

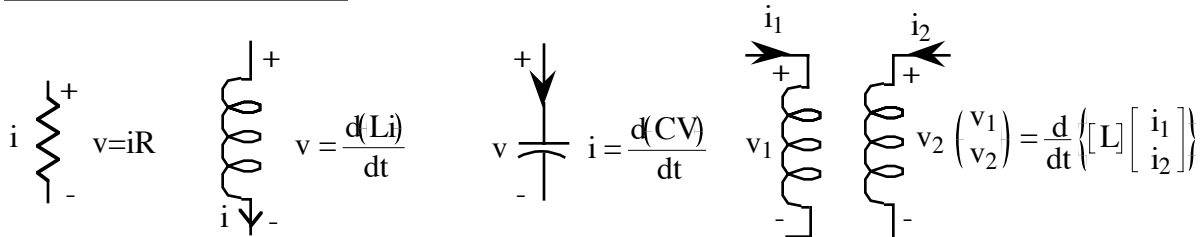
TRANSIENT SIMULATION: LECTURE I

Methods For Formulation Of Equations For Lumped Linear Networks

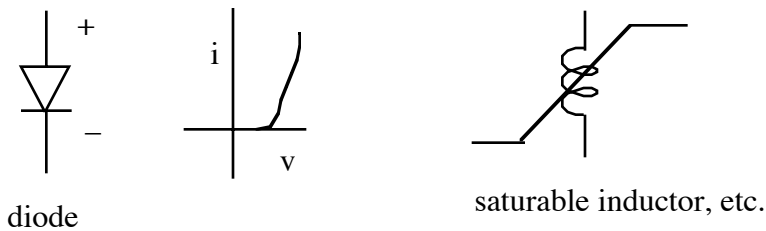
1.1 Examples of Power System Components

Shown below are some examples of typical power system components

Linear (Lumped Elements)

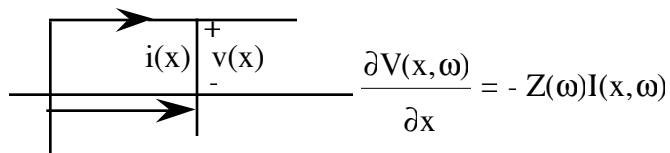


Nonlinear Elements:



Linear Distributed Elements:

Transmission Line, etc.



1.2 Circuits with Linear Elements

In this section we consider the modeling of lumped linear passive elements such as in the circuit shown below:

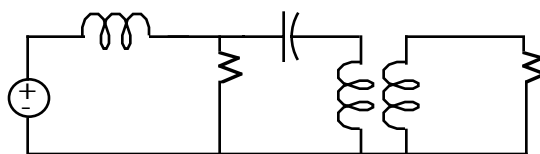


Fig. 1.1 Typical lumped, linear circuit.

How do we write the equations for these?

a) By inspection

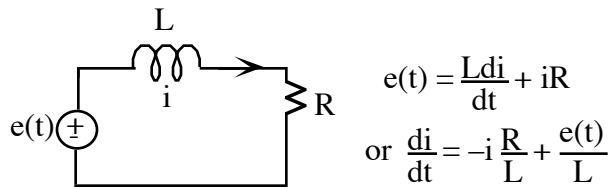


Fig. 1.2. Simple R-L Circuit

and we solve by analytic or numerical methods.

b) By systematic approaches like the state variable approach [1] which result in equations like:

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{u}(t) \quad (\text{or } = \underline{f}(\underline{x}, u) \text{ for the general case})$$

\downarrow \swarrow \downarrow
 state variable input variable

Advantages:

- minimal set of variables to be solved for
- automatic generation of equations is possible
- a large body of mathematics dealing with stability, etc. is available
- $\underline{x}(t)$ is a continuous function of time if $\underline{f}(\underline{x}, u)$ is finite. An important factor in numerical integration
- more later.

c) By transformation of the problem into a purely resistive circuit problem with the help of trapezoidal integration and Admittance Matrices.

