1 Problem 1

(a) $a_i = b_i c_{jj} + d_{jj}$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1 c_{jj} + d_{jj} \\ b_2 c_{jj} + d_{jj} \\ b_3 c_{jj} + d_{jj} \end{bmatrix} =$$

$$\begin{bmatrix} b_1c_{11} + b_1c_{22} + b_1c_{33} \\ b_2c_{11} + b_2c_{22} + b_2c_{33} \\ b_3c_{11} + b_3c_{22} + b_3c_{33} \end{bmatrix} + \begin{bmatrix} d_{11} + d_{22} + d_{33} \\ d_{11} + d_{22} + d_{33} \\ d_{11} + d_{22} + d_{33} \end{bmatrix} =$$

$$\begin{bmatrix} b_1c_{11} + d_{11} + b_1c_{22} + d_{22} + b_1c_{33} + d_{33} \\ b_2c_{11} + d_{11} + b_2c_{22} + d_{22} + b_2c_{33} + d_{33} \\ b_3c_{11} + d_{11} + b_3c_{22} + d_{22} + b_3c_{33} + d_{33} \end{bmatrix} = a_i$$

(b) $s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$

$$s_{ij} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$$

$s_{11} = \sigma_{11} - \frac{1}{3}\sigma_{kk}\delta_{11}$
$s_{12} = \sigma_{12} - \frac{1}{3}\sigma_{kk}\delta_{12}$

Since for $\delta_{12}, i \neq j$, the delta term is zero. Therefore s_{12} can be written as, $s_{12} = \sigma_{12}$. For terms when $i \neq j$, $s_{ij} = \sigma_{ij}$.

For terms with $i = j$, such as s_{11}, s_{22} and onwards, they can be represented as:

$$\begin{aligned} s_{11} &= \sigma_{11} - \frac{1}{3}\sigma_{kk}\delta_{11} \\ &= \sigma_{11} - \frac{1}{3}\left[\sigma_{11}\delta_{11} + \sigma_{22}\delta_{11} + \sigma_{33}\delta_{11}\right] \\ &= \sigma_{11} - \frac{1}{3}\delta_{11}\left[\sigma_{11} + \sigma_{22} + \sigma_{33}\right] \end{aligned}$$

$$\begin{aligned} s_{22} &= \sigma_{22} - \frac{1}{3}\sigma_{kk}\delta_{22} \\ &= \sigma_{22} - \frac{1}{3}\left[\sigma_{11}\delta_{22} + \sigma_{22}\delta_{22} + \sigma_{33}\delta_{22}\right] \\ &= \sigma_{22} - \frac{1}{3}\delta_{22}\left[\sigma_{11} + \sigma_{22} + \sigma_{33}\right] \end{aligned}$$

$$\begin{bmatrix} \sigma_{11} - \frac{1}{3}\left[\sigma_{11}+\sigma_{22}+\sigma_{33}\right] & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \frac{1}{3}\left[\sigma_{11}+\sigma_{22}+\sigma_{33}\right] & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \frac{1}{3}\left[\sigma_{11}+\sigma_{22}+\sigma_{33}\right] \end{bmatrix}$$

(c) $s = \frac{1}{2}\left[\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}\right]$

When $i = j, s = 0$. When $i \neq j$, the terms can be represented as:

$$\begin{aligned} s = \frac{1}{2} & \Big[\sigma_{11}\left[\sigma_{22}+\sigma_{33}\right] - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} \\ & \sigma_{22}\left[\sigma_{11}+\sigma_{33}\right] - \sigma_{21}\sigma_{12} - \sigma_{23}\sigma_{32} \\ & \sigma_{33}\left[\sigma_{11}+\sigma_{22}\right] - \sigma_{31}\sigma_{13} - \sigma_{32}\sigma_{23}\Big] \\ = \frac{1}{2} & \Big[\sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33} - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} \\ & \sigma_{22}\sigma_{11} + \sigma_{22}\sigma_{33} - \sigma_{21}\sigma_{12} - \sigma_{23}\sigma_{32} \\ & \sigma_{33}\sigma_{11} + \sigma_{33}\sigma_{22} - \sigma_{31}\sigma_{13} - \sigma_{32}\sigma_{23}\Big] \end{aligned}$$

$$\begin{aligned} s &= \frac{1}{2}\left[2\sigma_{11}\sigma_{22} + 2\sigma_{11}\sigma_{33} + 2\sigma_{22}\sigma_{33} - 2\sigma_{12}\sigma_{21} - 2\sigma_{13}\sigma_{31} - 2\sigma_{32}\sigma_{23}\right] \\ &= \sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33} + \sigma_{22}\sigma_{33} - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} - \sigma_{32}\sigma_{23} \end{aligned}$$

(d) $(b_i a_j - a_i b_j)\,\varepsilon_{jkl} c_k d_l$

Permutation	(j,k,l)	ε_{jkl}
Positive	(1,2,3) , (2,3,1) , (3,1,2)	+1
Negative	(3,2,1) , (2,1,3) , (1,3,2)	−1
All others	Any other order	0

When $i = j$, the terms are equal to zero. Scenarios in this case were omitted in the expansion of terms.

