# Filtering of Noisy Frequency Measurements Using Coherence between Multiple Measurements

Kaustav Dey[†] and A. M. Kulkarni[*]
Department of Electrical Engineering
Indian Institute of Technology Bombay
Mumbai, India
Email: [†]kaustavd@iitb.ac.in, [*]anil@ee.iitb.ac.in

***Abstract*—Distributed energy sources that are interfaced to the grid using power electronic converters and are connected to the medium and low voltage network, can potentially play an important role in improving the electro-mechanical stability of a grid. The bus frequency signal is the most convenient local feedback signal that can be used for this purpose. The bus voltage in a distribution system can be significantly distorted; therefore the frequency measurements are noisy and require heavy filtering. In this paper we propose the use of coherence functions of measurements taken at multiple locations to design the parameters (in non real-time) that result in very low phase delay. The filter designed using this method is found to be resilient to sudden spurious data (periodic spikes or glitches). The filtering algorithm has been tested on the real-life distribution system frequency measurements of the Indian grid.**

***Index terms*—Wide Area Measurement System, Distributed Damping Controller, Power System Stability***

## I. Introduction

Electro-mechanical stability can be improved by modulation of the controllable devices in the system. Distributed energy sources, including wind and solar systems [1], storage devices, and controllable loads [2] can also be used for this purpose. The control is facilitated by the power electronic converters which interface these systems to the electrical grid. The use of a large number of distributed controllers (deployed at the medium and low voltage network) is advantageous because they offer controllability of a larger number of electro-mechanical modes, which are observable at various locations in the network to different extents. This has motivated our research in distributed damping controllers across the network.

Modulation of the drawn power in proportion to the frequency deviation is a simple and robust strategy for introducing damping into the network [2], [3]. The frequency signal is also used for various control and protection applications like power-frequency droop control and frequency based load shedding. The bus voltage waveforms are significantly distorted due to harmonics and noise at the distribution level. This may make frequency based modulation control difficult [2], as the signal-to-noise ratio (SNR) may be very low. Noisy frequency measurements, if given to the damping controller can cause unnecessary control activation and can even saturate the actuators. A filter is required to remove noise and higher frequency transients. The output of the filter should be non-responsive to sudden/periodic spikes and glitches. While a simple low pass filter can be used for the filtering, the need for heavy filtering implies that there will be a significant phase shift in the output.

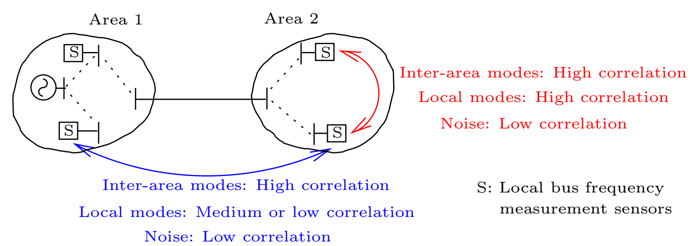



Fig. 1: Correlation of the modes between measurements

In order to avoid the phase-delays, the filtering algorithm should be able to "learn" and "identify" the characteristics of both the noise and the signals of interest, and use this information in a predictive fashion. Since noise tends to be localized, it is possible to isolate it by comparing the signal with the frequency signals at other locations. On the other hand, the electro-mechanical modes are correlated across the different measurements. Frequency measurements taken at distant locations will show high correlation for the inter-area swing modes as well as the common (centre-of-inertia) mode. This is depicted in Fig. 1. It is feasible to obtain the time-synchronized frequency measurements from various locations through local and wide-area measurement systems [4], [5]. The non-local frequency measurements are used only to obtain the parameters of the filtering algorithm and are not used directly in the filtering. The design of the parameters is not a real-time function (adaptation of parameters can be done slowly). Thus, the communication delays in obtaining the non-local signals do not affect the filtering. A schematic block diagram of the filtering algorithm is shown in Fig. 2.