1.3 Admittance Matrix Solutions for Resistive Networks (Note Voltage Method) [1]

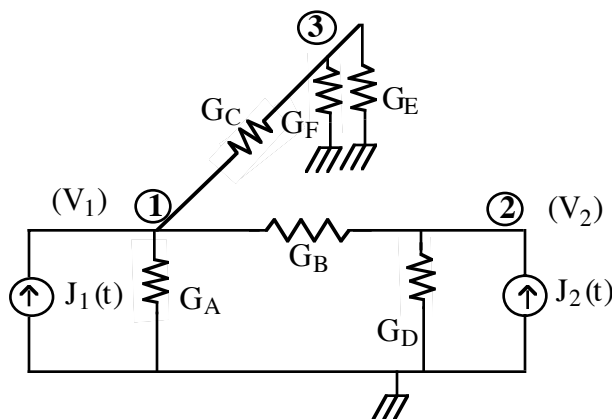


Fig. 1.3. Typical resistive circuit.

KCL at node 1 gives:

$$v_1 \cdot G_A + (v_1 - v_2) G_B + (v_1 - v_3) \cdot G_C = +J_1(t)$$

or

$$v_1 (G_A + G_B + G_C) - v_2 \cdot G_B - v_3 \cdot G_C = J_1(t)$$

or v_1 [sum of all conductances incident at node 1]

$$+ v_2 [-\text{conductance between 1 \& 2}]$$

$$+ v_3 [-\text{conductance between 2 \& 3}]$$

$$= J_1(t) = \text{current source incident at node 1}$$

Doing this for the other two nodes,

$$\begin{bmatrix} G_A + G_B + G_C & -G_B & -G_C \\ -G_B & G_B + G_D & 0 \\ -G_C & 0 & G_F + G_C + G_E \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} J_1(t) \\ J_2(t) \\ 0 \end{bmatrix} \quad (1)$$

$$\text{or } \mathbf{Y} \mathbf{v} = \mathbf{J} \quad (2)$$

we can automatically form the admittance matrix equation:

$$\mathbf{Y} \mathbf{v} = \mathbf{J}$$

where $\mathbf{v} = [v_1, v_2, \dots, v_n]^T$, $\mathbf{J} = [J_1, J_2, \dots, J_n]^T$

and $\mathbf{Y} = [y_{ij}]_{i=1, n}^{j=1, n}$

$$\text{where } y_{ii} = \sum_{j=1, n} g_{ij} \quad (2)$$

$$\text{and } y_{ij} \underset{(i \neq j)}{=} -g_{ij}$$

where g_{ij} is the conductance between nodes i and j ,
and g_{ii} is the conductance from node i to ground.

We can then solve for the voltages \mathbf{v} as

$$\mathbf{v} = [\mathbf{Y}]^{-1} \mathbf{J} \quad (3)$$

Properties of $[\mathbf{Y}]$

- | | |
|--|----------------------------|
| $\left. \begin{array}{l} \text{Y is symmetric } (y_{ij} = y_{ji}) \\ \text{Y may be quite sparse, for example} \\ \text{if there is no connection between} \\ \text{nodes } i \& j, y_{ij} = 0 \end{array} \right\}$ | storage implications |
| $\left. \begin{array}{l} \text{If J changes (say } J = J(t)\text{), we need not} \\ \text{recalculate } Y^{-1}\text{, because Y remains} \\ \text{unchanged} \end{array} \right\}$ | computational implications |

NOTE: We need not explicitly calculate Y^{-1} , we could, for example, use the method of LU factorization [2] where Y can be partitioned into an upper and lower triangular matrix:

$$\text{or } \mathbf{L} \mathbf{v}' = \mathbf{J} \quad \text{where } \mathbf{v}' = \mathbf{U} \mathbf{v}$$

and solve for $v'_n = \frac{J_1}{u_{11}}$

$$v'_k = \frac{J_k - L_{k1} V'_1 \dots L_{k,k-1} V'_{k-1}}{L_{kk}}, k > 1$$

and after \underline{v}' is determined, we similarly solve for $U \ v = v'$ by back substitution.

Again, once the LU factorization is obtained, we don't have to re-obtain it unless the network changes.

Example 1.

Write the admittance matrix form equation for the circuit below and solve:

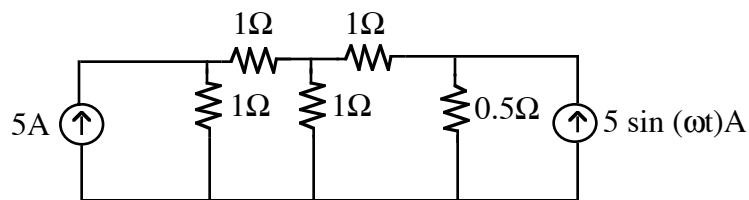
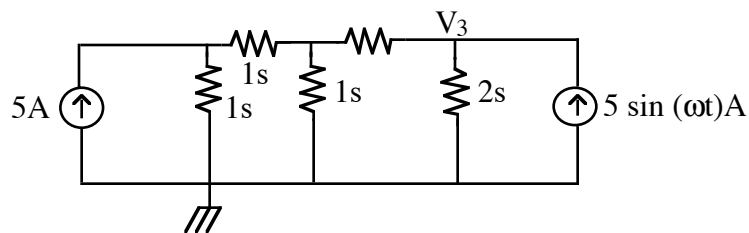


Fig. 1.3. Circuit for Example 1.

