Some examples on domain and range

Note. Here solve exercises 3 and 4 of section 1.1 in class.

The sign of a linear form $y = ax + b$ is determined by the following rule:

		$-\frac{b}{a}$	
y=ax+b	**opposite to the sign of a**		**the same sign as a**

Example. Find the domain of the function $f(t) = \sqrt{\dfrac{-1+2t}{3t-6}}$

Solution.

We must have $\dfrac{-1+2t}{3t-6} \geq 0$ but under the restriction $t \neq 2$ to make sure that the denominator is non-zero. We must determine the signs of both $-1+2t$ and $3t-6$ to be able to determine where the ration $\dfrac{-1+2t}{3t-6}$ is non-negative.

		$\frac{1}{2}$		2	
$-1+2t$	$-$	$\bullet$	$+$		$+$
$3t-6$	$-$		$-$	$\bullet$	$+$
ratio	$+$		$-$		$+$

So the domain on which $\dfrac{-1+2t}{3t-6} \geq 0$ excluding where the denominator is zero is the set

$$D_f = (-\infty\,,\,\frac{1}{2}] \cup (2\,,\,\infty)$$

Example (section 1.1 exercise 31). Find the domain of $f(x) = \dfrac{x+4}{x^2-9}$.

Solution. The points $x = \pm 3$ must be excluded as they make the denominator zero.

$$D_f = \mathbb{R} - \{-3\ ,\ 3\}$$

Example (section 1.1 exercise 32). Find the domain of $f(x) = \dfrac{2x^3-5}{x^2+x-6}$.

Solution. The points satisfying $x^2+x-6=0$ must be excluded as they make the denominator zero.

$$x^2+x-6=0 \quad\Rightarrow\quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} -3 \\ 2 \end{cases}$$

$$D_f = \mathbb{R} - \{-3\ ,\ 2\}$$

Example (section 1.1 exercise 33). Find the domain of $f(t) = \sqrt[3]{2t-1}$.

Solution. As the operator $\sqrt[3]{\cdot}$ can act on all numbers (positive and negative), the value $\sqrt[3]{2t-1}$ can be calculated for all t, so

$$D_f = \mathbb{R}$$

Example (section 1.1 exercise 34). Find the domain of $g(t) = \sqrt{3-t} - \sqrt{2+t}$.

Solution. We must have both $3-t \geq 0$ and $2+t \geq 0$. So, we must have both $3 \geq t$ and $t \geq -2$. So the domain is the interval $[-2\ ,\ 3]$

Example (section 1.1 exercise 35). Find the domain of $h(x) = \dfrac{1}{\sqrt[4]{x^2-5x}}$.

Solution. We must have $x^2 - 5x > 0$. But $x^2 - 5x = x(x-5)$ and

		0		5	
x	$-$	$\bullet$	$+$		$+$
$x-5$	$-$		$-$	$\bullet$	$+$
product	$+$		$-$		$+$

So, the domain is:

$$D_h = (-\infty\,,0) \cup (5\,,\infty)$$

Example (section 1.1 exercise 38). Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4-x^2}$.

Solution.

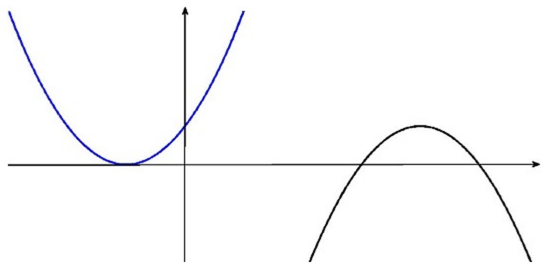
$$5 - x^2 \geq 0 \quad \Leftrightarrow \quad 5 \geq x^2 \quad \overset{\text{square root}}{\Leftrightarrow} \quad \sqrt{5} \geq |x| \quad \Leftrightarrow \quad -\sqrt{5} \leq x \leq \sqrt{5}$$

$$D_h = [-\sqrt{5}\ ,\ \sqrt{5}]$$

By putting $y = \sqrt{4-x^2}$ we get (after squaring) $\begin{cases} y \geq 0 \\ x^2 + y^2 = 4 \end{cases}$. Therefore the points (x,y) are on the upper semicircle described above. So, by projecting on the y-axis we get the range being equal to $R_h = [0\,,2]$.

Example (section 1.1 exercise 40). Find the domain and range of the function $F(x) = x^2 - 2x + 1$.

Solution. There is no restriction on x , therefore the domain is $\mathbb{R}$. As for the range, note that the quadratic functions $f(x) = ax^2 + bx + c$ are either of the following:



By completing the square one can find out about the maximum value or the minimum value of the function from which the range can be determined (more details were given in class). Back to our example:

$$x^2 - 2x + 1 = (x - 1)^2 \geq 0$$

The minimum for this function is zero, so the range is $[0\ ,\ \infty)$.

Example (section 1.1 exercise 41). Find the domain and range of the function $f(t) = 2t + t^2$.

Solution. There is no restriction on t , therefore the domain is $\mathbb{R}$. As for the range, we complete the square to get:

$$2t + t^2 = (t + 1)^2 - 1 \geq -1$$

The minimum for this function is -1, so the range is $[-1\ ,\ \infty)$. ✓

Example. Answer the same question for the quadratic function $g(x) = -3x^2 + 2x - 1$.

Solution. There is no restriction on x , therefore the domain is $\mathbb{R}$. As for the range, we complete the square to get:

$$-3x^2+2x-1=-3(x^2-\frac{2}{3}x)-1=-3\left[(x-\frac{1}{3})^2-\frac{1}{9}\right]-1=-3(x-\frac{1}{3})^2+\frac{3}{9}-1$$

$$=-3(x-\frac{1}{3})^2-\frac{2}{3}\leq-\frac{2}{3} \quad \text{(the maximum value of } -\tfrac{2}{3} \text{ is reached at } x=\tfrac{1}{3}\text{).}$$

The maximum for this function is $-\frac{2}{3}$, so the range is $(-\infty\,,\,-\frac{2}{3}]$. ✓

Here solve exercises 44, 45, and 49 of section 1.1

Here solve exercises 27 to 30 of section 1.1

Here solve exercises 47 to 50 of section 1.3