Frequency estimation under the presence of harmonics has been attempted using modified Prony method [6], least mean square error [7] and other methods. The results show that these methods also capture the fast system variations, which reinstates the need for filtering. Note that in contrast to other approaches like filtering using extended Kalman filter [8], this approach is a measurement based model and therefore does not require the entire system model for the filtering. The filtering algorithm has been tested on ambient and ringdown waveforms on the bus frequency measurements of the Indian grid. The filtering is found to improve the SNR and track the frequency measurements accurately without any delay in both ambient

978-1-5386-6159-8/18/$31.00 © 2018 IEEE

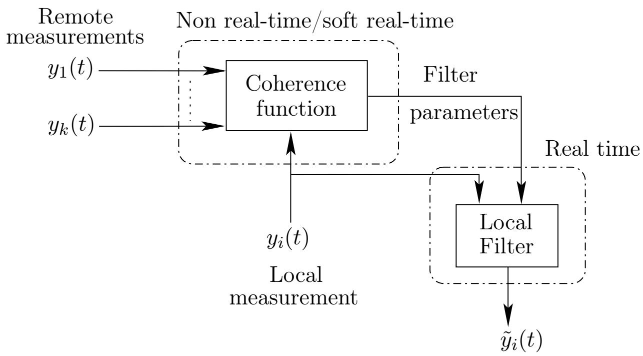



Fig. 2: Schematic diagram of filtering process at $i^{th}$ location

and ringdown measurements.

The paper is organized as follows: The limitation of traditional filters and the understanding of the coherence function is explained in Section II. An algorithm is designed to utilize this filtering scheme, which is shown in Section III. A real-time application in power system requiring such filtering and a reduced-order computation for real-time applications are explained in Section IV. The practical issues to be considered for real-time applications and their remedies are discussed in Section V. Section VI reports the case studies performed on the frequency measurements on the Indian grid and the conclusions are presented in Section VII.

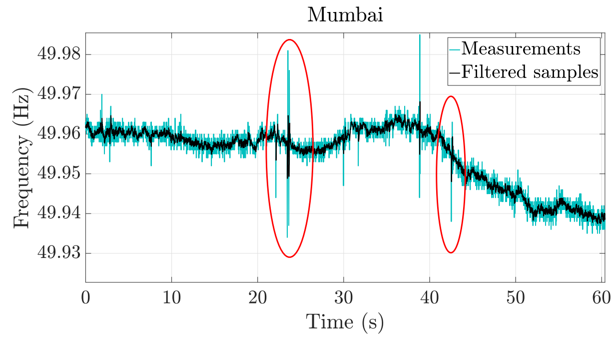
## II. Magnitude squared coherence function

For control applications, it is desired that the phase shift introduced by the filter should be minimal (not more than a few degrees) in the frequency range of interest. A traditional low pass filter (a) will introduce a significant phase shift in the desired frequency range and (b) desired stop-band attenuation cannot be obtained with a lower order. The filtering of noisy bus frequency measurements by using a simple low pass filter is shown here. It is visible in Fig. 3a that the low pass filter is responsive to sudden spikes (circled in red) in the measurements. It can be seen in Fig. 3b that a first order low pass filter with cut-off frequency around 3 Hz introduces a time lag of $\approx$ 70 ms (phase lag of $\approx 25°$ at 1 Hz (swing frequency range), highlighted in red). While the desired stop-band attenuation can be achieved using higher order filters, the increased phase shift will be unsuitable for control applications. Instead of simple low pass filtering, we attempt to identify the correlated frequency components of all the measurements and we allow these frequencies to pass unchanged, while rejecting the uncorrelated components. This does not introduce any phase shift in the filter passband. The coherence function is now discussed.

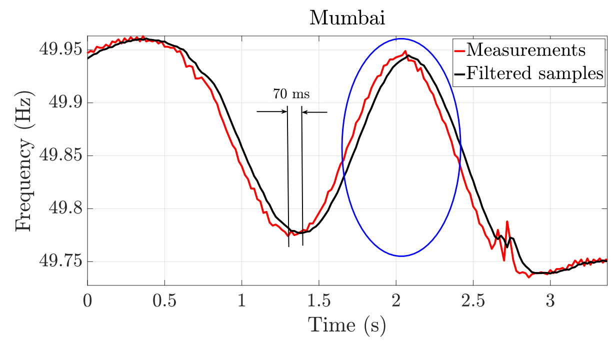
Let us consider that the dynamical system is measured at $k$ locations, which are represented as $y_1$, $y_2$,...., $y_k$. If the true response of the system are $y_{o1}$, $y_{o2}$,.., $y_{ok}$, then

$$y_i(t) = y_{oi}(t) + e_i(t) \quad \text{for } i = 1, ..., k \tag{1}$$

where $e_i(t)$ is the additive white Gaussian noise (measurement error) at the $i^{th}$ sensor ($y_{oi}$) at time $t$. The normalized coherence function [9] between two outputs, say $y_1(t)$ and




