MATH 1520 **Quiz 3** **February 27, 2008**

1. Find y' for the following functions. [Do *not* simplify your answers.]

[2] (a) $y = 3x^{1/3} - 2x^{1/2}$

Solution:

$$y' = 3\left(\frac{1}{3}\right)x^{1/3-1} - 2\left(\right)x^{1/2-1}$$
$$= \underbrace{x^{-2/3}}_{[1]} - \underbrace{x^{-1/2}}_{[1]}$$

[2] (b) $y = 4x^5 - \dfrac{2}{x^2} + \sqrt{x}$

Solution:

$$y = 4x^5 - 2x^{-2} + x^{1/2}$$
$$y' = 4(5)x^{5-1} - 2(-2)x^{-2-1} + \frac{1}{2}x^{1/2-1}$$
$$y' = \underbrace{20x^4}_{[1]} + \underbrace{4x^{-3} + \frac{1}{2}x^{-1/2}}_{[1]}$$

[3] (c) $y = (x^3 + x)(x + e)$

Solution:

$$y = f(x)g(x) \qquad y' = f'(x)g(x) + f(x)g'(x) \qquad [1]$$
$$y = \underbrace{\left(3x^2 1\right)(x + e)}_{[1]} + \underbrace{\left(x^3 + x\right)(1)}_{[1]}$$

Students need not write the rule, but give one mark for the pattern of the derivative.

[3] (d) $y = (2x^2 + e)\left(\sqrt{x} + \dfrac{1}{x}\right)$

Solution:

$$y = f(x)g(x) \qquad y' = f'(x)g(x) + f(x)g'(x) \qquad [1]$$
$$y = \underbrace{(4x)\left(\sqrt{x} + \frac{1}{x}\right)}_{[1]} + \underbrace{\left(2x^2 + e\right)\left(\frac{1}{2}x^{-1/2} - x^{-2}\right)}_{[1]}$$

Students need not write the rule, but give one mark for the pattern of the derivative.

[4] (e) $y = \dfrac{\sqrt{x}}{x^3+1}$

Solution:

$$y = \frac{f(x)}{g(x)} \qquad y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad [1]$$

$$y' = \frac{\overbrace{\left(\frac{1}{2}x^{-1/2}\right)\left(x^3+1\right)}^{[1]} - \overbrace{\left(x^{1/2}\right)\left(3x^2\right)}^{[1]}}{\underbrace{\left(x^3+1\right)^2}_{[1]}}$$

Students need not write the rule, but give one mark for the pattern of the derivative.

[4] (f) $y = \dfrac{4x^3-6x}{3x-5}$

Solution:

$$y = \frac{f(x)}{g(x)} \qquad y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad [1]$$

$$y' = \frac{\overbrace{\left(12x^2-6\right)\left(3x-5\right)}^{[1]} - \overbrace{\left(4x^3-6x\right)\left(3\right)}^{[1]}}{\underbrace{\left(3x-5\right)^2}_{[1]}}$$

Students need not write the rule, but give one mark for the pattern of the derivative.

[4] 2. Let $f(x) = x^2 + 2x$. Write the equation of the line tangent to f at the point where $x = 1$.

Solution:

$$\begin{aligned} f'(x) &= 2x+2 && [1] \\ m = f'(1) &= 2(1)+2 = 4 && [1] \\ y_0 = f(1) &= (1)^2 + 2(1) = 3 && [1] \\ y - y_0 &= m\,(x - x_0) && \\ y - 3 &= 4\,(x-1) && [1] \end{aligned}$$

[4] 3. Let $f(x) = x^3 + 3x^2 + 1$. Write the equation of the line tangent to f at the point where $x = 1$.

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x && [1] \\ m = f'(1) &= 3(1)^2 + 6(1) = 9 && [1] \\ y_0 = f(1) &= (1)^3 + 3(1)^2 + 1 = 5 && [1] \\ y - y_0 &= m\,(x - x_0) \\ y - 5 &= 9\,(x - 1) && [1] \end{aligned}$$

[7] 4. Let $f(x) = \sqrt{1-x}$. Using *only* the definition of the derivative, find $f'(x)$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} && [1] \\ &= \lim_{h\to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h} && [1] \\ &= \lim_{h\to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h} \times \frac{\sqrt{1-x-h} + \sqrt{1-x}}{\sqrt{1-x-h} + \sqrt{1-x}} && [1] \\ &= \lim_{h\to 0} \frac{(1-x-h) - (1-x)}{h\left(\sqrt{1-x-h} + \sqrt{1-x}\right)} && [1] \\ &= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{1-x-h} + \sqrt{1-x}\right)} && [1] \\ &= \lim_{h\to 0} \frac{-1}{\sqrt{1-x-h} + \sqrt{1-x}} && [1] \\ &= \frac{-1}{\sqrt{1-x-0} + \sqrt{1-x}} = \frac{-1}{2\sqrt{1-x}} && [1] \end{aligned}$$

[7] 5. Let $f(x) = \sqrt{2+3x}$. Using *only* the definition of the derivative, find $f'(x)$.

Solution:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} && [1] \\
&= \lim_{h\to 0} \frac{\sqrt{2+3(x+h)}-\sqrt{2+3x}}{h} && [1] \\
&= \lim_{h\to 0} \frac{\sqrt{2+3(x+h)}-\sqrt{2+3x}}{h} \times \frac{\sqrt{2+3x+3h}+\sqrt{2+3x}}{\sqrt{2+3x+3h}+\sqrt{2+3x}} && [1] \\
&= \lim_{h\to 0} \frac{(2+3x+3h)-(2+3x)}{h\left(\sqrt{2+3x+3h}+\sqrt{2+3x}\right)} && [1] \\
&= \lim_{h\to 0} \frac{3h}{h\left(\sqrt{2+3x+3h}=\sqrt{2+3x}\right)} && [1] \\
&= \lim_{h\to 0} \frac{3}{\sqrt{2+3x+3h}=\sqrt{2+3x}} && [1] \\
&= \frac{-1}{\sqrt{2+3x+0}=\sqrt{2+3x}} = \frac{-1}{2\sqrt{2+3x}} && [1]
\end{aligned}
$$