Pricing Credit Default Swaps with a Random Recovery Rate by a Double Inverse Fourier Transform

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Abstract

We evaluate the par spread for a single-name credit default swap with a random recovery rate. It is carried out under the framework of a structural default model in which the asset-value process is of infinite activity but finite variation. The recovery rate is assumed to depend on the undershoot of the asset value below the default threshold when default occurs. The key part is to evaluate a generalized expected discounted penalty function, which is a special case of the so-called Gerber-Shiu function in actuarial ruin theory. We first obtain its double Laplace transform in time and in spatial variable, and then implement a numerical Fourier inversion integration. Numerical experiments show that our algorithm gives accurate results within reasonable time and different shapes of spread curve can be obtained.

Keywords: credit default swap; infinite activity; Lévy process; random recovery rate; structural model

JEL Classification: C10, G13

1 Introduction

Since its birth in the 1990s, credit default swap (CDS) has become the most widely used credit risk derivative to hedge credit risk. Insurance companies may be exposed as both protection buyers and protection sellers in the CDS market. On the one hand, insurance companies, as well as pension funds, are usually big holders of corporate bonds and hence may buy a CDS as a hedge against losses due to bond default. On the other hand, an insurance company may enter into a CDS as a protection seller since the CDS pays a stream of premiums that is a consistent source of investment income for the company. So it is important for insurance companies to understand how a CDS is priced.

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Pricing a CDS is essentially quantifying the risks that are transferred from the protection buyer to the protection seller. In this paper we ignore the counterparty credit risk, i.e., both parties do not default. Then, by entering a CDS contract, the protection buyer completely transfers both default risk and recovery risk of the reference entity to the protection seller. The former comes from the uncertainty on when default occurs, while the latter is due to the randomness of the recovery rate. Most default models in financial practice assume the default time and the recovery rate to be independent or simply assume a fixed recovery rate. However, the recovery rate is not fixed in reality. For example, a bond's recovery rate is not determined at issuance and it is natural to link the recovery rate to the status of the bond issuer's asset value at default because what bondholders recover upon default heavily depends on the remaining value of the bond issuer. Therefore, a default model that can handle the time and severity of default simultaneously is of great advantage for taking into account such a dependence between the default time and the recovery rate. In this paper we use a structural model with downward jumps, which was first proposed by Madan and Schoutens (2008).

Following the seminal paper of Black and Cox (1976), most of today's structural default models define default time as the first-passage time of the asset-value process below a certain threshold level. While the distribution of default time has been extensively investigated in the literature, it is much more difficult to consider the joint distribution of default time and the asset value at default due to its complexity. One idea is to include downward jumps in the asset-value process, which allows for situations where the default threshold is not just hit but crossed by a jump. This provides the possibility to connect the recovery rate with the undershoot of the asset-value process below the default threshold. Applying this idea, Zhou (2001) suggested modeling the logarithm of the asset-value process as the superposition of a diffusion component and a jump component with normally distributed jumps. Assuming the recovery rate as a function of the asset value at default, Zhou (2001) provided a Monte Carlo algorithm for pricing defaultable bonds. Chen and Kou (2009) used a double exponential jump-diffusion model to study credit spreads, optimal capital structure, and implied volatility of equity options. Ruf and Scherer (2011), pointing out that Monte Carlo algorithms are computationally expensive and may imply a systematic bias, calculated the price of a defaultable bond in a geometric jump-diffusion structural model with two-sided jumps by using an improved Brownian-bridge algorithm. Their method provides an efficient and unbiased Monte Carlo simulation for the computation of bond prices. However, all these above papers focused on Poissonian jumps only and did not address the important issue of infinite activity. In this paper we consider asset-value processes that are of infinite activity but finite variation and evaluate the CDS par spreads. Our algorithm does not involve Monte Carlo simulations and fairly accurate results can be obtained.

It is worthwhile to emphasize on the importance of processes of infinite activity and finite variation in modeling asset values. According to Carr *et al.* (2002), who empirically investigated the fine structure of asset returns, there is evidence from market prices of equity that both physical and risk-neutral processes for equity prices seem to be pure-jump processes of infinite activity and finite variation. Some of their findings are summarized here: (1) index returns tend to be pure-jump processes of infinite activity and finite variation, both physically and risk-neutrally. A diffusion component appears to be statistically insignificant, while it may be present in individual equity returns. (2) Jump components consistently account for significant skewness levels from equity prices. (3) The shape of the mean corrected density for asset returns appears to be a long spike near zero conjoined with two convex curves describing large returns. It apparently departs from that of a normal distribution, which is always concave within one standard deviation of the mean. In contrast, the densities of processes with infinite activity and finite variation are consistent with equity prices.

Since the asset-value process is of infinite activity, it is very difficult, if not impossible, to capture a default event continuously via Monte Carlo simulations. Instead, we evaluate CDS spreads by calculating a generalized expected discounted penalty function (EDPF) within a finite-time horizon. The original concept of EDPF was introduced by the classical paper of Gerber and Shiu (1998). EDPF is also called Gerber-Shiu function, which is a functional of the ruin time, the surplus prior to ruin, and the deficit at ruin. Later on, Biffis and Morales (2010) generalized the EDPF to include the surplus at the last minimum before ruin. Kuznetsov and Morales (2014) further introduced the generalized finite-time EDPF and showed that it is computationally tractable for the evaluation of risk measures such as Value at Risk when the risk process is from the beta and theta families of Lévy processes. The generalized finite-time EDPF fits our needs because it shares the same form as the present value of the loss leg of a CDS contract. It enables us to apply the quintuple law at first passage to obtain an explicit expression for the double Laplace transform of the loss leg, in time and in the spatial variable.

The method we use to invert the double Laplace transform is due to Dubner and Abate (1968) and Hosono (1981), developed by Abate and Whitt (1992, 1995), and extended to the multidimensional setting by Choudhury *et al.* (1994). As Rogers (2000) commented, "the idea of the method is basically a Fourier inversion integral, performed by integrating up a suitably-chosen contour $a + \mathrm{i}\mathbb{R}$, where the integral is approximated by a trapezoidal-rule sum, with equally-spaced points." The choices of the spacing of the points and the parameter a allow good control over the approximation errors. Euler sum is used as an acceleration technique. Rogers (2000) applied this method on calculating first-passage probabilities for general spectrally one-sided Lévy processes. Madan and Schoutens (2008) proposed a Lévy default model and used the algorithm from Rogers (2000) to calculate CDS spreads under the assumption of a constant recovery rate. In this paper, we assume the recovery rate to be random in the default model by Madan and Schoutens (2008) and revise the algorithm of Rogers (2000) so that CDS spreads can also be calculated.

This paper is organized as follows. We describe the structural model and the general formula for CDS spreads in Section 2. Then we derive the double Laplace transform for the loss leg of a CDS contract as a generalized EDPF in Section 3. We show how to invert the double Laplace transform through a Fourier inversion integral in Section 4. Finally, we conclude in Section 5 with some numerical experiments concerning runtime and accuracy of our algorithm and showing different shapes of CDS spread curve.

2 Model description and CDS spread

2.1 Asset-value model

We model the asset-value process of the reference entity of a CDS by a stochastic process $V = \{V_t, t \geq 0\}$ on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, where

$$V_t = V_0\, e^{Z_t}, \quad t \geq 0.$$

Throughout this paper we work under the pricing measure $\mathbb{Q}$. The process $Z = \{Z_t, t \geq 0\}$ takes the form of a positive drift minus a pure-jump subordinator. Specifically,

$$Z_t = \mu t - S_t, \quad t \geq 0, \tag{2.1}$$

with the Laplace exponent

$$\psi(s) := \ln \mathbb{E}(e^{sZ_1}) = \mu s + \int_{-\infty}^{0} (e^{sx} - 1)\, \Pi(\mathrm{d}x),$$

where $\mu > 0$ and the Lévy measure $\Pi(\cdot)$ defined on $(-\infty, 0)$ satisfies

$$\Pi((-\infty, 0)) = \infty \quad \text{and} \quad \int_{-1}^{0} |x|\Pi(\mathrm{d}x) < \infty. \tag{2.2}$$

So the process Z has paths of infinite activity and bounded variation.