(a) Filtering of ambient measurements




(b) Filtering of ringdown measurements

Fig. 3: Filtering of noisy frequency measurements using simple low pass filter

$y_2(t)$, denoted by $C_{y_1y_2}(f)$ at frequency $f$ is defined as

$$C_{y_1y_2}(f) = \frac{|R_{y_1y_2}(f)|^2}{R_{y_1}(f)R_{y_2}(f)} \tag{2}$$

where $R_{y_1y_2}(f)$ is the cross power-spectral density between the signals $y_1(t)$ and $y_2(t)$. $R_{y_1}(f)$ and $R_{y_2}(f)$ are the individual power spectral densities of $y_1(t)$ and $y_2(t)$ respectively, where spectral power density represents the distribution of the power into the frequency components of the signal.

From the spectral-power inequality [9], it can be inferred that $C_{y_1y_2}(f) \in [0, 1]$. It denotes the fraction of power contributed by the correlated components between the outputs at frequency $f$. The remaining fraction is contributed by uncorrelated components (typically white noise). This implies that higher the coherence function at a particular frequency, lesser is the fraction of power contributed by noise.

A coherence function is defined to identify the correlated frequency components at a particular measurement location. The coherence function of a particular output is constructed by checking its coherence with all other outputs. If a particular frequency component is correlated and thus it appears in all measurements, the coherence function should be high between all pairs of measurements at that frequency. The coherence function at location $i$, $(C_{y_i}(f))$ is calculated as given in (3).

$$C_{y_i}(f) = \prod_{j=1}^{k} C_{y_iy_j}(f) \tag{3}$$

A threshold is set to identify the correlated frequency components. An output $y_i$ is considered to be correlated at a frequency $f^*$, if more fraction of power at that frequency is correlated i,e, if $C_{y_i}(f^*) \geq 0.5$. A logical filter $C_{di}(f)$ is calculated from the coherence function as given in (4).

$$C_{di}(f) = \begin{cases} 1, & \text{if } C_{y_i}(f) \geq 0.5 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

The frequency resolution (smallest distinguishable frequency components) for estimation of the coherence function is dependent on the number of data points that are considered to calculate the power spectra of the measurements. If the measurements are sampled at $f_s$ Hz and a frequency resolution of $f_{res}$ Hz is desired, then, the coherence function needs to be evaluated at $n = (\frac{f_s}{2f_{res}} + 1)$ points (including dc component). This necessitates the computation of the two-sided Fourier transform at $r = (2n - 1)$ points, since the zero frequency (dc component) appears only once. We shall now discuss the coherence based filtering algorithm in application to a non-linear system, namely the power system.

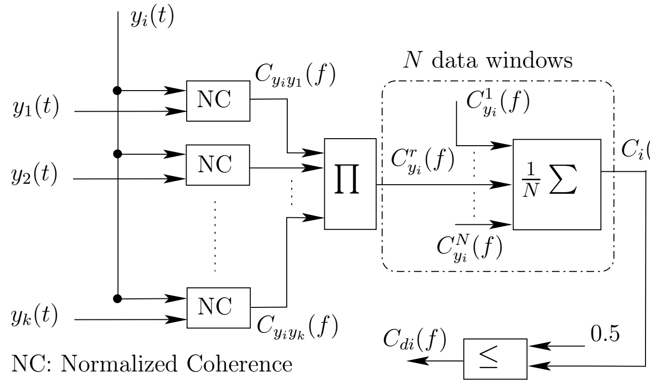
## III. Coherence based filtering algorithm

In multi-machine power systems, the inter-area modes are observable across the entire network. Since these modes are usually very poorly damped, the damping controllers are designed to primarily improve the damping of these modes. However, the local modes are usually not observable in wide-area measurements and therefore, they are not expected to be highly correlated between the wide-area measurements. If any operator wishes to improve the damping of the local modes as well as the inter-area modes, then the filtering strategy (as explained in Fig. 4) can be used either by taking local (nearby) measurements only or select the threshold appropriately such that the local modes (less correlated than the inter-area modes) appears in the passband. The different modes namely inter-area modes, local modes and high frequency modes (typically noise), and their correlation between different sets of measurements across the system are shown in Fig. 1.