Select an appropriate node numbering scheme:



Thus:
$$\begin{bmatrix} 1+1 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 1+2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5A \\ 0A \\ 5 \sin(\omega t)A \end{bmatrix}$$

or
$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 5 \sin(\omega t) \end{bmatrix} A$$

or
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.615 & 0.231 & 0.077 \\ 0.231 & 0.462 & 0.154 \\ 0.077 & 0.154 & 0.385 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \sin(\omega t) \end{bmatrix} 1$$

or $v_1(t) = 3.075 + 0.315 \sin \omega t \text{ V}$

$v_2(t) = 1.155 + 0.77 \sin \omega t \text{ V}$

$v_3(t) = 0.315 + 1.925 \sin \omega t \text{ V}$

1.4 Admittance Matrix Formulations (due to Dommel) for Dynamic Networks [3]

Inductance

Shown below is a schematic diagram and the differential equation governing the voltage-current relationship for an inductor.

$$\begin{array}{c}
 i \\
 + \\
 \text{L} \quad \text{v} \quad v = \frac{L di}{dt} \\
 -
 \end{array}$$

$$i(t) = \frac{1}{L} \int_{\tau=0}^t v \, d\tau + i(0)$$

Solution Using the Trapezoidal Rule

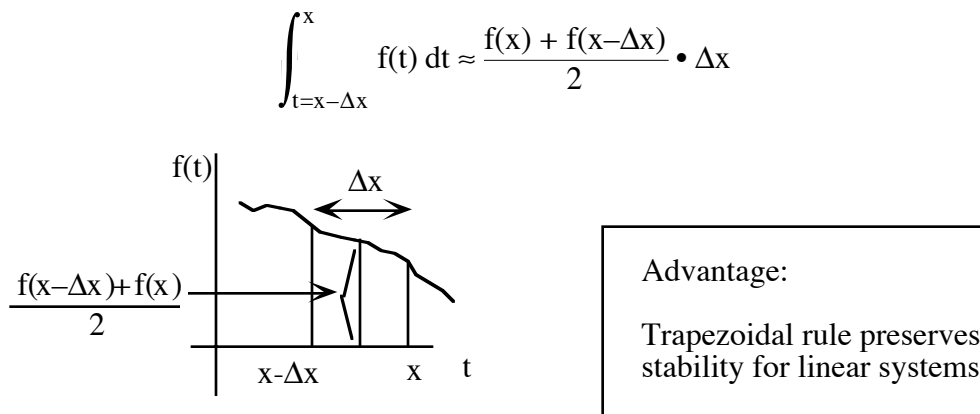


Fig. 1.4. Illustration of trapezoidal rule.

Thus:

$$\begin{aligned}
 i &= \frac{1}{L} \int v \, d\tau \quad \text{or} \quad i(t) = i(t-\Delta t) + \frac{1}{L} \int_{t-\Delta t}^t v(\tau) \, d(\tau) \\
 &\approx i(t-\Delta t) + \frac{1}{L} \left\{ \frac{v(t) + v(t-\Delta t)}{2} \right\} \cdot \Delta t
 \end{aligned}$$

or

$$i(t) = \left(\frac{\Delta t}{2L} \right) v(t) + \left\{ i(t-\Delta t) + \frac{\Delta t}{2L} v(t-\Delta t) \right\} \quad (4)$$

or

$$i(t) = g_L \cdot v(t) + I_L(t-\Delta t) \quad (5)$$

where $g_L = \frac{\Delta t}{2L}$, $I_L = \underbrace{i(t-\Delta t) + g_L v(t-\Delta t)}_{\text{'HISTORY TERM'}}$

Note the two components of $i(t)$, one which depends on present voltage and the other a 'history term' dependent on past conditions.

