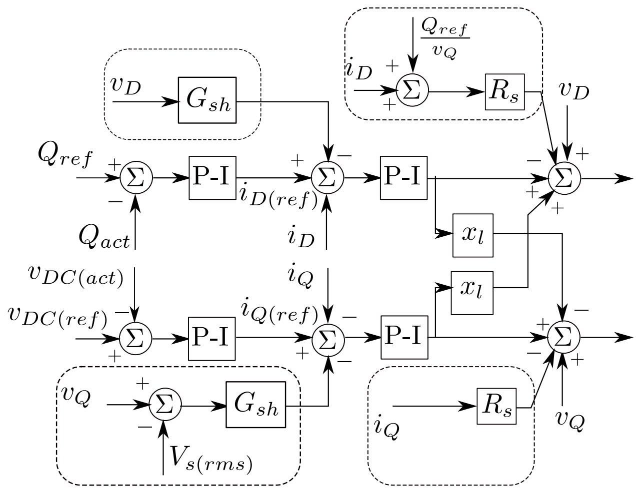

Figure 4. Modified Type-I controller for STATCOM

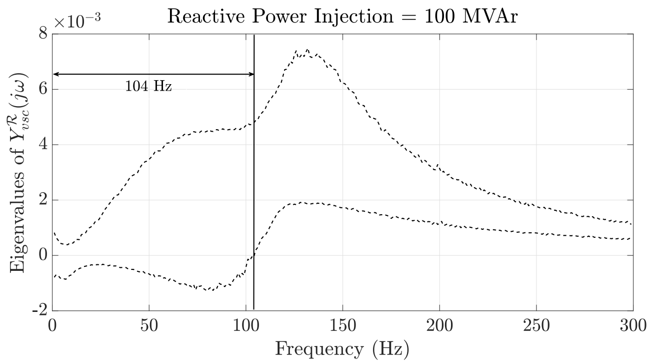

Figure 5. STATCOM frequency scan with basic controller (Q = 100 MVAr)

of the eigenvalues of $Y^{\mathcal{R}}_{vsc}(j\omega)$ with frequency ω, with the modified controller are shown in Figure 6 (for G_{sh} = 25 pu, R_s = 2.5 pu). It can be seen that the STATCOM now behaves like a passive block for $\omega \geq 4$ Hz. The frequency range over which the STATCOM is not passive has been reduced from 104 Hz to 4 Hz. However, the device is not passive over the entire frequency range and therefore, by definition, it is not positive real.

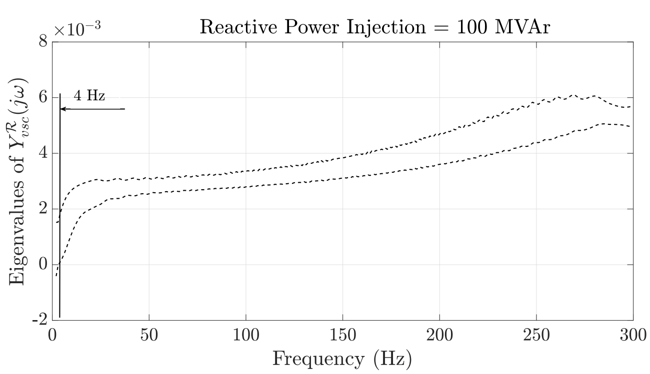

Figure 6. STATCOM frequency scan: modified controller (Q = 100 MVAr)

The relative comparison of the network and the STATCOM admittances are now shown. It is assumed that a 500 kV bus will have a connectivity to 2-3 other buses through HV transmission lines in a realistic grid. From (1), it can be seen that the closed loop transfer function can be approximated by the open loop transfer function if the admittance of the shunt connected system is much smaller than the network self-admittance at that bus. The ratio of the self-admittance at the STATCOM bus (a 2×2 transfer function matrix) to the STATCOM admittance matrix (ratio of each term) is shown in Figure 7.

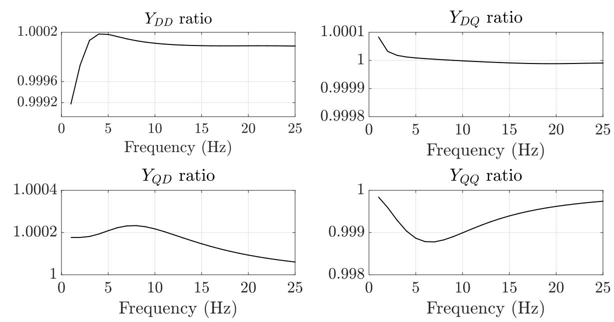

Figure 7. Ratio of admittances at low frequency

It has been observd here that for low frequencies (upto 5–6 Hz) where the STATCOM on itself does not behave as a passive device, the ratio of the admittances ≈ 1, indicating that the closed loop system is approximated by the open loop transfer function matrix $(Z_{DQn}(s))$ in the low frequency range, thereby making the closed loop system positive real. Therefore, the closed loop system is ensured to be small-signal stable.