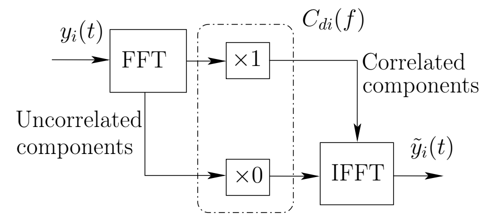
As the power system is essentially non-linear, the frequency of these oscillations are not fixed, but are functions of the equilibrium point. The coherence of the outputs are therefore required to be computed with multiple data windows (past measurements that are available in data archival of system operators) to include the different operating points. The data windows should include both ringdown and ambient conditions, to identify the different frequency components of the system. The coherence function is updated over several windows of data and thereby, the coherent frequency components of the system are identified. If the coherence function of the $i^{th}$ output over all data windows ($N$ data windows are taken here) be denoted by $C_i(f)$, then

$$C_i(f) = \frac{\sum_{r=1}^{N} C_{y_i}^r(f)}{N} \tag{5}$$

where $C_{y_i}^r(f)$ is the coherence function of $y_i$ at frequency $f$ with $r^{th}$ data window. The logical coherence function of $i^{th}$ output, denoted as $C_{di}(f)$ is constructed from $C_i(f)$ using the threshold logic given in (4). $C_{di}(f)$ determines the passband of the Finite Impulse Response (FIR) filter at the $i^{th}$ output. Once the coherence function is decided, the filtering can be performed at individual locations in real-time using local measurements only. A block diagram to determine the filter parameters at a particular location is shown in Fig. 4a.




(a) Identification of coherent components for $i^{th}$ location




(b) Filtering of frequency measurements at $i^{th}$ location

Fig. 4: Block diagram of coherence based filter design and real-time filtering

The temporal measurements are transformed into the frequency domain. The correlated frequencies (identified from the coherence function) are passed unchanged and the uncorrelated frequencies are attenuated using the logical filter $C_{di}(f)$. Since the filtering does not modify any components in the passband, it does not add any phase shift (advantageous for control applications). The modified frequency spectrum is reconstructed to obtain the filtered time domain response. A block diagram representation of the real-time filtering using correlated frequency components is shown in Fig. 4b. It can be seen that the wide-area measurements are required only during the filter design phase (offline computation for the determination of the filter parameters). A practical application of this filtering scheme for grid operation is now discussed.

## IV. Real-time applications in power systems

Filtering of noisy frequency measurements are required in real-time as well as in near real-time applications. We discuss here power oscillation damping control (real-time) as a potential application of this filtering scheme. Control and protection applications like power swing damping controllers, inertial controllers, frequency based load-shedding, rate-of-change-of-frequency (ROCOF) relays etc require local bus frequency and ROCOF measurements. These applications are time-critical and cannot withstand any time delay. To avoid time delay, these applications mostly use local measurements. Due to lack of redundant measurements, noisy and spiky frequency measurements for such applications can cause (i) unwanted spikes at the controller outputs which can saturate the actuators, (ii) cause unnecssary generation/loads trip through $df/dt$ relays. A typical block diagram of distributed damping controller using controlled power injection devices and local frequency measurement is shown in Fig. 5. Since the distributed energy sources are expected to operate close to the maximum power point (MPP), removal of spiky components in the measurements can help it to operate closer to the MPP. Faster variations in the real power injection (control input) can be filtered using coherence based filter. The computation of the filtering process can be reduced for real-time applications, which is discussed next.

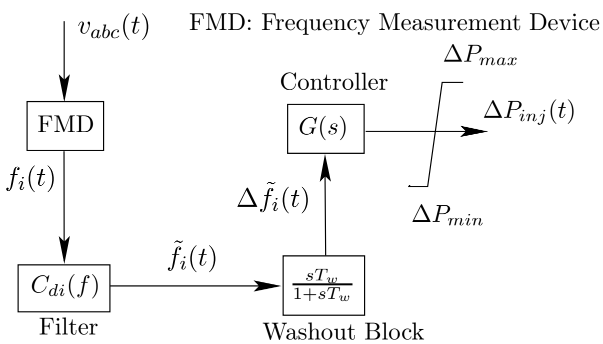



Fig. 5: Distributed damping controller at $i^{th}$ location

For real-time control applications, the computations need to be minimized so that it can be performed almost instantaneously and the output can be given to the controller with minimal time delay. Since Discrete Fourier Transformation (for a fixed sampling rate and for a fixed number of samples in the data window) can be expressed as a linear transformation, the entire filtering process can be reduced to a linear transformation. The overall filtering can be expressed in the form of a FIR filter, whose co-efficients are computed from the coherence function of the measurements (offline computations). The filtered samples can therefore be expressed as linear combinations of the inputs as given in (6).

