Implementation of single-port admittance in emtp type program

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Main Reference:
EMTP Theory Book,
Hermann W. Dommel, Second Edition,
Microtran Power System Analysis Corporation, Vancouver, BC, April 1996.

In frequency domain

$$I(s) = Y(s)V(s) \tag{1}$$

$$s = j\omega$$

(1) What is the time domain form of (1)?

In theory, the product of two frequency domain functions $(Y(s)\ \&V(s))$ becomes convolution in time domain.

$$i(t) = y(t) * v(t) \tag{2}$$

Note that, lower case letters represent time domain form of corresponding upper case frequency domain functions and "*" denotes convolution.

(2) How do we evaluate the convolution?

One of the typical problems in time domain modelling is to evaluate the time domain convolution. The **recursive convolution** is a numerically efficient technique used to compute convolutions.

(3) Requirements for **Recursive Convolution?**

Either y(t) or v(t) must be expressed as a summation of exponentials.

$$y(t) \; or \; v(t) = \sum_{i=1}^{n} c_i e^{a_i t} \tag{3}$$

(4) Frequency domain form of (3)?

$$V(s) \; or \; Y(s) = \sum_{i=1}^{n} \frac{c_i}{s - a_i} \tag{4}$$

Assume one of the frequency domain functions in (1) can be written in the pole residue form (**rational function**) as in (4), so that in time domain, this function can be expressed as a summation of exponentials (see equation 3).

(5) Finding the rational form for the admittance function

If the physical network is known (e.g. consisting of capacitance, inductance, resistance), then admittance function can be calculated.

Sometimes, the admittance function can be calculated (or available) as a function of frequency. In such cases, a technique like Vector Fitting can be used to find the rational form (4).

<u>(6) Recursive convolution</u>

$$i(t) = y(t) * v(t) \tag{5}$$

AND

$$y(t) = e^{a_i t} \tag{6}$$

The convolution can be written in the integral form,

$$i(t) = \int_0^{\infty} v(t-u) e^{au} du \tag{7}$$

With discrete time domain with time step Δt, above integration can be written as,

$$i(t) = \int_{0}^{\Delta t} v(t-u)e^{au}du + \int_{\Delta t}^{\infty} v(t-u)e^{au}du \tag{8}$$

$$= \int_{0}^{\Delta t} v(t-u)e^{au}du + e^{a(\Delta t)}i(t-\Delta t)$$

Assume that *v(t-u)* behaves linearly between *u = 0* and *u = dt*

$$v(t-u) = \left[\frac{v(t-\Delta t)-v(t)}{\Delta t}\right]u + v(t) \tag{9}$$

$$i(t) = \int_0^{\Delta t} \left[\left\{ \frac{v(t-\Delta t) - v(t)}{\Delta t} \right\} u + v(t) \right] e^{au} du \tag{10}$$
$$+ e^{a(\Delta t)} i(t - \Delta t)$$

Then discrete form of $i(t)$ is shown in (11) in terms of past values of $i(t)$ and $v(t)$, and the present value of $v(t)$.

$$i(t) = k_1 v(t) + k_2 v(t-\Delta t) + k_3 i(t - \Delta t) \tag{11}$$

The constants k_1, k_2, k_3 in (11) are,

$$k_1 = -\frac{1}{a}\left(1 + \frac{1 - e^{a\Delta t}}{a\Delta t}\right) \tag{12}$$
$$k_2 = \frac{1}{a}\left(e^{a\Delta t} + \frac{1 - e^{a\Delta t}}{a\Delta t}\right)$$
$$k_3 = e^{a\Delta t}$$

In time domain, equation 11 represents a time domain equivalent circuit.

Stability of the Recursive Convolution

The BIBO stability of any discrete recursive formula can be predicted using Z transform. As for any transfer function, the poles of the denominator should be within unit circle for stability. Lets assume the delay is approximately an integer multiple of time step (as required for discrete simulations). After applying Z transform,

$$I(z) = k_1 V(z) + k_2 z^{-1} V(z) + k_3 z^{-1} I(z) \tag{13}$$

Then (13) becomes,

$$\frac{I(z)}{V(z)} = \frac{(zk_1 + k_2)}{z - k_3} \tag{14}$$

The stability is guaranteed, if k_3 lies inside a unit circle in the complex plane. For recursive convolution, $k_3 = e^{a\Delta t}$ and for stable poles $(a < 0)$, $k_3 < 1$ and the simulation is stable (BIBO).

(7) Find equivalent circuit if the admittance of a network is expressed in a general form

$$Y(s) = \sum_{i=1}^{n} \frac{c_i}{s - a_i} + d \tag{15}$$

Time domain form

$$y(t) = \sum_{i=1}^{n} c_i e^{a_i t} + d\delta(t) \tag{16}$$

The convolution $i(t) = y(t) * v(t)$ now becomes,

$$i(t) = y(t) * v(t) = \sum_{i=1}^{n} s_i(t) + dv(t) \tag{17}$$

where,

$$s_i\left(t\right) = k_1{}^i v(t) + k_2{}^i v(t-\Delta t) + k_3{}^i s_i(t-\Delta t) \quad (18)$$

$$i(t) = y_{eq} v\left(t\right) + hist(t-\Delta t) \quad (19)$$

Multi-port system

$$I(s) = Y(s)V(s) = \begin{bmatrix} Y_1(s) & Y_2(s) \\ Y_3(s) & Y_4(s) \end{bmatrix} \begin{bmatrix} V_1\left(s\right) \\ V_2\left(s\right) \end{bmatrix} \quad (20)$$

$$y(t) * v(t) = \begin{bmatrix} y_1(t) & y_2(t) \\ y_3(t) & y_4(t) \end{bmatrix} * \begin{bmatrix} v_1\left(t\right) \\ v_2\left(t\right) \end{bmatrix} \quad (21)$$

$$= \begin{bmatrix} y_1(t) * v_1(t) + y_2(t) * v_2(t) \\ y_3(t) * v_1(t) + y_4(t) * v_2(t) \end{bmatrix} \tag{22}$$

$$= \begin{bmatrix} y_{eq}^{1} & y_{eq}^{2} \\ y_{eq}^{3} & y_{eq}^{4} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

$$+ \begin{bmatrix} hist^1(t-\Delta t) + hist^2(t-\Delta t) \\ hist^3(t-\Delta t) + hist^4(t-\Delta t) \end{bmatrix}$$

Example

$$I(s) = Y(s)V(s)$$

$$Y(s) = 3/(s+2)$$

$$v(t) = u(t)$$

$$find \quad i(t)?$$

Theoretical solution

$$I(s) = \frac{1.5}{s} - \frac{1.5}{(s+2)}$$

$$i(t) = 1.5u(t) - 1.5e^{-2t}$$

Discrete solution,

$$i(t) = y(t) * u(t)$$

$$i(t) = k_1 u(t) + k_2 u(t - \Delta t) + k_3 i(t - \Delta t)$$

$$k_1 = -\frac{c}{a}\left(1 + \frac{1 - e^{a\Delta t}}{a\Delta t}\right)$$

$$k_2 = \frac{c}{a}\left(e^{a\Delta t} + \frac{1 - e^{a\Delta t}}{a\Delta t}\right)$$

$$k_3 = e^{a\Delta t}$$

$$i(t) = 0.8515u(t) + 0.4455u(t - \Delta t) + 0.1353i(t - \Delta t)$$