The reference entity defaults when V_t falls below the default trigger point L, which is known at time 0 to be some value less than V_0. Denote by τ the default time, i.e.,

$$\tau = \inf\{t : V_t \le L\} = \inf\{t : \ln(V_0/L) + Z_t \le 0\}.$$

For convenience, we denote

$$X_t := \ln(V_0/L) + Z_t, \quad t \ge 0.$$

Then τ is actually the ruin time of process $X = \{X_t, t \ge 0\}$. We use X_τ, the deficit at ruin of X, to specify the default severity.

One important feature of our modeling is that the reference entity has a random recovery rate which depends on the default severity. Precisely, as long as $\tau \le T$ the protection seller is required to pay the protection buyer $1 - R(-X_\tau)$ for every unit notional amount at the moment of default, where $R(\cdot) \in [0, 1]$ is a non-negative and non-increasing function defined on $[0, \infty)$.

Example 2.1 One example of Z in (2.1) is the family of so-called shifted CMY processes, in which process $S = \{S_t, t \ge 0\}$ is from the family of CMY processes. The CMY process S with parameters C, M and Y is the Lévy process that starts at zero and has stationary and independent CMY-distributed increments. More precisely, S_t follows the CMY(Ct, M, Y) distribution. Recall that the characteristic function of the CMY distribution with parameters $C, M > 0$ and $0 \le Y < 1$ is given by

$$\varphi_{CMY}(u; C, M, Y) = \exp\left\{C\Gamma(-Y)\left((M - \mathrm{i}u)^Y - M^Y\right)\right\}.$$

Thus, the Lévy measure and the Laplace exponent of Z are, respectively,

$$\Pi(\mathrm{d}x) = Ce^{Mx}(-x)^{-1-Y}\mathrm{d}x, \quad x < 0,$$

and

$$\psi(s) = \mu s + C\Gamma(-Y)\left((M + s)^Y - M^Y\right). \tag{2.3}$$

Note that a shifted CMY process with $Y = 0$ reduces to a shifted gamma process with $\psi(s) = \mu s - C\ln(1 + s/M)$, and a shifted CMY process with $Y = 0.5$ reduces to a shifted inverse Gaussian process with $\psi(s) = \mu s - 2\sqrt{\pi}C(\sqrt{s + M} - \sqrt{M})$. For a shifted CMY process, it is clear that its Lévy measure satisfies the conditions in (2.2), and hence it has paths of infinite activity and finite variation. See Carr *et al.* (2002) and Madan and Schoutens (2008) for more properties of the CMY processes.

2.2 CDS spread

Assume a constant risk-free continuously compounded interest rate $r \geq 0$. Since $\mathbb{E}(V_t) = V_0 e^{rt}$ for $t \geq 0$ under $\mathbb{Q}$, we immediately have

$$r = \psi(1).$$

Without loss of generality we assume that the CDS has a unit notional amount. If c is the constant yearly continuous par spread, then the value of the CDS can be expressed as

$$\mathbb{E}\left[e^{-r\tau}(1 - R(-X_\tau))\mathbf{1}_{\{\tau \leq T\}}\right] - \mathbb{E}\left[\frac{c}{r}\left(1 - e^{-r(\tau\wedge T)}\right)\right],$$

where the first and second expectations respectively correspond to the present values of the so-called loss leg and premium leg of the CDS contract. Pricing the CDS is to find the par spread c which makes the present value of the loss leg equal to that of the premium leg:

$$c = \frac{r\mathbb{E}\left[e^{-r\tau}(1 - R(-X_\tau))\mathbf{1}_{\{\tau \leq T\}}\right]}{\mathbb{E}\left[1 - e^{-r(\tau\wedge T)}\right]}. \tag{2.4}$$

Note that although in reality premiums are paid at discrete times we, in this paper, follow the convention in the literature and focus on the mathematically friendly continuous par spread.

Let $\delta(t)$ denote the probability that no ruin occurs by time t, i.e.,

$$\delta(t) := \mathbb{Q}(\tau > t).$$

It is easy to see that the premium leg is completely determined by $\delta(t)$, $t \in [0, T]$. But theoretically we need to know the joint distribution of τ and $-X_\tau$ to calculate the loss leg. This joint distribution is embedded in the so-called quintuple law at first passage for the dual process $\tilde{Z}_t = -Z_t$, $t \geq 0$. However, as shown in the proof of Proposition 3.1, the quintuple law in general does not necessarily provide explicit formulae for special examples of Lévy processes due to the indirect involvement of the quantities κ and $\hat{\kappa}$, which themselves are embedded into the Wiener-Hopf factorization. One exception is for spectrally positive processes, for which one may make reasonable progress into making the Laplace transform of the law more explicit. See, for example, Section 7.3 of Kyprianou (2014) for details about the quintuple law at first passage. We show in Section 3 details on how to use the quintuple law to obtain an explicit formula for the double Laplace transform of the generalized EDPF in our model.

Remark 2.1 When the recovery rate R is a constant in $(0,1)$, the expression of c in (2.4) can be simplified as

$$c = \frac{(1-R)\left(-\int_0^T e^{-rt}\mathrm{d}\delta(t)\right)}{\int_0^T e^{-rt}\delta(t)\mathrm{d}t} = (1-R)\left(\frac{1-e^{-rT}\delta(T)}{\int_0^T e^{-rt}\delta(t)\mathrm{d}t} - r\right).$$

So all we need to know is $\delta(t)$, $t \in (0,T]$. See Madan and Schoutens (2008), Schoutens and Cariboni (2009, Section 3.1.1) and Hao *et al.* (2013) for more details on how to calculate $\delta(t)$ and c in this special case.

3 Generalized Expected Discounted Penalty Function

In this section we consider the generalized EDPF for a more general process

$$X_t := x + Z_t, \quad t \geq 0, \tag{3.1}$$

where $X_0 = x \geq 0$ and Z is defined as in (2.1). Let τ still denote the ruin time of X. It turns out that the generalized EDPF is the key quantity in calculating the CDS spread c.

The original concept of EDPF was introduced into actuarial ruin theory by Gerber and Shiu (1998). They studied ruin in the classical compound Poisson risk process by analyzing the joint law of τ, $-X_\tau$, and $X_{\tau-}$ in one single object, the EDPF. Since the process Z we consider here is more general than a compound Poisson process and, more importantly, we want to apply its quintuple law at first passage, we define two generalized EDPFs for the process X as follows.

Definition 3.1 *For the process X in (3.1), the generalized EDPF ϕ is*

$$\phi(x;r) := \mathbb{E}\left[e^{-r\tau}w(-X_\tau, X_{\tau-}, \underline{X}_{\tau-})\mathbf{1}_{\{\tau<\infty\}}\middle| X_0 = x\right],$$

and the generalized finite-time EDPF ϕ_t is

$$\phi_t(x;r) := \mathbb{E}\left[e^{-r\tau}w(-X_\tau, X_{\tau-}, \underline{X}_{\tau-})\mathbf{1}_{\{\tau<t\}}\middle| X_0 = x\right],$$

with $r \geq 0$ and w a bounded measurable function on $\mathbb{R}_+^3 = [0,\infty)^3$.