$$\tilde{y}(t) = \mathcal{F}^H \mathcal{C} \mathcal{F} y(t) \tag{6}$$

Here $\mathcal{F} \in \mathbb{C}^{r\times r}$, $\mathcal{C} \in \mathbb{R}^{r\times r}$ where $r$ represents the length of the data window in terms of the samples. $\mathcal{F}$ denotes the unitary Fourier transform operator and is given as $\mathcal{F}(p,q) = \frac{1}{\sqrt{r}}\exp\left(-j\frac{2\pi pq}{r}\right)$ and $\mathcal{F}^H$ denotes the conjugate transpose of $\mathcal{F}$. $\mathcal{C}$ is the binary coherence filter, which is a diagonal matrix of entries 0 and 1 given as:

$$\mathcal{C}(j,j) = \begin{cases} 1 & \text{if } j^{th} \text{ frequency component is correlated} \\ 0 & \text{else} \end{cases} \tag{7}$$

Since the frequency interval is $[-\frac{f_s}{2}, \frac{f_s}{2}]$, thus $\mathcal{C}$ will have a non-zero entry for a particular frequency (if correlated at that frequency) and for its additive inverse also. It is to be noted that once the correlated frequencies are identified for a particular location, $\mathcal{C}$ is fixed and thus, $\mathcal{F}^H\mathcal{C}\mathcal{F} = \mathcal{A}$ is fixed. For a moving window type application, the final sample only gets appended to the earlier filtered measurements. In every data window, the final sample only needs to be updated. The co-efficients of the FIR filter will be the entries of the last row of $\mathcal{A}$, corresponding to the last sample of the filtered data window. Since $\mathcal{A}^H = \mathcal{A}$, $\mathcal{A}$ is real and therefore, the filter co-efficients are real and the filter is practically realizable. For a data window of $r$ samples, the filtered sample at $k^{th}$ instant ($\tilde{y}[k]$) is given in (8).

$$\tilde{y}[k] = \sum_{i=1}^{r} \mathcal{A}(r,i) y[k-r+i] \tag{8}$$

where $y$ is the vector of raw measurements in the current data window. This reduces the computations to $2r \approx O(r)$ flops ($r$ additions and $r$ multiplications). This reduced computation can be performed in real-time using a DSP processor at the sensor locations and can be given as input to the controller with minimal filtering delay.

## V. Discussions

(1) The main requirement for the filtering algorithm is that it should be non-responsive to sudden glitches and spikes (periodic or aperiodic). Since the sudden spikes at an individual location is expected to have poor correlation across the different outputs, they are not tracked by the coherence filter. However, if glitches appear in all the measurements and have good correlation among them in the filter passband (coordinated false data injection), the filter may fail to identify these spurious components in the measurements.

(2) It has been observed that the filtering gives very satisfactory results in near real-time applications (requiring future data). For a finite length data window, the output data seems to have some spurious oscillations at the beginning and at the end of the window. Thus, for real-time applications, the filter gives unsatisfactory outputs at the current instant. The problem vanishes when the filtering is performed in near real-time, with future measurements. A few second delay in filtering can be tolerated for applications like state estimation and modal estimation but it is not acceptable for control and protection applications. This issue has been resolved by extending the current data window with "approximate" future measurements so that the current sample gets filtered properly. The future samples are extrapolated from the current sample with superposition of a zero mean Gaussian random noise of pre-determined variance. It has been assumed that the system output cannot change very much over a small time, which is consistent with the frequency range of the system transients. The extended samples are computed as given in (9).

$$y_i[k+j] = \frac{y_i[k-1] + y_i[k]}{2} + \epsilon_i[j] \tag{9}$$

where $j = 1,..,m$ for $m$ extrapolated samples, $\epsilon_i[j]$ represents the $j^{th}$ sample of the zero mean Gaussian noise at location $i$.