This provides us an alternative (weaker) stability criterion: If the shunt connected systems are not passive only in frequency ranges where their admittance is negligible as compared to the electrical network admittance, the closed loop system will be stable.
This criterion requires some prior information about the network frequency response.

The modified controller has been tested for its behaviour under transient conditions and a step test is performed to check the same. A step increase and a subsequent step decrease in the reactive power reference is applied at 0.5 s and at 2 s respectively. The actual reactive power flow and the DC capacitor voltage variations are shown in Figure 8 for both the basic controller and the modified controller. It can be seen that the steady-state response due the modified controller has not changed significantly. It can be seen that the closed loop system has been made passive, therefore ensuring stable grid-connected operation.

VI. Conclusions

A sufficient passivity based criterion for stable grid-connected operation of controlled real and reactive power injection devices is presented. The small-signal frequency scan

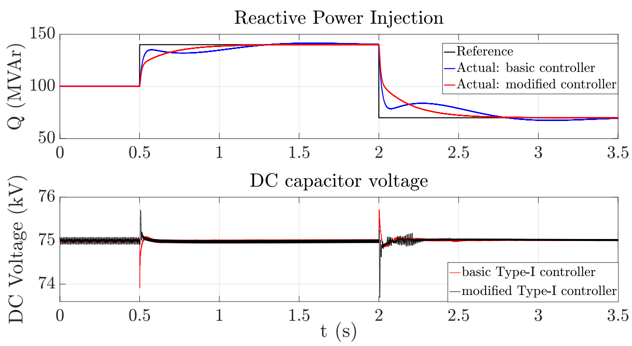

Figure 8. Step response of STATCOM Type-I controller (Q = 100 MVAr)

of these systems may be used for checking the criterion. It has been shown here that for VSC based devices, the existing controllers can be modified to make the system passive. The modified control strategies have been tested on a detailed VSC based STATCOM. Although this paper focuses on the passivity of shunt-connected VSC based devices, the passivity of other controlled devices needs further examination. It is hoped that this will lead to a "simple-to-use" local grid-connectivity criterion for ensuring stability.

VII. Acknowledgements

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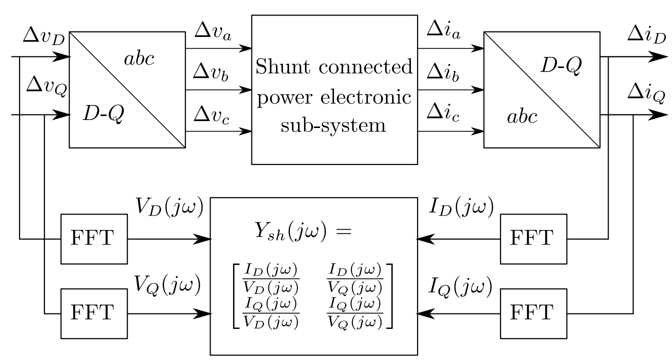
Appendix A
Frequency Scanning [4], [5]

The frequency response of the admittance function of VSC based devices is sometimes quite difficult to obtain analytically. An alternative is to identify the transfer function of a system by injecting small amplitude probing signals about that point and deriving the frequency response from the steady-state inputs and outputs. The system is excited by a wide-band signal containing multiple frequency sinusoids as given in (5).

$$u(t) = \sum_{k=1}^{N} a_k \sin(2\pi k f_{res} t + \phi_k) \tag{5}$$

The magnitude of the input signal should be small enough to ensure that the operating point does not change but at the same time should have enough frequency components to obtain the transfer function. This can be achieved if the phase of the sinusoids ϕ_k are chosen as shown in [15]. The schematic of the frequency scanning procedure to obtain the admittance of the VSC devices is shown in Figure 9.

It should be noted that frequency scanning is useful to



$$Y_{sh}(j\omega) = \begin{bmatrix} Y_{DD}(j\omega) & Y_{DQ}(j\omega) \\ Y_{QD}(j\omega) & Y_{QQ}(j\omega) \end{bmatrix}$$

Figure 9. Frequency Scanning of shunt connected devices

determine a small-signal linearised model of a system about an equilibrium point. It is therefore implicit that the system under consideration should be stable at the equilibrium point where frequency scanning is intended to be performed.