# 1 Problem 1

(a) 
$$a_i = b_i c_{jj} + d_{jj}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1c_{jj} + d_{jj} \\ b_2c_{jj} + d_{jj} \\ b_3c_{jj} + d_{jj} \end{bmatrix} =$$

$$\begin{bmatrix} b_1c_{11} + b_1c_{22} + b_1c_{33} \\ b_2c_{11} + b_2c_{22} + b_2c_{33} \\ b_3c_{11} + b_3c_{22} + b_3c_{33} \end{bmatrix} + \begin{bmatrix} d_{11} + d_{22} + d_{33} \\ d_{11} + d_{22} + d_{33} \\ d_{11} + d_{22} + d_{33} \end{bmatrix} =$$

$$\begin{bmatrix} b_1c_{11} + d_{11} + b_1c_{22} + d_{22} + b_1c_{33} + d_{33} \\ b_2c_{11} + d_{11} + b_2c_{22} + d_{22} + b_2c_{33} + d_{33} \\ b_3c_{11} + d_{11} + b_3c_{22} + d_{22} + b_3c_{33} + d_{33} \end{bmatrix} = a_i$$

(b) 
$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$$

$$s_{ij} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

$$s_{11} = \sigma_{11} - \frac{1}{3}\sigma_{kk}\delta_{11}$$
  
 $s_{12} = \sigma_{12} - \frac{1}{3}\sigma_{kk}\delta_{12}$ 

Since for  $\delta_{12}$ ,  $i \neq j$ , the delta term is zero. Therefore  $s_{12}$  can be written as,  $s_{12} = \sigma_{12}$ . For terms when  $i \neq j$ ,  $s_{ij} = \sigma_{ij}$ .

For terms with i = j, such as  $s_{11}$ ,  $s_{22}$  and onwards, they can be represented as:

$$s_{11} = \sigma_{11} - \frac{1}{3}\sigma_{kk}\delta_{11}$$

$$= \sigma_{11} - \frac{1}{3}\left[\sigma_{11}\delta_{11} + \sigma_{22}\delta_{11} + \sigma_{33}\delta_{11}\right]$$

$$= \sigma_{11} - \frac{1}{3}\delta_{11}\left[\sigma_{11} + \sigma_{22} + \sigma_{33}\right]$$

$$s_{22} = \sigma_{22} - \frac{1}{3}\sigma_{kk}\delta_{22}$$

$$= \sigma_{22} - \frac{1}{3}\left[\sigma_{11}\delta_{22} + \sigma_{22}\delta_{22} + \sigma_{33}\delta_{22}\right]$$

$$= \sigma_{22} - \frac{1}{3}\delta_{22}\left[\sigma_{11} + \sigma_{22} + \sigma_{33}\right]$$

$$\begin{bmatrix} \sigma_{11} - \frac{1}{3} \left[ \sigma_{11} + \sigma_{22} + \sigma_{33} \right] & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \frac{1}{3} \left[ \sigma_{11} + \sigma_{22} + \sigma_{33} \right] & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \frac{1}{3} \left[ \sigma_{11} + \sigma_{22} + \sigma_{33} \right] \end{bmatrix}$$

(c)  $s = \frac{1}{2} \left[ \sigma_{ii} \sigma_{jj} - \sigma_{ij} \sigma_{ji} \right]$ 

When i = j, s = 0. When  $i \neq j$ , the terms can be represented as:

$$s = \frac{1}{2} \qquad \left[ \sigma_{11} \left[ \sigma_{22} + \sigma_{33} \right] - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} \right.$$

$$\sigma_{22} \left[ \sigma_{11} + \sigma_{33} \right] - \sigma_{21}\sigma_{12} - \sigma_{23}\sigma_{32}$$

$$\sigma_{33} \left[ \sigma_{11} + \sigma_{22} \right] - \sigma_{31}\sigma_{13} - \sigma_{32}\sigma_{23} \right]$$

$$= \frac{1}{2} \qquad \left[ \sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33} - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} \right.$$

$$\sigma_{22}\sigma_{11} + \sigma_{22}\sigma_{33} - \sigma_{21}\sigma_{12} - \sigma_{23}\sigma_{32}$$

$$\sigma_{33}\sigma_{11} + \sigma_{33}\sigma_{22} - \sigma_{31}\sigma_{13} - \sigma_{32}\sigma_{23} \right]$$

$$s = \frac{1}{2} \left[ 2\sigma_{11}\sigma_{22} + 2\sigma_{11}\sigma_{33} + 2\sigma_{22}\sigma_{33} - 2\sigma_{12}\sigma_{21} - 2\sigma_{13}\sigma_{31} - 2\sigma_{32}\sigma_{23} \right]$$

$$= \sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33} + \sigma_{22}\sigma_{33} - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} - \sigma_{32}\sigma_{23}$$

# (d) $(b_i a_j - a_i b_j) \varepsilon_{jkl} c_k d_l$

Permutation	(j,k,l)	$arepsilon_{jkl}$	
Positive	(1,2,3) , $(2,3,1)$ , $(3,1,2)$	+1	
Negative	(3,2,1) , $(2,1,3)$ , $(1,3,2)$	-1	
All others	Any other order	0	

When i = j, the terms are equal to zero. Scenarios in this case were omitted in the expansion of terms.