Actually the generalized EDPFs $\phi(x;r)$ and $\phi_t(x;r)$ have been recently introduced and studied in the actuarial literature. See, for example, Biffis and Morales (2010) and Kuznetsov and Morales (2014).

It is obvious that the present value of the loss leg is exactly $\phi_T(x;r)$ with $x = \ln(V_0/L)$ and $w(-X_\tau, X_{\tau-}, \underline{X}_{\tau-})$ reduces to $w(-X_\tau) = 1 - R(-X_\tau)$. To calculate this $\phi_T(x;r)$ we first derive an explicit expression for its double Laplace transform in Proposition 3.1. Then we show how to invert the double Laplace transform in Section 4.

Proposition 3.1 *Let X be the process in (3.1). For $r \geq 0$, the double Laplace transform of $\phi_t(x;r)$ defined as*

$$g(\lambda, z) = \int_{x=0}^{\infty} \int_{t=0}^{\infty} e^{-\lambda t - zx} \phi_t(x;r) \mathrm{d}t \mathrm{d}x, \quad \lambda, z > 0,$$

has the following formula

$$g(\lambda, z) = \frac{1}{\lambda(r + \lambda - \psi(z))} \int_{v=0}^{\infty} \int_{u=0}^{\infty} w(v) \Pi(-u - \mathrm{d}v) \left(e^{-zu} - e^{-\psi^{[-1]}(r+\lambda)u} \right) \mathrm{d}u, \tag{3.2}$$

where $\psi^{[-1]}(q)$ is the right inverse of ψ, i.e., $\psi^{[-1]}(q) = \sup\{s \geq 0 : \psi(s) = q\}$, $q \geq 0$.

Proof. It is easy to see that

$$\begin{aligned}
\int_{t=0}^{\infty} e^{-\lambda t} \phi_t(x;r) \mathrm{d}t &= \int_{t=0}^{\infty} e^{-\lambda t} \int_{s=0}^{t} e^{-rs} \mathbb{E}\left[w(-X_\tau) | \tau = s, X_0 = x \right] \mathbb{Q}(\tau \in \mathrm{d}s | X_0 = x) \mathrm{d}t \\
&= \frac{1}{\lambda} \int_{s=0}^{\infty} e^{-(\lambda+r)s} \mathbb{E}\left[w(-X_\tau) | \tau = s, X_0 = x \right] \mathbb{Q}(\tau \in \mathrm{d}s | X_0 = x) \\
&= \frac{\phi(x; r + \lambda)}{\lambda},
\end{aligned}$$

which implies

$$g(\lambda, z) = \frac{1}{\lambda} \int_{x=0}^{\infty} e^{-zx} \phi(x; r + \lambda) \mathrm{d}x. \tag{3.3}$$

Thus finding the double Laplace transform of $\phi_t(x;r)$ in both t and x is equivalently to finding the Laplace transform of $\phi(x; r + \lambda)$ in x.

Now we derive a formula for $\phi(x; r+\lambda)$. This part is essentially a special case of Lemma 2 of Kuznetsov *et al.* (2012). See also Proposition 4 of Kuznetsov and Morales (2014). We apply the so-called Gerber-Shiu measure defined as

$$\mathbb{P}^{x,r}(\mathrm{d}v, \mathrm{d}u, \mathrm{d}y) = \mathbb{E}\left[e^{-r\tau} \mathbf{1}_{\left\{ -X_\tau \in \mathrm{d}v, X_{\tau-} \in \mathrm{d}u, \underline{X}_{\tau-} \in \mathrm{d}y \right\}} \right]$$

for $r \geq 0$, $x > 0$, $v > 0$, $u > 0$, and $y \in (0, x \wedge u)$. Note that we denote the running maximum and running minimum for a stochastic process X by $\overline{X}_t = \sup_{0 \leq s \leq t} X_s$ and $\underline{X}_t = \inf_{0 \leq s \leq t} X_s$ respectively. We introduce the dual process $\tilde{Z}_t = -Z_t$, $t \geq 0$. Then the process $\tilde{Z}$ is a pure-jump subordinator plus a negative drift with Lévy measure $\tilde{\Pi}(\mathrm{d}x) = \Pi(-\mathrm{d}x)$, $x > 0$. It is obvious that $\tau = \inf\{t : \tilde{Z}_t \geq x\}$. Using the quintuple law at first passage for the process $\tilde{Z}$ we obtain that

$$\begin{aligned}
&\mathbb{E}\left[e^{-r\tau} \mathbf{1}_{\left\{ \tilde{Z}_\tau - x \in \mathrm{d}v, x - \tilde{Z}_{\tau-} \in \mathrm{d}u, x - \overline{\tilde{Z}}_{\tau-} \in \mathrm{d}y \right\}} \right] \\
&= \int_{s=0}^{\infty} e^{-rs} \mathcal{U}(\mathrm{d}s, x - \mathrm{d}y) \int_{t=0}^{\infty} e^{-rt} \hat{\mathcal{U}}(\mathrm{d}t, \mathrm{d}u - y) \tilde{\Pi}(u + \mathrm{d}v),
\end{aligned}$$

where $\mathcal{U}$ and $\hat{\mathcal{U}}$ are the bivariate measures associated with the ascending and descending ladder processes of $\tilde{Z}$ respectively. See Theorem 7.7 of Kyprianou (2014) and references therein for the details on the quintuple law at first passage for general Lévy processes. By the definitions of the bivariate measures we have

$$\int_{s=0}^{\infty} e^{-rs}\mathcal{U}(\mathrm{d}s, x-\mathrm{d}y) = \frac{1}{\kappa(r,0)}\mathbb{Q}\left(\overline{\tilde{Z}}_{\mathrm{e}_r} \in x-\mathrm{d}y\right)$$

and

$$\int_{t=0}^{\infty} e^{-rt}\hat{\mathcal{U}}(\mathrm{d}t, \mathrm{d}u-y) = \frac{1}{\hat{\kappa}(r,0)}\mathbb{Q}\left(-\underline{\tilde{Z}}_{\mathrm{e}_r} \in \mathrm{d}u-y\right),$$

where e_q represents an independent exponentially distributed random variable with rate $q>0$ and $\kappa(r,0)$ and $\hat{\kappa}(r,0)$ are the Laplace exponents of the ascending and descending ladder processes of $\tilde{Z}$ satisfying $\kappa(r,0)\hat{\kappa}(r,0)=r$. Thus,

$$\begin{aligned}
&\mathbb{E}\left[e^{-r\tau}\mathbf{1}_{\left\{\tilde{Z}_\tau - x\in\mathrm{d}v, x-\tilde{Z}_{\tau-}\in\mathrm{d}u, x-\overline{\tilde{Z}}_{\tau-}\in\mathrm{d}y\right\}}\right] \\
&= \frac{1}{r}\mathbb{Q}\left(\overline{\tilde{Z}}_{\mathrm{e}_r} \in x-\mathrm{d}y\right)\mathbb{Q}\left(-\underline{\tilde{Z}}_{\mathrm{e}_r} \in \mathrm{d}u-y\right)\tilde{\Pi}(u+\mathrm{d}v),
\end{aligned}$$

which is obviously equivalent with

$$\mathbb{P}^{x,r}(\mathrm{d}v,\mathrm{d}u,\mathrm{d}y) = \frac{1}{r}\mathbb{Q}\left(-\underline{Z}_{\mathrm{e}_r} \in x-\mathrm{d}y\right)\mathbb{Q}\left(\overline{Z}_{\mathrm{e}_r} \in \mathrm{d}u-y\right)\Pi(-u-\mathrm{d}v).$$