Eqn. 5 has an equivalent network representation as

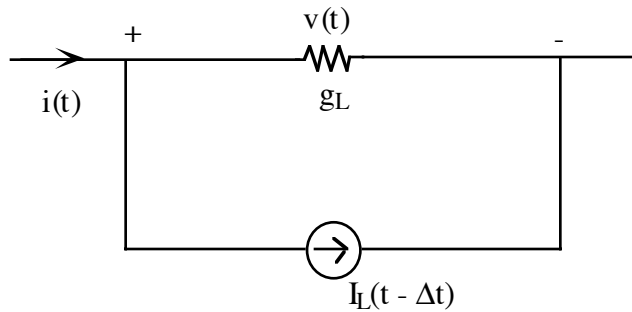


Fig. 1.4. Equivalent circuit for inductor valid during a timestep.

This model can help us to solve the dynamic equation at time t . Note $I_L(t-\Delta t)$ is based on past information (history) and is thus known at time t .

Capacitance: A similar derivation exists for the capacitor:

$$V = \frac{1}{C} \int idz$$

or

$$v(t) = \frac{1}{C} \left[\frac{i(t) + i(t-\Delta t)}{2} \right] \Delta t + v(t-\Delta t)$$

or

$$i(t) = g_c v(t) + I_C(t-\Delta t) \quad (6)$$

where $g_c = \frac{2C}{\Delta t}$, $I_C = \underbrace{-i(t-\Delta t) - g_c v(t-\Delta t)}_{\text{'HISTORY TERM'}}$

or

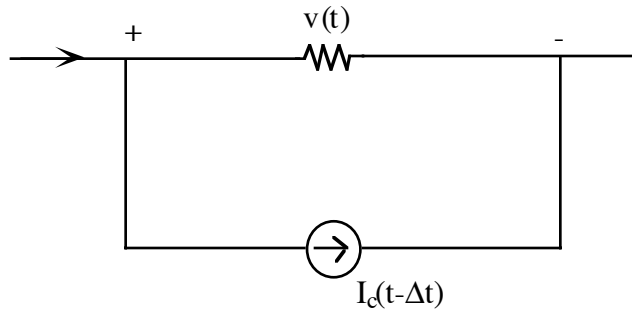


Fig. 1.5: Equivalent Circuit for Capacitor

Example (2):

Calculate $v_1(t)$ in the circuit below (Use $\Delta t = 0.01$ s):

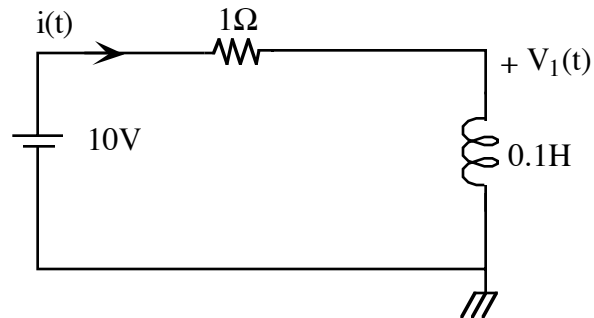
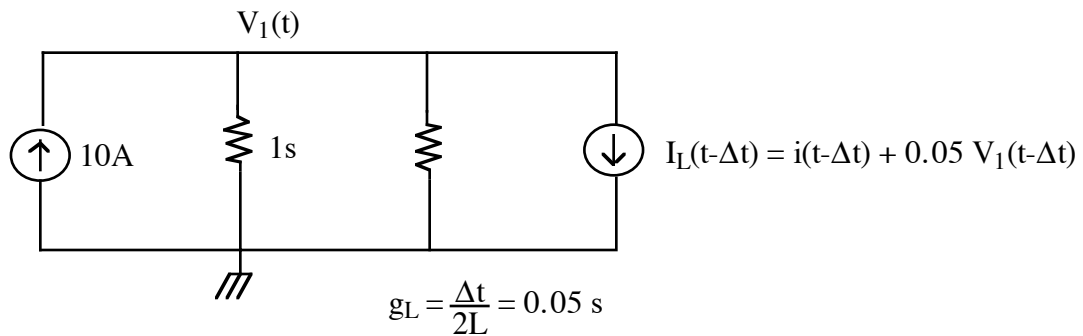
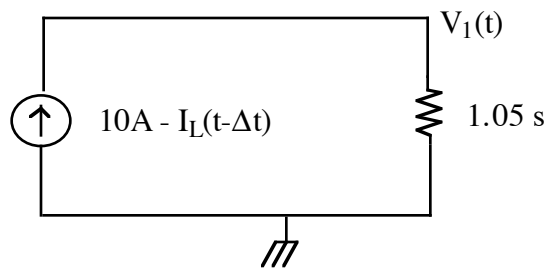