(3) Once the coherence function is constructed, wide-area measurements (from other locations) are not required for the filter. The filter can be implemented in a dedicated processor with real-time computation capability at the sensor location. This makes the filtering scheme robust to loss of communication and cyber attacks.

(4) The filter can be continuously tuned for better performance. The coherence functions of each location can be updated at a central control centre, using the measurements of all locations. The filter parameters for each location can be updated at regular pre-decided intervals using this scheme.

The updated filter co-efficients can be used to tune the digital controllers regularly for better performance. An updated system information will also help the controllers to operate better under large disturbances which may even split the system into islands. Inertial controllers are very useful to stabilize the frequency transients in an islanded system (lower inertia).

(5) The real-time filtering can also be done by using the non-local measurements. However, for large grids and geographically dispersed sensors, there will be a delay associated with the communication network .The electro-mechanical oscillations also may not be instantaneously correlated at far locations but propagates with a time delay (similar to wave propagation delay in transmission lines). This will reject the observable disturbances at a single location and the controller will not operate. Thus, it is advisable to use the local measurements only for real-time applications. We shall now present some practical illustrations of the filtering alogrithm on both ambient and ringdown measurements on the distribution level frequency measurements of the Indian grid.

## VI. Case Studies

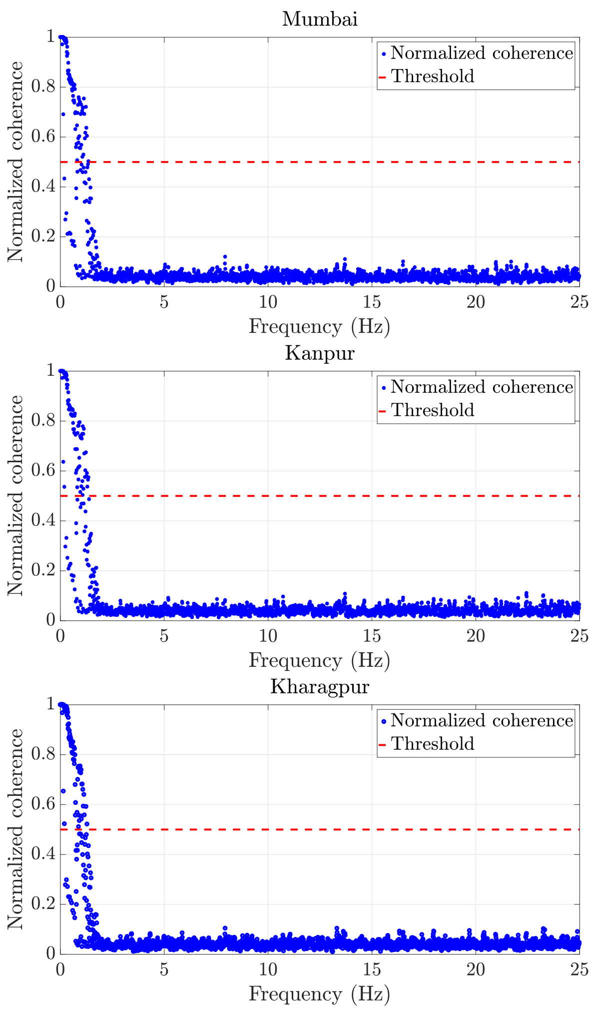



Fig. 6: Magnitude squared coherence at different locations

The filtering scheme is tested on the bus frequency measurements taken at different locations of the Indian power grid. The measurements are obtained from the bus voltages at distribution level using network time synchronized frequency measurement devices (FMD), as proposed in [5]. The measurements are sampled at 50 Hz, time synchronized and are stored in a server at IIT Bombay. As the sampling frequency is 50 Hz, the maximum allowable (Nyquist) frequency is 25 Hz. It is therefore meaningful to evaluate the spectral power densities upto 25 Hz. Three locations namely Mumbai (western region), Kanpur (northern region) and Kharagpur (eastern region) are selected for the calculation of coherence function, as the availability of data is maximum for these locations. Since the locations are geographically far, it is expected that the inter-area modes will only appear to be correlated as the local modes of each location are not expected to be seen elsewhere.