So we have

$$\begin{aligned}
\phi(x;r+\lambda) &= \int_{v=0}^{\infty}\int_{u=0}^{\infty}\int_{y=0}^{x\wedge u} w(v)\mathbb{P}^{x,r+\lambda}(\mathrm{d}v,\mathrm{d}u,\mathrm{d}y) \\
&= \int_{v=0}^{\infty}\int_{u=0}^{\infty}\int_{y=0}^{x\wedge u} \frac{w(v)}{r+\lambda}\mathbb{Q}\left(-\underline{Z}_{\mathrm{e}_{r+\lambda}} \in x-\mathrm{d}y\right)\mathbb{Q}\left(\overline{Z}_{\mathrm{e}_{r+\lambda}} \in \mathrm{d}u-y\right)\Pi(-u-\mathrm{d}v).
\end{aligned}$$

Then we plug the above expression into (3.3) and obtain

$$\begin{aligned}
&g(\lambda,z) \\
&= \int_{v=0}^{\infty}\int_{y=0}^{\infty}\int_{u=y}^{\infty}\int_{x=y}^{\infty} \frac{e^{-zx}w(v)}{\lambda(r+\lambda)}\mathbb{Q}\left(-\underline{Z}_{\mathrm{e}_{r+\lambda}} \in x-\mathrm{d}y\right)\mathbb{Q}\left(\overline{Z}_{\mathrm{e}_{r+\lambda}} \in \mathrm{d}u-y\right)\Pi(-u-\mathrm{d}v)\mathrm{d}x.
\end{aligned} \tag{3.4}$$

The well-known Wiener-Hopf factorization for the spectrally negative process Z tells us that $\overline{Z}_{\mathrm{e}_q}$ is exponentially distributed with rate $\psi^{[-1]}(q)$ and

$$\mathbb{E}\left(e^{t\underline{Z}_{\mathrm{e}_q}}\right) = \frac{q}{\psi^{[-1]}(q)}\frac{\psi^{[-1]}(q)-t}{q-\psi(t)}.$$

See, for example, Chapter 6 of Kyprianou (2014) for details about the Wiener-Hopf factorization. Using these facts we are able to simplify the formula in (3.4). By first calculating the integral with respect to x we get

$$\begin{aligned} & g(\lambda, z) \\ = & \frac{\psi^{[-1]}(r+\lambda) - z}{\lambda \psi^{[-1]}(r+\lambda)(r+\lambda-\psi(z))} \int_{v=0}^{\infty} \int_{y=0}^{\infty} \int_{u=y}^{\infty} w(v) e^{-zy} \mathbb{Q}\left(\overline{Z}_{\mathrm{e}_{r+\lambda}} \in \mathrm{d}u - y\right) \Pi(-u - \mathrm{d}v) \mathrm{d}y. \end{aligned}$$

Then by calculating the integral with respect to y we immediately obtain formula (3.2). ■

4 Double Inverse Fourier Transform

The reason why a double inverse Fourier transform is relevant is because $g(\lambda, z)$ obtained in Proposition 3.1 is analytic in the region where both arguments have strictly positive real parts. Madan and Schoutens (2008) made a very clear presentation on it. For the sake of completeness, we show their presentation in this paragraph. Let $\lambda_1, \lambda_2, z_1, z_2$ be real numbers with $\lambda_1, z_1 > 0$. Then by definition,

$$g(\lambda_1 - \mathrm{i}\lambda_2, z_1 - \mathrm{i}z_2) = \int_{x=0}^{\infty} \int_{t=0}^{\infty} \exp\{-\lambda_1 t + \mathrm{i}\lambda_2 t - z_1 x + \mathrm{i} z_2 x\} \phi_t(x; r) \mathrm{d}t \mathrm{d}x.$$

So $g(\lambda_1 - \mathrm{i}\lambda_2, z_1 - \mathrm{i}z_2)$ is the double Fourier transform of $\exp\{-\lambda_1 t - z_1 x\}\phi_t(x; r)$. By the inverse Fourier transform, we have

$$\exp\{-\lambda_1 t - z_1 x\}\phi_t(x; r) = \frac{1}{4\pi^2} \int_{\lambda_2=-\infty}^{\infty} \int_{z_2=-\infty}^{\infty} \exp\{-\mathrm{i}\lambda_2 t - \mathrm{i} z_2 x\} g(\lambda_1 - \mathrm{i}\lambda_2, z_1 - \mathrm{i}z_2) \mathrm{d}z_2 \mathrm{d}\lambda_2,$$

or, equivalently,

$$\begin{aligned} \phi_t(x; r) &= \frac{1}{4\pi^2} \int_{\lambda_2=-\infty}^{\infty} \int_{z_2=-\infty}^{\infty} \exp\{(\lambda_1 - \mathrm{i}\lambda_2)t + (z_1 - \mathrm{i}z_2)x\} g(\lambda_1 - \mathrm{i}\lambda_2, z_1 - \mathrm{i}z_2) \mathrm{d}z_2 \mathrm{d}\lambda_2 \\ &= -\frac{1}{4\pi^2} \int_{\Gamma_1} \int_{\Gamma_2} \exp\{\lambda t + zx\} g(\lambda, z) \mathrm{d}\lambda \mathrm{d}z, \end{aligned} \tag{4.1}$$

where the contour $\Gamma_1 = \lambda_1 + \mathrm{i}\mathbb{R}$ and the contour $\Gamma_2 = z_1 + \mathrm{i}\mathbb{R}$.

The difficulty in implementing the Fourier integration (4.1) is calculating $\psi^{[-1]}(r + \lambda)$ with $\lambda \in \Gamma_1$. Rogers (2000) suggested to alter contour Γ_1 to another contour Γ_0 such that it is easy to solve for $\psi^{[-1]}$ on the new contour and, more importantly, the integral up Γ_0 agrees with the integral up Γ_1 in (4.1). According to formula (3.2) of $g(\lambda, z)$, we can alter the contour Γ_1 to the contour

$$\Gamma_0 = \psi(\Gamma_1/\mu) - r.$$

Now for a point $\eta = \psi(\lambda/\mu) - r \in \Gamma_0$ with $\lambda \in \Gamma_1$, it is obvious that $\psi^{[-1]}(r+\eta) = \lambda/\mu$. Then we show in the next proposition under what conditions this contour altering does not change the value of the Fourier integration in (4.1).

Proposition 4.1 *Consider the Fourier integration in (4.1), where $g(\lambda, z)$ is from Proposition 3.1. Assume that $\psi(\cdot)$, the Laplace exponent of process Z, satisfies the following three conditions: for $s \in \mathbb{C}$ with $\mathrm{Re}(s) > 0$,*

C1: $(\psi(s) - \mu s)/s \to 0$ *as* $|s| \to \infty$,

C2: $|\psi^{[-1]}(s)| \to \infty$ *as* $|s| \to \infty$,

C3: $\mathrm{Re}(\psi^{[-1]}(s)) > 0$.

Then, integrating on contour Γ_0 instead of on contour Γ_1 in (4.1) does not change the value of the Fourier integration.