Fig. 1.6: Simple R-L Circuit

Represent all network elements as current sources and conductances.



or:



thus: $v_1(t) = [10A - I_L(t-\Delta t)] \cdot 1.05\text{ V}$

or $i(t) = g_L \cdot v_1(t) + I_L(t-\Delta t)\text{ A}$
(in inductor)

The code written below in FORTRAN models the given situation.

Fig. 1.7 shows the resulting waveforms. Also plotted for comparison is the theoretical solution $v_1(t) = 10.0e^{-Rt/L}\text{ V}$, $i(t) = 10.0(1 - e^{-Rt/L})\text{ A}$. Solutions for three different values of Δt are shown.

In a similar manner, it's possible to represent most network elements current source-conductance equivalents, where the current sources are functions of the past history of the network. Thus any general network reduces to one with current sources and conductances, for which the admittance matrix equations are readily written, resulting in an equation of the form:

$[Y]\underline{V} = \underline{J} + \underline{I}(t-\Delta t)$, where \underline{J} is the vector of external (real) current sources, whereas $\underline{I}(t-\Delta t)$ are the (fictitious) history sources introduced in the algorithm. This equation is then readily solved as:

$$\underline{V} = [Y]^{-1} [\underline{J} + \underline{I}(t-\Delta t)]$$


```

C*****
C Solution of Simple ckt by Admittance methods:
C*****
      real delT,R,L,fintim,prtime,v1,I1,i1,v1t,ilt
C
C -----
C open datafile for reading and output file for answer:
      open(unit=10,file='inp_file')
      open(unit=11,file='out_file')
      write(11,*) 'This is the solution of a simple R-L circuit:'
C
C -----
C Reading the datafile:
C
      read (10,*) delT,fintim,prtime
      read(10,*)R,L
C
C -----
C Initialization
      gr=1/R
      gl= delT/(2*L)
      v1=10.0
      i1=0.0
      v1t = 10.0
      ilt = 0.0
      time=0.0
      write(11,*) time, v1, i1, v1t, ilt
C
C -----
C Setting the loop on time:
      nstep=fintim/delT
      nprt = prtime/delT
C
C -----
C The numerical solution:
C
      Do 1 J=1,nstep+1
C Calculating the history term:
      I1 = gl*v1 + i1
C (Note: at this time the values of v1 and I1 on the right hand side
C are from thge previous timestep (t-delt) as the soln for time=t has
C as yet not been found!)
C
C -----
C Now the solution:
C
      v1 = (10 - I1 )/ (gr +gl)
      i1= gl*v1 + I1
C
C -----
C Updating time:
C
      time=time+delT
C
C -----
C The theoretical Solution for comparison:
      v1t = 10.0 * exp(-R/L*time )
      ilt = 10.0/R*(1.0 - exp(-R/L*time) )
C Writing the soln. to an output file:
C
      if(mod(J-1,npnt).eq.0) write(11,*) time, v1, i1, v1t, ilt

1      continue
C.....Now loop back for the next timestep.
C
C -----
      write(6,*)'End of Simulation!'
      stop
      end

