136.152 Final

Calculus for Management & Social Sciences

April, 2006

PART A: Multiple Choice

[3] 1. The graph of the function f(x) is shown below.



Which of the following statements is **NOT** true?

A. $\lim_{x \to 1} f(x) = 0$ B. $\lim_{x \to \infty} f(x) = \infty$ C. f(1) = 0D. f'(1) does not exist E. f is discontinuous at x = 1 $1 + x^4$

[3] 2. Decide whether the limit $\lim_{x\to\infty} \frac{1+x^4}{x-x^4}$ exists. If it exists, find its value. A. 1 B. -1 C. ∞ D. $-\infty$ E. does not exist

[3] 3. Let
$$f(t) = 2e^{-t^2}$$
. Find $f''(0)$
A. -4 B. -2 C. 1 D. 4 E. none of these

[3] 4. The graph of the function f(x) is shown below.



Which of the following statements is true?

- A. f has a discontinuity
- B. f has a relative maximum
- C. f does not have a relative minimum that coincides with the absolute minimum
- D. f has an inflection point
- E. f has an interval of concave downward

[3] 5. Let
$$f(x,y) = \frac{\sqrt{e^{2x-1} + 10y}}{\log_2 y}$$
. Find the value of $f\left(\frac{1}{2}, 8\right)$.
A. -3 B. 3 C. $\frac{80}{3}$ D. 27 E. none of these

[3] 6. Let
$$f(x, y) = \ln (x^2 + 3y^4)$$
. Find the value of $f_x(1, 1)$.
A. 0 B. $\frac{1}{2}$ C. -2 D. $\frac{7}{2}$ E. none of these

[3] 7. Let
$$f(x, y) = e^{x^2 - y^2} - x^2 + y^2$$
. Find $2f_y(x, y)$.
A. $4y$
B. $-2ye^{x^2 - y^2} + 2y$
C. $-4ye^{x^2 - y^2} + 4y$
D. $-4xye^{x^2 - y^2} + 4xy$
E. none of the above

[3] 8. A manager has found that the cost in dollars of manufacturing x items is given by C(x) = 25x + 150 and the revenue function is given by $R(x) = 60x - x^2$, where x is the number of products manufactured (0 < x < 20). Find the break-even quantity.

A. 5 B. $\frac{95}{12}$ C. $\frac{35}{4}$ D. -30 E. none of these

- [3] 9. The marginal profit of a small fast-food stand is given by $P'(x) = 3x^2 + 20x$, where x is the sales volume in thousands of hamburgers. The "profit" is -\$50 when no hamburgers are sold. Find the profit function P(x).
 - A. $P(x) = x^3 + 10x^2 50$ B. $P(x) = 3x^2 + 10x^2 - 50$ C. $P(x) = x^3 + 10x^2 + 50$
 - D. $P(x) = 3x^2 + 10x^2 + 50$
 - E. none of the above
- [3] 10. The graph of a function f(x) is shown below.



A. -1 B. 0 C. 1 D. 2 E. none of these

PART B: Long Answer

Answer the following questions in the space provided. Show details of your work for full marks.

12. Differentiate the following functions. DO NOT SIMPLIFY.

[3] (a)
$$f(x) = \sqrt{2x} - 2^x + e^{1/x}$$

[4] (b)
$$g(x) = (3x^3 + \ln x)^3$$

[5] (c)
$$h(x) = \frac{\ln(x^5 + \pi^5)}{\sqrt[5]{x + \pi}}$$

[10] 13. Find all the absolute extrema of the function

$$f(x) = (x+2)(x-1)^2$$
 on $[-3,2].$

- [12] 14. A farmer wants to fence an area of 600 square metres in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. What is the minimum length of fencing needed? Justify your answer.
 - 15. A company manufactures and sells the iPod Shuffle. The production manager has determined that the cost and demand functions are given by

$$C(x) = 14x + 110$$
 and $p = 50 - 2x$

where $x \ (x \ge 0)$ is the number of units sold and p is the price per unit of an iPod Shuffle.

- [3] (a) Find the revenue function R(x) as a function of x.
- [2] (b) Find the profit function P(x) as a function of x.
- [5] (c) Find the maximum possible **profit**. Justify your answer.

16. Let

$$f(x) = \frac{-2}{(2-x)(2+x)}, \quad f'(x) = \frac{-4x}{(2-x)^2(2+x)^2} \text{ and } f''(x) = \frac{-4(4+3x^2)}{(2-x)^3(2+x)^3}.$$

[18]

(a) Provide the following requested information about the function f(x) and its graph. GIVE ANSWERS ONLY; do rough work on the attached blank pages. Answer "NONE" for any item that does not exist.

Domain	
Equation(s) of horizontal asymptote(s)	
Equation(s) of vertical asymptote(s)	
Critical number(s)	
Open interval(s) where f is increasing	
Open interval(s) where f is decreasing	
x and y coordinates of all relative maxima	
x and y coordinates of all relative minima	
Open interval(s) where f is concave upward	
Open interval(s) where f is concave downward	
x and y coordinates of all inflection point(s)	

[6] (b) Sketch the graph of the function $f(x) = \frac{-2}{(2-x)(2+x)}$ labelling all asymptote(s), relative extrema, and inflection point(s).



17. Find each of the following indefinite integrals.

[4] (a)
$$\int \left(\sqrt{t} - \frac{9}{t}\right) dt$$

[5] (b) $\int \left(\pi x^2 + e^{-2x} - 1\right) dx$