when 
$$i = 1$$

$$(b_1a_2 - a_1b_2)(+1)c_3d_1$$

$$(b_1a_3 - a_1b_3)(+1)c_1d_2$$

$$(b_1a_3 - a_1b_3)(-1)c_2d_1$$

$$(b_1a_2 - a_1b_2)(-1)c_1d_3$$

$$[i = 1 \text{ and } (j, k, l) = (2, 3, 1)]$$

$$[i = 1 \text{ and } (j, k, l) = (3, 1, 2)]$$

$$[i = 1 \text{ and } (j, k, l) = (3, 2, 1)]$$

when 
$$i = 2$$

$$(b_2a_1 - a_2b_1)(+1)c_2d_3$$

$$(b_2a_3 - a_2b_3)(+1)c_1d_2$$

$$(b_2a_3 - a_2b_3)(-1)c_2d_1$$

$$(b_2a_1 - a_2b_1)(-1)c_3d_2$$

$$[i = 2 \text{ and } (j, k, l) = (3, 1, 2)]$$

$$[i = 2 \text{ and } (j, k, l) = (3, 2, 1)]$$

$$[i = 2 \text{ and } (j, k, l) = (3, 2, 1)]$$

when 
$$i = 3$$
 
$$(b_3a_1 - a_3b_1)(+1)c_2d_3 \qquad [i = 3 \text{ and } (j, k, l) = (1, 2, 3)]$$
 
$$(b_3a_2 - a_3b_2)(+1)c_3d_1 \qquad [i = 3 \text{ and } (j, k, l) = (2, 3, 1)]$$
 
$$(b_3a_2 - a_3b_2)(-1)c_1d_3 \qquad [i = 3 \text{ and } (j, k, l) = (2, 1, 3)]$$
 
$$(b_3a_1 - a_3b_1)(-1)c_3d_2 \qquad [i = 3 \text{ and } (j, k, l) = (1, 3, 2)]$$

Simplification

when 
$$i = 1$$

$$(b_1a_2 - a_1b_2)(c_3d_1 - c_1d_3)$$

$$(b_1a_3 - a_1b_3)(c_1d_2 - c_2d_1)$$
when  $i = 2$ 

$$(b_2a_1 - a_2b_1)(c_2d_3 - c_3d_2)$$

$$(b_2a_3 - a_2b_3)(c_1d_2 - c_2d_1)$$
when  $i = 3$ 

$$(b_3a_1 - a_3b_1)(c_2d_3 - c_3d_2)$$

$$(b_3a_2 - a_3b_2)(c_3d_1 - c_3d_2)$$

The terms can then be represented as:

$$x_i = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (b_1a_2 - a_1b_2)(c_3d_1 - c_1d_3) + (b_1a_3 - a_1b_3)(c_1d_2 - c_2d_1) \\ (b_2a_1 - a_2b_1)(c_2d_3 - c_3d_2) + (b_2a_3 - a_2b_3)(c_1d_2 - c_2d_1) \\ (b_3a_1 - a_3b_1)(c_2d_3 - c_3d_2) + (b_3a_2 - a_3b_2)(c_3d_1 - c_3d_2) \end{bmatrix}$$

# 2 Problem 2

### (a) Dot product of two vectors

A	A	В	С	D
1		х	У	Z
2	Vector 1	1	2	3
3	Vector 2	2	2	2
4				
5	Dot Product	12		

Figure 1: The Excel function used is =SUMPRODUCT(B2:D2,B3:D3).

#### (b) Multiplication between a matrix and a vector

1	В	С	D	E	F	G	H
1	3x3	Matrix			Vector	-	Result
2	2	2	0		2		10
3	0	1	0		3		3
4	0	0	1		4		4

Figure 2: The Excel function used is =MMULT(A,B). The 3x3 matrix was defined as A, and the vector defined as B. A 1x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

#### (c) Multiplication between two matrices

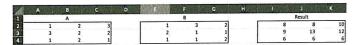


Figure 3: The Excel function used is =MMULT(A2:C4,E2:G4). A 3x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

### (d) Inverse of a matrix

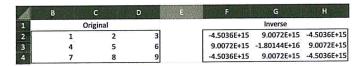


Figure 4: The Excel function used is =MINVERSE(B2:D4). A 3x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

# (e) Determinant of a matrix

A	Α	В	С	D	E
1		3x3 Matrix			Determinant
2	1	2	1		5
3	2	1	3		
4	2	2	1		

Figure 5: The Excel function used is =MDETERM(A2:C4)

# 3 Problem 3

$$x' = 0.650945, 0.650945, 0.390567$$
  
 $y' = 0.245256, 0.306570, -0.919709$   
 $z' = -0.718415, 0.694468, 0.039912$ 

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta_{x'x} & \cos \theta_{x'y} & \cos \theta_{x'z} \\ \cos \theta_{y'x} & \cos \theta_{y'y} & \cos \theta_{y'z} \\ \cos \theta_{z'x} & \cos \theta_{z'y} & \cos \theta_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The angle between the new coordinate system and the standard [100],[010],[001] coordinate system was obtained. Since the magnitude of x', y', and z' are 1, the following angles were determined. Thus, the transformation matrix can be seen as,

$$= \begin{bmatrix} \cos(49.39) & \cos(49.39) & \cos(67.01) \\ \cos(79.80) & \cos(72.15) & \cos(156.88) \\ \cos(135.92) & \cos(46.02) & \cos(97.71) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$