```

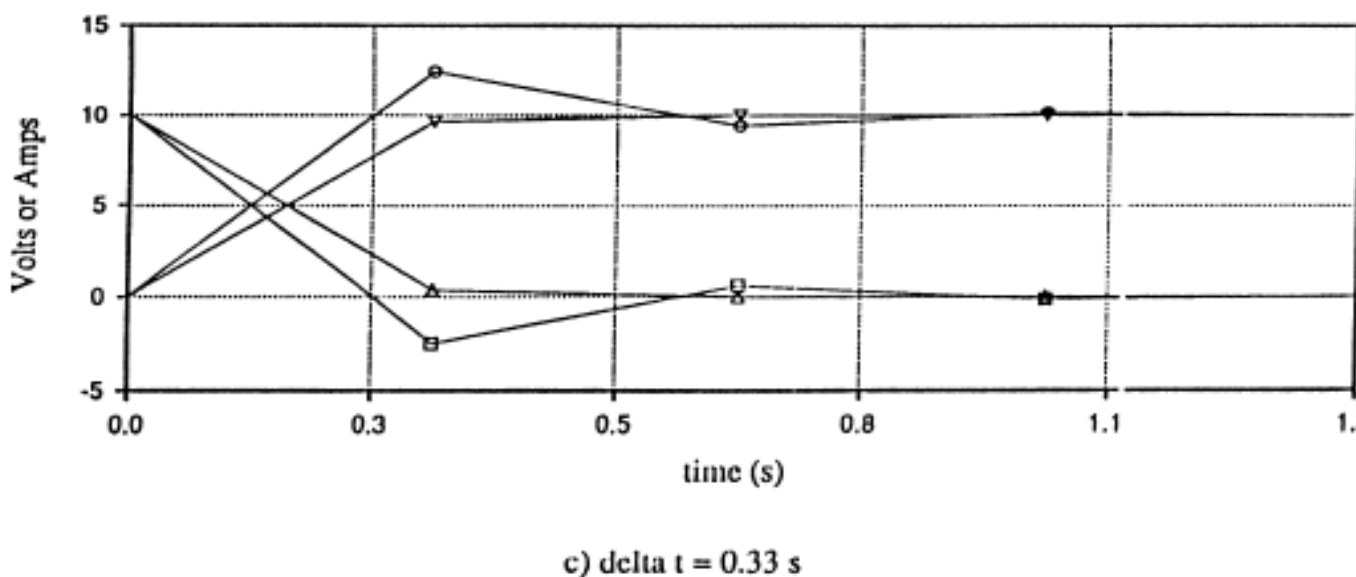
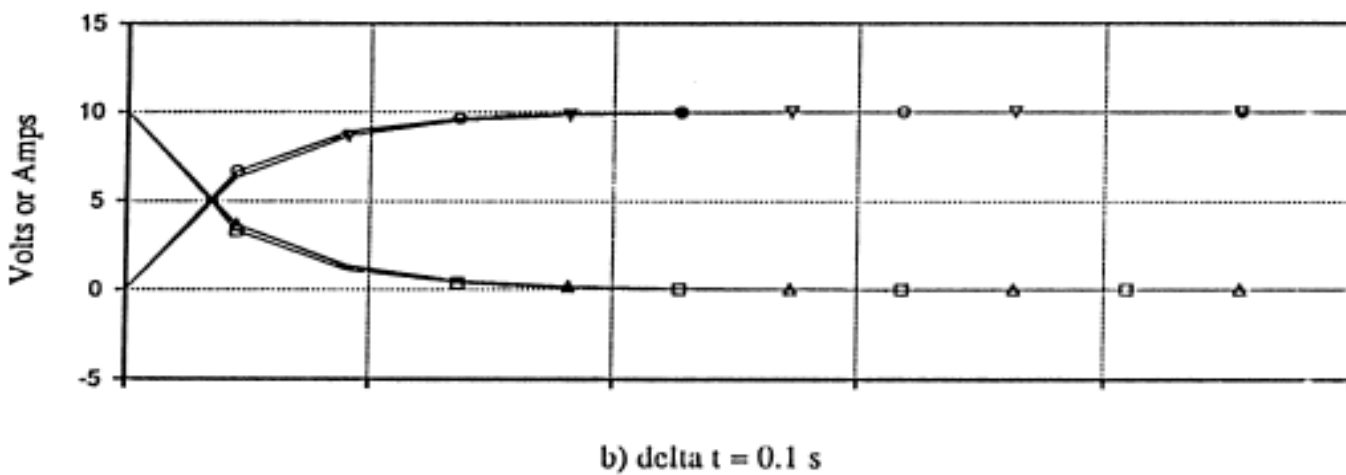
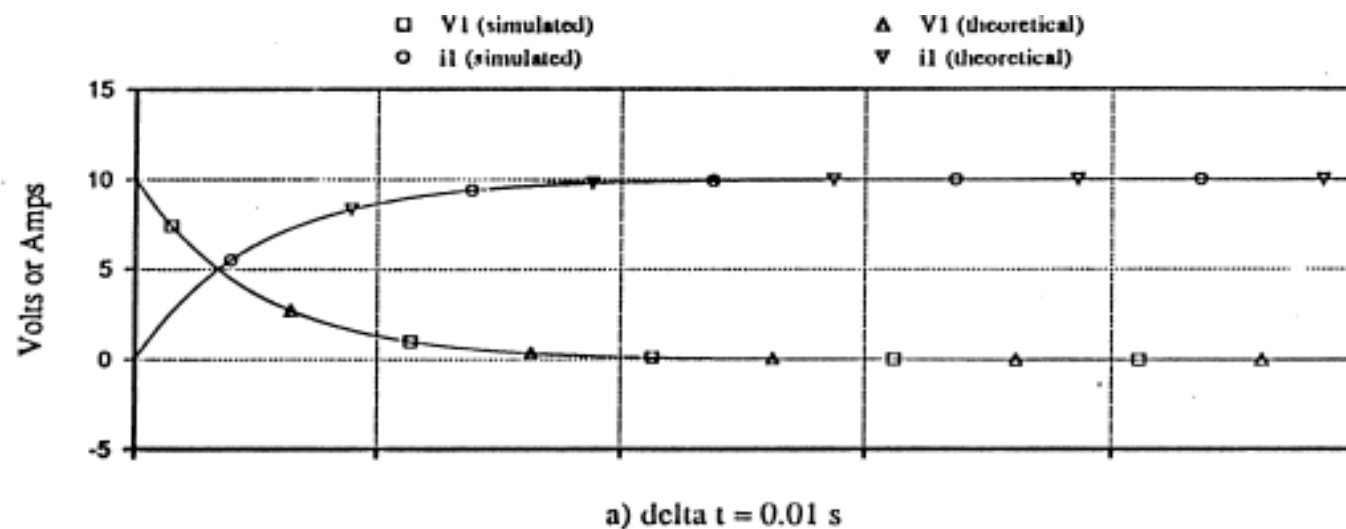


