MATH 1520 Practice Final

Calculus for Management & Social Sciences

May, 2007

PART A/100 marks

[20] 1. Find the derivatives y' for the following [DO NOT SIMPLIFY]

(a) $y = x^{\pi} + \pi^2 + \dfrac{1}{x^4} - \sqrt{x}$

(b) $y = log_2(x^3 - 7x + 1)$

(c) $y = (2^{3x+1})(e^{x^2})$

(d) $y = \dfrac{ln(x^3+1)}{x^2+x}$

[18] 2. Evaluate the following integrals.

(a) $\displaystyle\int (\frac{1}{x^2} + x)dx$

(b) $\displaystyle\int_1^e (\frac{1}{x} + e^x)dx$ (Simplify as much as possible).

(c) $\displaystyle\int (6x^3 - e^{7x})dx$

(d) $\displaystyle\int \sqrt{x}(x^2 + x)dx$

[25] 3. Identify the following for the function below, if one of the properties does not exist, state "none".

$f(x) = \dfrac{x}{x^2+4}$ $\qquad$ $f'(x) = \dfrac{-x^2+4}{(x^2+4)^2}$ $\qquad$ $f''(x) = \dfrac{2x(x^2-12)}{(x^2+4)^3}$

(a) Domain.

(b) x and y intercepts.

(c) Equation(s) of any horizontal asymptote(s).

(d) Coordinates of all critical points.

(e) Interval(s) where the curve is increasing and where it is decreasing.

(f) The x and y coordinates of any relative maximum and/or any relative minimum values. Identify each as a relative maximum and/or relative minimum.

(g) Interval(s) where the curve is concave up and where it is concave down

(h) The x and y coordinates of any inflection points.

(i) Use the above information to sketch $f(x)$.

[7] 4. Sketch a graph that has the following properties:

1) Vertical asymptote at $x = 0$.

2) No x or y intercepts.

3) $f''(x) < 0$ on $(-\infty, 0)$.

4) $f''(x) > 0$ on $(0, \infty)$.

5) $f'(x) > 0$ on $(-\infty, -1) \cup (1, \infty)$.

6) $f'(x) < 0$ on $(-1, 0) \cup (0, 1)$.

7) Relative max of -2 at $x = -1$.

8) Relative min of 2 at $x = 1$.

[8] 5. For $f(x, y) = 2x^3y + \dfrac{x}{y}$, find:

(a) $f_x(x, y)$

(b) $f_y(x, y)$

(c) $f_y(1, 1)$

(d) $f_{xy}(x, y)$

[5] 6. If acceleration (in m/s^2) of an object is given by $x(t) = 2t^2 + t + 1$ for $t \geq 0$, find the velocity of this object after 2 seconds. It is known that the object's velocity is 3 m/s after 1 second.

[7] 7. A <u>closed</u> rectangular box has height twice its width. If we have 24 m^2 of material to work with, what are the dimensions of the box with maximal volume that we could build?

[10] 8. Make a rough sketch of the function $f(x) = x^2 - 1$. Shade the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = 0$ and $x = 2$. Find the area of the shaded region.

PART B/20 marks (2 marks per question)

[2] 9. (a) What is the domain of $\ln(x^2+1)$?

(b) If the cost function for x shirts is $C(x) = 3x^2 + 6$ dollars and the selling price per shirt is \$10 per shirt, find:

[2] i. The revenue function $R(x)$.

[2] ii. Profit function $P(x)$.

[2] iii. The marginal profit function.

[2] (c) Find $\lim_{x\to\infty} \dfrac{x^7+12x-1}{7x^7+3x^3+9}$.

[2] (d) Re-write the expression $6e^{\frac{\ln 8}{3}t}$ to contain no logarithms.

[2] (e) Find $f(4,3)$ if $f(x) = \sqrt{x^2+y^2}$.

(f) If $f(x) = 3x+1$ and $g(x) = x^2 + x$, find a simplified expression for

[2] i. $g(f(x))$.

[2] ii. $f(g(x))$.

[2] iii. $g(f(1))$.