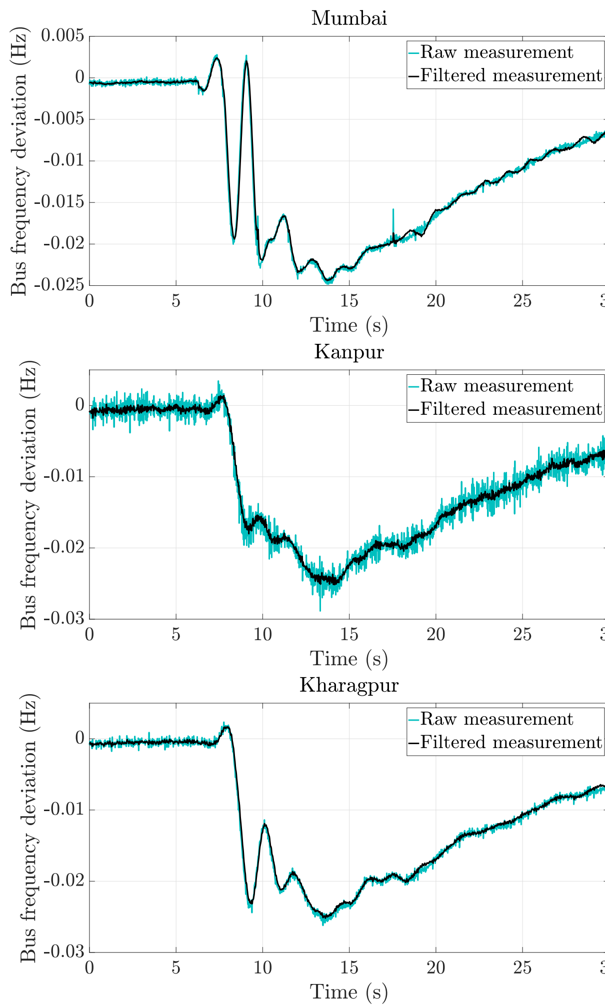



Fig. 7: Frequency deviations following ringdown event (Time window: 7:21:52 PM - 7:22:22 PM on 12.03.2014)

The data window is taken to be of 100 s (5001 data samples) to achieve a frequency resolution of 0.01 Hz, as explained earlier. Several windows of data are taken for these locations and the coherence functions for each location (sensor) are calculated as explained in Section III. The coherence functions for the three locations along with the threshold are shown in Fig. 6. It can be seen that the measurements have significantly correlated components (filter passband) only at low frequencies (upto about 2 Hz), which is consistent with the frequency spectrum of the electro-mechanical transients.

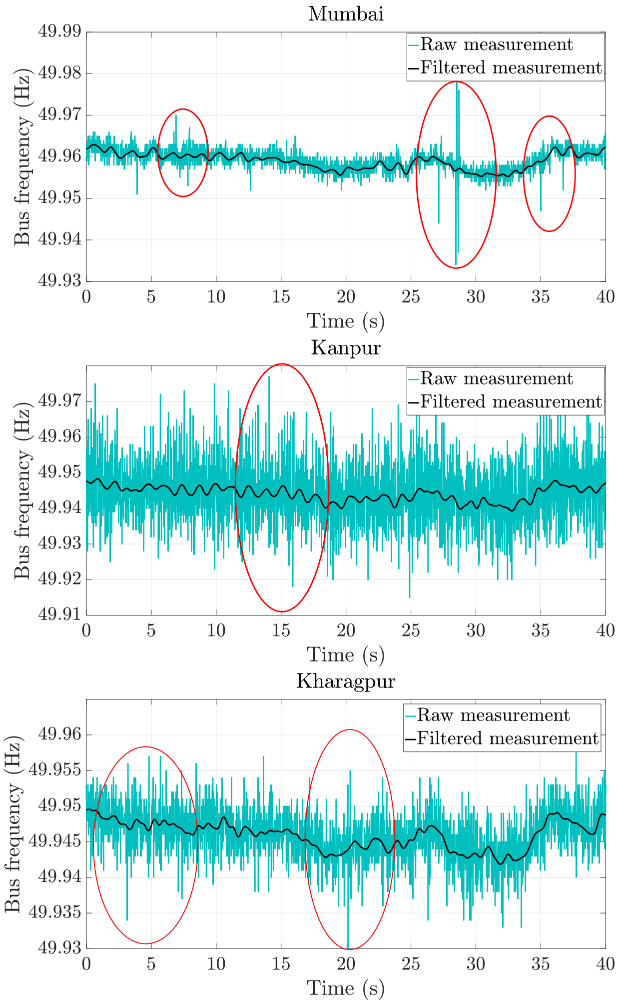



Fig. 8: Frequency measurements in ambient condition (Time window: 7:21:00 PM - 7:21:40 PM on 12.03.2014)