Proof. By Cauchy's integral theorem, if we integrate up Γ_1 from $\lambda_1 - \mathrm{i}\lambda_2$ to $\lambda_1 + \mathrm{i}\lambda_2$ then cross over to Γ_0 and integrate down from $\psi((\lambda_1 + \mathrm{i}\lambda_2)/\mu) - r$ to $\psi((\lambda_1 - \mathrm{i}\lambda_2)/\mu) - r$ and then cross back to $\lambda_1 - \mathrm{i}\lambda_2$, we get 0. So to show that the integral up Γ_0 agrees with the integral up Γ_1 in (4.1), we only need to prove that the contribution of the two crossings is negligible as $\lambda_2 \to \infty$. Indeed, for each fixed $z \in \Gamma_2$,

$$|g(\lambda, z)\,(\lambda - \psi(\lambda/\mu) + r)| = |\lambda g(\lambda, z)| \left| \frac{\lambda - \psi(\lambda/\mu) + r}{\lambda} \right| := I_1(\lambda) I_2(\lambda), \quad \lambda = \lambda_1 + \mathrm{i}\lambda_2 \in \Gamma_1.$$

Obviously, $I_2(\lambda) \to 0$ as $\lambda_2 \to \pm\infty$ by condition C1. By formula (3.2) and denoting $\xi = \psi^{[-1]}(r + \lambda)$ we have

$$\begin{aligned}
I_1(\lambda) &\le \frac{\left|\int_{v=0}^{\infty} \int_{u=0}^{\infty} \Pi(-u - \mathrm{d}v) \left(e^{-zu} - e^{-\xi u}\right) \mathrm{d}u\right|}{|r + \lambda - \psi(z)|} \\
&= \frac{\left|\int_{s=0}^{\infty} \Pi(-\mathrm{d}s) \int_{u=0}^{s} \left(e^{-zu} - e^{-\xi u}\right) \mathrm{d}u\right|}{|r + \lambda - \psi(z)|} \\
&= \frac{\left|\int_{s=0}^{\infty} \Pi(-\mathrm{d}s) \left(\left(1 - e^{-zs}\right)/z - \left(1 - e^{-\xi s}\right)/\xi\right)\right|}{|r + \lambda - \psi(z)|} \\
&\le \frac{\left|\int_{s=0}^{\infty} \Pi(-\mathrm{d}s)\left(1 - e^{-zs}\right)/z\right| + \left|\int_{s=0}^{\infty} \Pi(-\mathrm{d}s)\left(1 - e^{-\xi s}\right)/\xi\right|}{|r + \lambda - \psi(z)|},
\end{aligned} \tag{4.2}$$

where

$$\begin{aligned}
\left|\int_{s=0}^{\infty} \Pi(-\mathrm{d}s)\left(1 - e^{-\xi s}\right)/\xi\right| &= \left|\left(\int_{s=0}^{1/|\xi|} + \int_{1/|\xi|}^{\infty}\right) \Pi(-\mathrm{d}s)\left(1 - e^{-\xi s}\right)/\xi\right| \\
&:= |J_1(\xi) + J_2(\xi)|.
\end{aligned} \tag{4.3}$$

By condition C2 and the bounded variation condition in (2.2),

$$|J_1(\xi)| = \left| \int_{s=0}^{1/|\xi|} \frac{1-e^{-\xi s}}{\xi s} s\Pi(-\mathrm{d}s) \right| \leq a \int_{s=0}^{1/|\xi|} s\Pi(-\mathrm{d}s) \to 0, \quad \text{as } \lambda_2 \to \pm\infty, \tag{4.4}$$

for some constant $a > 0$. Note that here we use the fact $(1-e^x)/x = 1 - x/2! + x^2/3! - \cdots$, $x \in \mathbb{C}$. Similarly, by conditions C2-C3 and the bounded variation condition in (2.2),

$$\begin{aligned} \limsup_{\lambda_2 \to \pm\infty} |J_2(\xi)| &\leq \limsup_{\lambda_2 \to \pm\infty} \frac{\sqrt{2^2+1^2}\,\Pi((-\infty, -1/|\xi|))}{|\xi|} \\ &\leq \sqrt{5} \left(\Pi((-\infty, -1]) + \int_{-1}^{0} |x| \Pi(\mathrm{d}x) \right) < \infty. \end{aligned} \tag{4.5}$$

Combining (4.2)-(4.5) we obtain that $I_1(\lambda) \to 0$ as $\lambda_2 \to \pm\infty$. So the contribution of the crossing from $\lambda_1 + \mathrm{i}\lambda_2$ to $\psi((\lambda_1 + \mathrm{i}\lambda_2)/\mu) - r$ is negligible as $\lambda_2 \to \pm\infty$. In conclusion, if conditions C1-C3 are satisfied, the integral up Γ_0 agrees with the integral up Γ_1 in (4.1). ■

Assuming that conditions C1-C3 hold, by defining

$$h(\lambda) = \psi(\lambda/\mu) - r : \Gamma_1 \to \Gamma_0$$

we have

$$\begin{aligned} \phi_t(x;r) &= -\frac{1}{4\pi^2} \int_{\Gamma_0} \int_{\Gamma_2} \exp\{\lambda t + zx\} g(\lambda, z) \mathrm{d}\lambda \mathrm{d}z \\ &= -\frac{1}{4\pi^2} \int_{\Gamma_1} \int_{\Gamma_2} \exp\{h(\lambda)t + zx\} g(h(\lambda), z) \mathrm{d}h(\lambda) \mathrm{d}z \\ &= -\frac{1}{4\pi^2} \int_{\Gamma_1} \int_{\Gamma_2} h'(\lambda) \exp\{h(\lambda)t + zx\} g(h(\lambda), z) \mathrm{d}\lambda \mathrm{d}z, \end{aligned} \tag{4.6}$$

where

$$\begin{aligned} g(h(\lambda), z) &= \frac{\int_{v=0}^{\infty} \int_{u=0}^{\infty} w(v) \Pi(-u - \mathrm{d}v) \left(e^{-zu} - e^{-\psi^{[-1]}(r+h(\lambda))u} \right) \mathrm{d}u}{h(\lambda)(r + h(\lambda) - \psi(z))} \\ &= \frac{\int_{v=0}^{\infty} \int_{u=0}^{\infty} w(v) \Pi(-u - \mathrm{d}v) \left(e^{-zu} - e^{-\lambda u/\mu} \right) \mathrm{d}u}{(\psi(\lambda/\mu) - r)(\psi(\lambda/\mu) - \psi(z))}. \end{aligned}$$

The right-hand side of (4.6) can be approximated by the double sum

$$\begin{aligned} S_N = \frac{h_1 h_2}{4\pi^2} &\sum_{n=-Nl_1}^{Nl_1} \sum_{m=-Nl_2}^{Nl_2} h'(a_1 + \mathrm{i}nh_1) g(h(a_1 + \mathrm{i}nh_1), a_2 + \mathrm{i}mh_2) \\ &\times \exp\left\{ th(a_1 + \mathrm{i}nh_1) + x(a_2 + \mathrm{i}mh_2) \right\}, \end{aligned} \tag{4.7}$$

and following Choudhury *et al.* (1994) we use

$$a_1 = \frac{A_1}{2tl_1}, \quad a_2 = \frac{A_2}{2xl_2}, \quad h_1 = \frac{\pi}{tl_1}, \quad h_2 = \frac{\pi}{xl_2},$$

where A_1, A_2 are two large positive real numbers and l_1, l_2 are two positive integers. Choudhury *et al.* (1994) further suggested using the following Euler sum to improve approximation accuracy:

$$\sum_{k=0}^{K} 2^{-K} \binom{K}{k} S_{N+k} \tag{4.8}$$

with N and K two positive integers. As we show in Section 5.1, we are able to properly choose A_1, A_2, l_1, l_2, N and K to control the approximation errors in (4.7).