Fig. 1.7 Solution to Ex. 2 for different timesteps

Assignment #1:

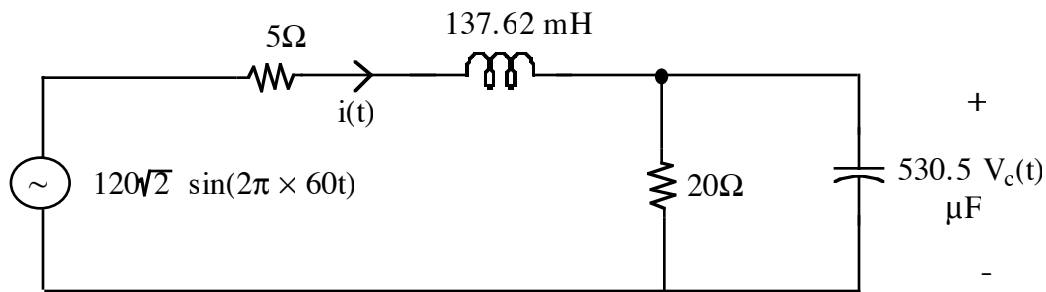


Fig. 1.8: Circuit for Assignment #1

For the above network, plot $i(t)$ and $v_c(t)$ as functions of time. Choose a suitable timestep. Continue solution for at least 4 cycles. Use a print-step of 1 ms. The following steps should be clearly shown:

- Represent the network in a circuit with only current sources and conductances.
- Directly form the admittance matrix.
- Develop the equation $\mathbf{Y} \mathbf{V} = \mathbf{I} - \mathbf{I}(t-\Delta t)$ and solve it, by inversion of \mathbf{Y} .
- Compare your answer with a phasor solution.

Timesteps:

From the solution of Example 2, we see that the timestep has an effect on the accuracy of solution. Typically a timestep which is less than about 10% of the smallest time constant or period in the circuit being solved is used as a rule of thumb. However, this is not always known a-priori, so the following procedure is recommended: a) Simulate the circuit with a given timestep. b) Reduce the timestep by 50% and repeat. If the results differ only insignificantly, the chosen timestep is adequate. We shall investigate the dependency of the stability of the solution further in this Chapter.

References:

- (1) N.Balbanian & T.Bickert, "Linear Network Theory, Analysis, Properties, Design and Synthesis", Matrix Publishers, 1981.
- (2) William H. Press, Saul A. Teukolsky, William T. Vetterling & Brian P. Flannery "Numerical Recipes in Fortran", Cambridge University Res., second edition, 1992, pp. 34-40.
- (3) H.W. Dommel, "Digital computation of Electromagnetic Transients in Single and Multi-phase Networks", IEEE Trans. PAS, Vol. PAS-88, No. 4, Apr. 1969, pp. 388-399.