The bus frequency measurements are passed through a washout filter as the distributed damping controllers require the bus frequency deviations as input, as shown in Fig. 5. A washout block with a time constant of 8 s is used so that it does not interfere with the frequency range of interest (0.2 - 2 Hz). The bus frequency deviations at the different locations following a ringdown (a large generation trip at Mundra (western region) at 7:21:59 PM on 12.03.2014, as given in [10]) is shown in Fig. 7. Use of local measurements only for the filtering is beneficial as the waveforms inidicate that the disturbance does not initiate instantaneously at all locations but shows a non-zero propagation delay.

It can be seen that (i) the filtered waveform tracks the original oscillations without any phase shift and (ii) slightly improves the SNR of the measurements, since the SNR of the measurements during ringdown is much higher than ambient condition. The phase alignment of the filtered waveform is very beneficial for the damping controllers, as no additional phase compensation has to be provided. The effect of the coherence based filter on ambient measurements is shown in Fig. 8. It can be seen that the filter improves the SNR significantly here and does not track sudden spikes in the raw measurements (highlighted in red). Periodic spikes can lead to the estimation of well damped spurious modes, thereby not identifying the true modes of the system. Sudden spikes will also cause surges in the real power output of the acutators, thereby shifting them further away from the MPP.

## VII. Conclusion

A filtering scheme for wide-area control and supervision using identification of coherent frequency components between multiple measurements is presented here. The filtering does not introduce any significant phase (time) lag, thereby, making it useful for control and protection applications. The practical issues for real-time implementation are discussed here. A reduced-computational approach for the filtering process has been shown, which is very useful for time-critical applications. Therefore, the filtering delay is negligible compared to the time period of the electro-mechanical oscillations. The filtering scheme is tested on the low voltage bus frequency measurements of the Indian power grid and the results indicate that the filter improves the SNR of the measurements, without introducing any additional phase shift in both ambient and ringdown conditions. The filter is robust to communication loss and is independent of the system operating point, size and topology.

## References


[1] M. Singh, A. Allen, E. Muljadi, and V. Gevorgian, "Oscillation damping: A comparison of wind and photovoltaic power plant capabilities," in *2014 IEEE Symposium on Power Electronics and Machines for Wind and Water Applications*, July 2014, pp. 1–7.

[2] O. Samuelsson, "Load modulation for damping of electro-mechanical oscillations," in *2001 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No.01CH37194)*, vol. 1, Jan 2001, pp. 241–246 vol.1.

[3] U. P. Mhaskar and A. M. Kulkarni, "Power oscillation damping using FACTS devices: modal controllability, observability in local signals, and location of transfer function zeros," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 285–294, Feb 2006.

[4] A. G. Phadke and J. S. Thorp, *Synchronized Phasor Measurements and their Applications*. Springer Science and Business Media, New York, 2008.

[5] K. A. Salunkhe, G. Gajjar, S. A. Soman, and A. M. Kulkarni, "Implementation and applications of a Wide Area Frequency Measurement System synchronized using Network Time Protocol," in *2014 IEEE PES General Meeting — Conference Exposition*, July 2014, pp. 1–5.

[6] T. Lobos, J. Rezmer, and J. Schegner, "Parameter estimation of distorted signals using Prony method," in *2003 IEEE Bologna Power Tech Conference Proceedings,*, vol. 4, 2003, pp. 1–5.

[7] A. K. Pradhan, A. Routray, and A. Basak, "Power system frequency estimation using least mean square technique," *IEEE Transactions on Power Delivery*, vol. 20, no. 3, pp. 1812–1816, July 2005.

[8] B. Li, J. Wang, H. Bao, and H. Zhang, "Islanding detection for microgrid based on frequency tracking using extended Kalman filter algorithm," *Journal of Applied Mathematics*, vol. 2014, 2014.

[9] J. S. Bendat and A. G. Piersol, *Random data: analysis and measurement procedures*, 4th ed. John Wiley & Sons, Hoboken, New Jersey, 2011.

[10] Wide Area Frequency Measurement System.[Online].Available:www.ee.iitb.ac.in/~anil.Accessed: 09-06-2018.