5 Numerical Experiments

In all numerical experiments in this section, we assume the process Z in (2.1) to be a shifted CMY process with $C, M > 0$ and $0 \leq Y < 1$. Before we proceed to apply the double inverse Fourier transform method described in Section 4 we need to check if $\psi(\cdot)$ satisfies conditions C1-C3. Actually, condition C1 is fulfilled by the fact $Y < 1$. Condition C2 can be easily proved by contradiction. As to condition C3, we observe that $\psi^{[-1]}(\cdot)$ is analytic on the half complex plane with positive real part. Since the process Z is a spectrally negative Lévy process, it is known that $\psi^{[-1]}(s_1) > 0$ at all real $s_1 > 0$. If condition C3 does not hold, then there must exist some $s_1 > 0$ and $s_2 \in \mathbb{R}$ such that $\psi^{[-1]}(s_1 + \mathrm{i}s_2)$ is on the axis of imaginaries. By plugging this solution into (2.3) and comparing the real parts on both sides, it turns out that there must exist some $\theta \in (-\pi/2, \pi/2)$ such that

$$\frac{\cos(Y\theta)}{(\cos\theta)^Y} = 1 + \frac{s_1}{C\Gamma(-Y)M^Y} < 1. \tag{5.1}$$

However, the left-hand side of (5.1), as a function of θ, is decreasing on $(-\pi/2, 0]$ and increasing on $[0, \pi)$. So it takes its local minimum 1 at $\theta = 0$, which contradicts the inequality in (5.1).

All numerical experiments in this section are implemented in R and share the following assumptions: the interest rate r is fixed at 0.03; the ratio of the default trigger point L over the asset value at time $t = 0$, L/V_0, is assumed to be 0.5.; and, the recovery rate is dependent upon the undershoot of the process X below 0 at ruin through the function $R(x) = 0.5\exp\{-x\}$, $x \geq 0$. We worked on a MacBook Pro computer equipped with a 2.4 GHz Intel Core i5 processor and a 4 GB 1333 MHz DDR3 memory.

5.1 Accuracy test

There are three sources of error in the approximation (4.7): the aliasing error, the roundoff error, and the truncation error. According to Choudhury *et al.* (1994, Section 2), we set the parameters in (4.7)-(4.8) as follows to control these errors. The aliasing error, also known as the discretization error, arises since we use a trapezoidal-rule form of numerical integration as in (4.7) to approximate $\phi_t(x; r)$. Since $\phi_t(x; r) \in (0, 1)$ by its definition, the aliasing error is bounded from above by approximately $e^{-A_1} + e^{-A_2}$. So we can limit the aliasing error to about 10^{-7} by choosing $A_1 = A_2 = 16.8$.

The roundoff error is due to multiplying large numbers by small ones. Specifically, it refers to the error due to a large value of the quantity $\exp\{A_1/(2l_1) + A_2/(2l_2)\}/(4l_1l_2tx)$, which can be taken out of the sums in (4.7). Since we have used A_1 and A_2 to control the aliasing error, we use l_1 and l_2 to control the roundoff error. Obviously increasing l_1 and l_2 will decrease $\exp\{A_1/(2l_1) + A_2/(2l_2)\}/(4l_1l_2tx)$ and thus reduce the roundoff error. However, from the sums in (4.7), choosing larger l_1 and l_2 costs more computation time which is proportional to the product of l_1 and l_2. Choudhury *et al.* (1994) suggests that for two-dimensional inversion usually $l_1 = l_2 = 2$ is adequate. Therefore we choose these values in our experiments.

As to the truncation error, unless computing the inverse transform near discontinuities, one can usually reduce the truncation error to 10^{-13} or lower by using the Euler sum with about 50 terms. When computing first-passage probabilities, Rogers (2000) used $N = 12$ and $K = 15$ and limited the truncation error to 10^{-4}. Our numerical experiments show that using $N = 12$ and $K = 15$ gives accurate enough results within reasonable runtime for our purpose as well.

We show 5-year CDS spreads and corresponding runtimes for different combinations of N and K in Table 1. We assume $C = 1$, $M = 5$, and $Y = 0$. We observe from the table that by decreasing N from 20 to 12 and K from 25 to 15 the runtime to calculate the CDS spread is shortened from more than 28 minutes to less than 8 minutes. However, we still can achieve an accuracy of 10^{-1} basis points (bps) at $N = 12$ and $K = 15$, which is accurate enough for credit spreads.

Table 1 is here.

In Table 2, 10-year CDS spreads and corresponding runtimes for $N = 12$ to 20 and $K = 15$ to 25 are shown. We change the values of C, M, and Y to 0.5, 4, and 0.25 respectively. We want to see if using $N = 12$ and $K = 15$ can still provide accurate CDS spreads under a different shifted CMY process and for a longer time horizon. It shows clearly that an accuracy of 10^{-1} bps can also be achieved.

Table 2 is here.

5.2 Term structure of CDS spreads

In a structural default model, the lower the credit quality of the firm, the closer it is to the default threshold, and hence the firm will face a higher probability of default over short maturities. For longer maturities, if no default occurs, the firm has a higher probability of credit improvement, and therefore the term structure of credit spreads is more likely to be humped or downward sloping. For high-quality firms, the reverse argument holds, and consequently, the term structure of credit spreads is more likely to be upward sloping. These typical shapes of the term structure of credit spreads are confirmed by the empirical work of Sarig and Warga (1989) and Fons (1994). Similar shapes are observed from the curve of CDS spreads. See, for example, Lando and Mortensen (2005) and Trück *et al.* (2004).

Our model is able to capture different styles of term structure of CDS spreads. In Figures 1-3 we show three examples of term structure demonstrating upward sloping, humped, and downward sloping, respectively. In Figure 1, the spreads are small and the curve is upward sloping, which is typical for an investment-grade reference entity. The spreads in Figure 2 are larger than those in Figure 1 and the curve is humped in shape. This style is more consistent with the term structure of a speculative-grade reference entity. Last, in Figure 3, the spreads are very large and the curve shows clearly a downward sloping. These are exactly the features of the CDS term structure of a reference entity with an extremely speculative grade.

Figures 1-3 are here.

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Table 1: CDS spreads c (in bps) and runtime (in minutes) for $T = 5$ with $C = 1$, $M = 5$, and $Y = 0$

	$N = 12$	13	14	15	16	17	18	19	20
$K = 15$	124.67 (07:49)	124.67 (08:29)	124.67 (09:18)	124.67 (10:00)	124.67 (10:51)	124.67 (11:40)	124.67 (12:45)	124.66 (13:50)	124.66 (14:36)
16	124.67 (08:34)	124.67 (09:13)	124.67 (09:59)	124.67 (10:51)	124.67 (11:41)	124.67 (12:35)	124.66 (13:38)	124.66 (14:40)	124.66 (15:43)
17	124.67 (09:15)	124.67 (09:59)	124.67 (10:49)	124.67 (11:45)	124.67 (12:39)	124.67 (13:37)	124.66 (14:38)	124.66 (15:56)	124.66 (16:55)
18	124.67 (10:02)	124.67 (10:50)	124.67 (11:41)	124.67 (12:37)	124.67 (13:37)	124.66 (14:36)	124.66 (15:47)	124.66 (17:20)	124.66 (18:07)
19	124.67 (10:51)	124.67 (11:43)	124.67 (12:36)	124.67 (13:37)	124.67 (14:42)	124.66 (15:44)	124.66 (16:57)	124.66 (18:10)	124.66 (19:24)
20	124.67 (11:50)	124.67 (12:37)	124.67 (13:35)	124.67 (14:38)	124.66 (15:50)	124.66 (16:54)	124.66 (18:13)	124.66 (19:37)	124.66 (20:46)
21	124.67 (12:40)	124.67 (13:37)	124.67 (14:37)	124.67 (15:45)	124.66 (17:06)	124.66 (18:08)	124.66 (19:25)	124.66 (20:50)	124.66 (22:11)
22	124.67 (13:40)	124.67 (14:38)	124.67 (15:45)	124.66 (16:55)	124.66 (18:13)	124.66 (19:30)	124.66 (20:45)	124.66 (22:12)	124.67 (23:41)
23	124.67 (14:43)	124.67 (15:48)	124.67 (16:55)	124.66 (18:09)	124.66 (19:27)	124.66 (21:01)	124.66 (22:21)	124.66 (23:40)	124.67 (25:08)
24	124.67 (15:52)	124.67 (16:58)	124.66 (18:08)	124.66 (19:28)	124.66 (20:46)	124.66 (22:11)	124.66 (23:55)	124.67 (25:09)	124.68 (26:42)
25	124.67 (17:09)	124.67 (18:10)	124.66 (37:36)	124.66 (20:53)	124.66 (22:12)	124.66 (23:41)	124.67 (25:14)	124.67 (26:42)	124.68 (28:25)

Table 2: CDS spreads c (in bps) and runtime (in minutes) for $T = 10$ with $C = 0.5$, $M = 4$, and $Y = 0.25$

	$N = 12$	13	14	15	16	17	18	19	20
$K = 15$	126.17 (07:42)	126.17 (07:43)	126.17 (08:56)	126.17 (09:17)	126.17 (10:09)	126.17 (11:11)	126.16 (16:48)	126.16 (18:07)	126.16 (27:55)
16	126.17 (07:54)	126.17 (12:04)	126.17 (09:13)	126.17 (10:05)	126.17 (11:00)	126.16 (12:05)	126.16 (18:06)	126.16 (19:46)	126.16 (21:06)
17	126.17 (08:33)	126.17 (09:08)	126.17 (10:03)	126.17 (10:55)	126.17 (11:53)	126.16 (13:03)	126.16 (19:31)	126.16 (21:55)	126.16 (22:17)
18	126.17 (09:05)	126.17 (09:55)	126.17 (10:50)	126.17 (11:50)	126.16 (12:49)	126.16 (14:02)	126.16 (20:59)	126.16 (22:38)	126.16 (23:55)
19	126.17 (09:51)	126.17 (10:43)	126.17 (11:42)	126.17 (12:45)	126.16 (13:46)	126.16 (15:04)	126.16 (22:54)	126.16 (24:40)	126.16 (25:25)
20	126.17 (10:38)	126.17 (11:36)	126.17 (12:37)	126.16 (13:44)	126.16 (14:49)	126.16 (16:59)	126.16 (24:04)	126.16 (26:20)	126.16 (27:25)
21	126.17 (11:31)	126.17 (12:31)	126.17 (13:35)	126.16 (14:45)	126.16 (15:54)	126.16 (23:27)	126.16 (25:15)	126.16 (28:36)	126.16 (29:20)
22	126.17 (12:27)	126.17 (13:27)	126.16 (14:35)	126.16 (15:47)	126.16 (17:06)	126.16 (25:43)	126.16 (26:59)	126.16 (30:03)	126.16 (31:14)
23	126.17 (13:17)	126.17 (14:35)	126.16 (15:37)	126.16 (16:56)	126.16 (18:22)	126.16 (27:31)	126.16 (29:02)	126.16 (30:52)	126.17 (33:17)
24	126.17 (14:20)	126.16 (16:32)	126.16 (16:43)	126.16 (18:08)	126.16 (19:42)	126.16 (29:22)	126.16 (30:26)	126.16 (32:03)	126.17 (35:29)
25	126.17 (15:22)	126.16 (19:23)	126.16 (17:54)	126.16 (19:22)	126.16 (21:01)	126.16 (31:23)	126.16 (32:50)	126.17 (35:26)	126.17 (37:27)

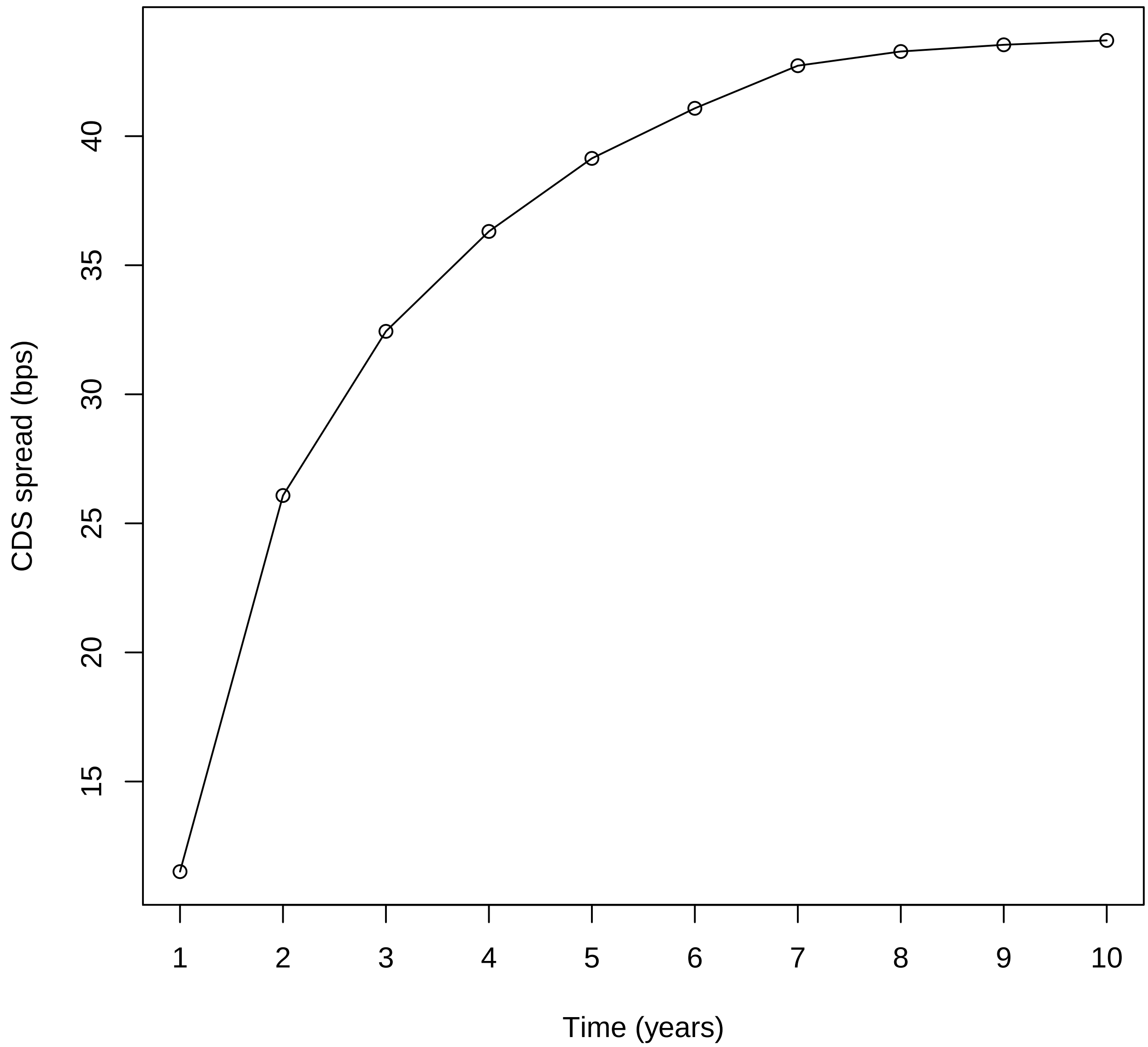

Figure 1: Term structure of CDS spreads assuming that the logarithm of the asset value follows a shifted CMY process with $C = 1$, $M = 7$, and $Y = 0$

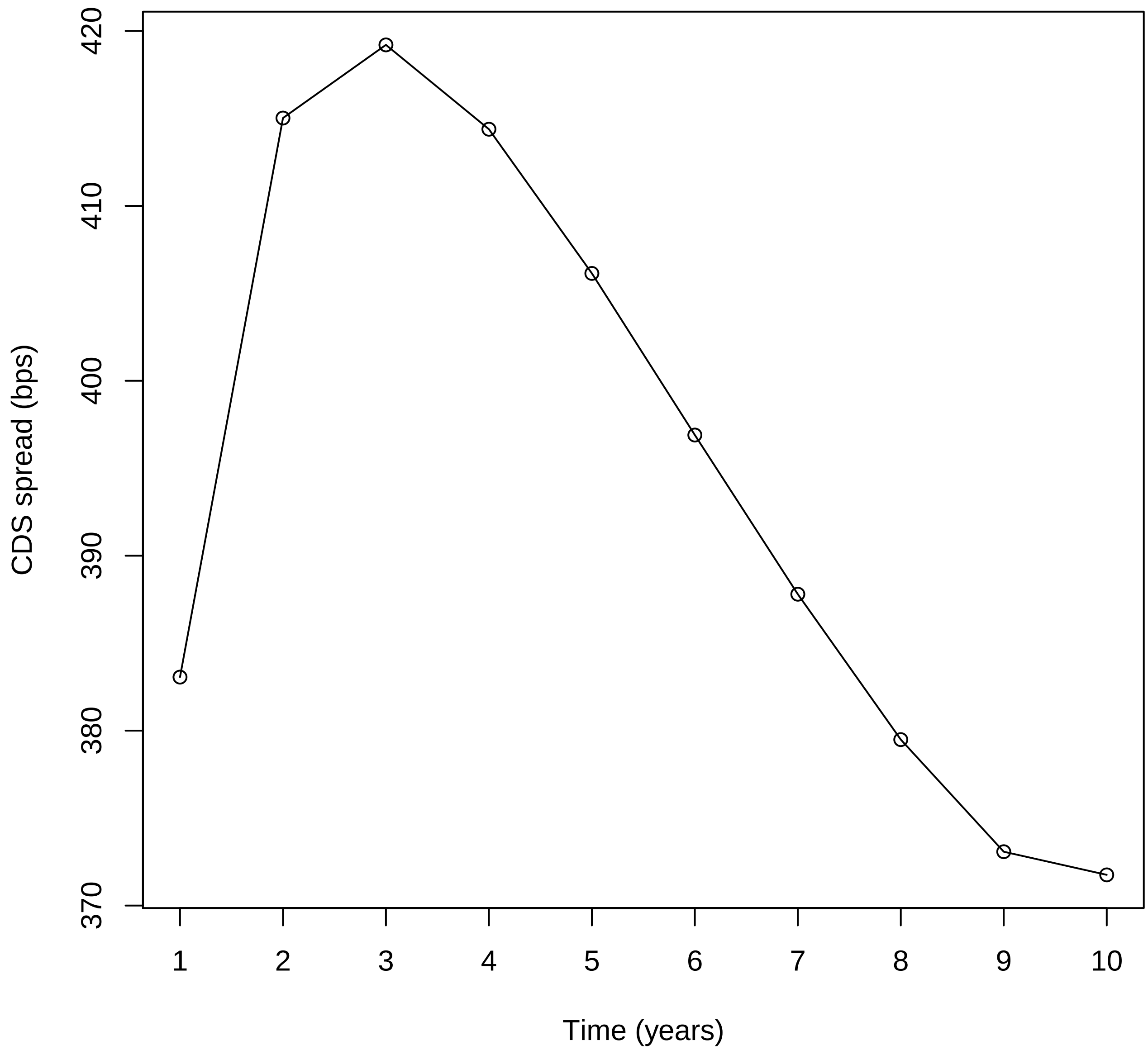

Figure 2: Term structure of CDS spreads assuming that the logarithm of the asset value follows a shifted CMY process with $C = 1$, $M = 3$, and $Y = 0$

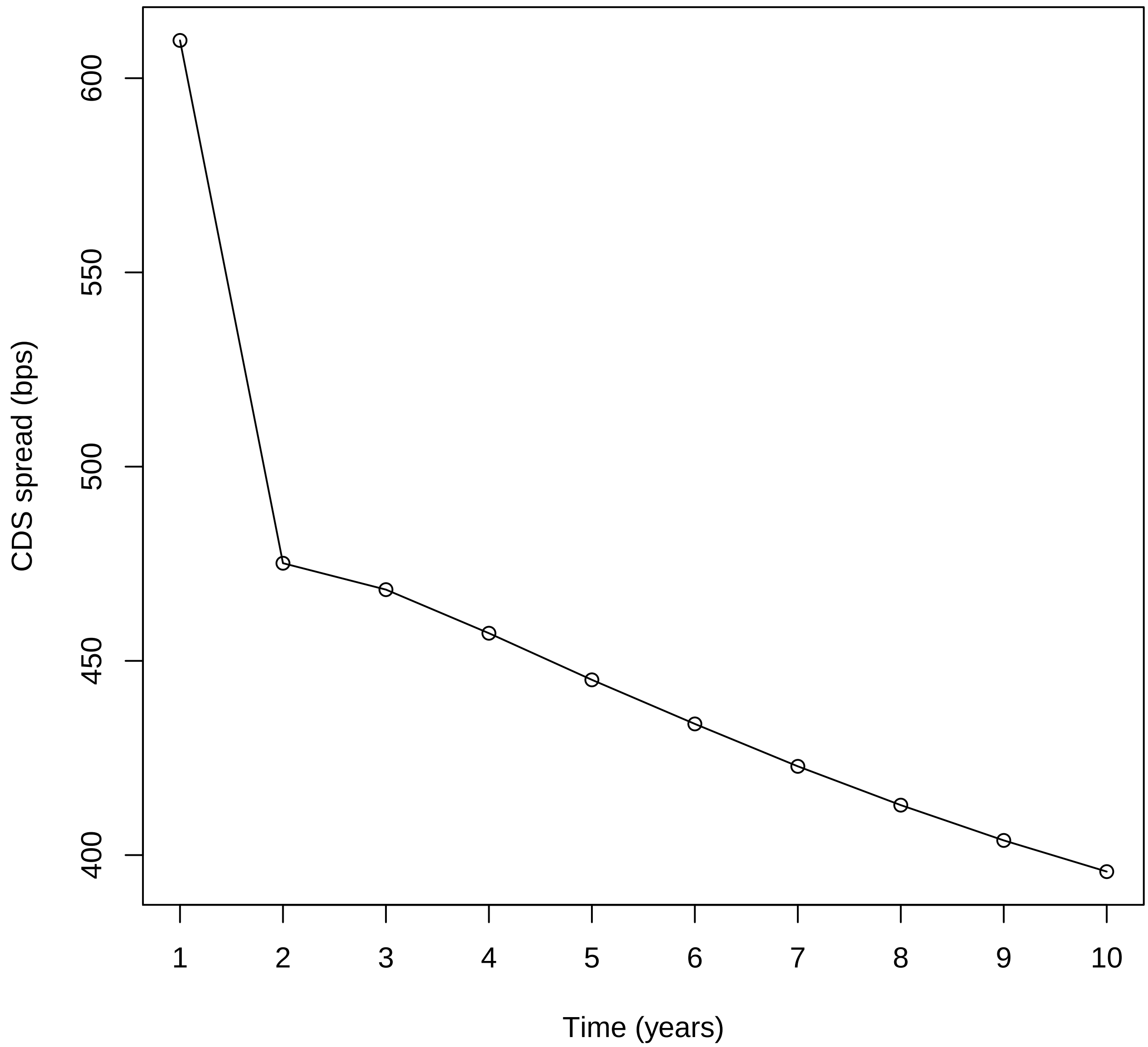

Figure 3: Term structure of CDS spreads assuming that the logarithm of the asset value follows a shifted CMY process with $C = 0.5$, $M = 1.9$, and $Y = 0$