when $i = 1$

$$(b_1a_2 - a_1b_2)(+1)c_3d_1 \qquad [i = 1 \text{ and } (j,k,l) = (2,3,1)]$$
$$(b_1a_3 - a_1b_3)(+1)c_1d_2 \qquad [i = 1 \text{ and } (j,k,l) = (3,1,2)]$$
$$(b_1a_3 - a_1b_3)(-1)c_2d_1 \qquad [i = 1 \text{ and } (j,k,l) = (3,2,1)]$$
$$(b_1a_2 - a_1b_2)(-1)c_1d_3 \qquad [i = 1 \text{ and } (j,k,l) = (2,1,3)]$$

when $i = 2$

$$(b_2a_1 - a_2b_1)(+1)c_2d_3 \qquad [i = 2 \text{ and } (j,k,l) = (1,2,3)]$$
$$(b_2a_3 - a_2b_3)(+1)c_1d_2 \qquad [i = 2 \text{ and } (j,k,l) = (3,1,2)]$$
$$(b_2a_3 - a_2b_3)(-1)c_2d_1 \qquad [i = 2 \text{ and } (j,k,l) = (3,2,1)]$$
$$(b_2a_1 - a_2b_1)(-1)c_3d_2 \qquad [i = 2 \text{ and } (j,k,l) = (1,3,2)]$$

when $i = 3$

$$(b_3a_1 - a_3b_1)(+1)c_2d_3 \qquad [i = 3 \text{ and } (j,k,l) = (1,2,3)]$$
$$(b_3a_2 - a_3b_2)(+1)c_3d_1 \qquad [i = 3 \text{ and } (j,k,l) = (2,3,1)]$$
$$(b_3a_2 - a_3b_2)(-1)c_1d_3 \qquad [i = 3 \text{ and } (j,k,l) = (2,1,3)]$$
$$(b_3a_1 - a_3b_1)(-1)c_3d_2 \qquad [i = 3 \text{ and } (j,k,l) = (1,3,2)]$$

Simplification

when $i = 1$

$$(b_1a_2 - a_1b_2)(c_3d_1 - c_1d_3)$$
$$(b_1a_3 - a_1b_3)(c_1d_2 - c_2d_1)$$

when $i = 2$

$$(b_2a_1 - a_2b_1)(c_2d_3 - c_3d_2)$$
$$(b_2a_3 - a_2b_3)(c_1d_2 - c_2d_1)$$

when $i = 3$

$$(b_3a_1 - a_3b_1)(c_2d_3 - c_3d_2)$$
$$(b_3a_2 - a_3b_2)(c_3d_1 - c_3d_2)$$

The terms can then be represented as:

$$x_i = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (b_1a_2 - a_1b_2)(c_3d_1 - c_1d_3) + (b_1a_3 - a_1b_3)(c_1d_2 - c_2d_1) \\ (b_2a_1 - a_2b_1)(c_2d_3 - c_3d_2) + (b_2a_3 - a_2b_3)(c_1d_2 - c_2d_1) \\ (b_3a_1 - a_3b_1)(c_2d_3 - c_3d_2) + (b_3a_2 - a_3b_2)(c_3d_1 - c_3d_2) \end{bmatrix}$$

2 Problem 2

(a) Dot product of two vectors

	A	B	C	D
1		x	y	z
2	Vector 1	1	2	3
3	Vector 2	2	2	2
4				
5	Dot Product	12		

Figure 1: The Excel function used is =SUMPRODUCT(B2:D2,B3:D3).

(b) Multiplication between a matrix and a vector

	B	C	D	E	F	G	H
1		3x3 Matrix			Vector		Result
2	2	2	0		2		10
3	0	1	0		3		3
4	0	0	1		4		4

Figure 2: The Excel function used is =MMULT(A,B). The 3x3 matrix was defined as A, and the vector defined as B. A 1x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

(c) Multiplication between two matrices

	A	B	C	D	E	F	G	H	I	J	K
1		A				B				Result	
2	1	2	3		1	3	2		8	8	10
3	3	2	2		2	1	1		9	13	12
4	1	2	1		1	1	2		6	6	6

Figure 3: The Excel function used is =MMULT(A2:C4,E2:G4). A 3x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

(d) Inverse of a matrix

	B	C	D	E	F	G	H
1		Original				Inverse	
2	1	2	3		-4.5036E+15	9.0072E+15	-4.5036E+15
3	4	5	6		9.0072E+15	-1.80144E+16	9.0072E+15
4	7	8	9		-4.5036E+15	9.0072E+15	-4.5036E+15

Figure 4: The Excel function used is =MINVERSE(B2:D4). A 3x3 box was highlighted before inputting the command. Once the command was typed, control, shift and enter keys were pressed.

(e) Determinant of a matrix

	A	B	C	D	E
1		3x3 Matrix			Determinant
2	1	2	1		5
3	2	1	3		
4	2	2	1		

Figure 5: The Excel function used is =MDETERM(A2:C4)

3 Problem 3

$$\begin{aligned} x' &= 0.650945, \quad 0.650945, \quad 0.390567 \\ y' &= 0.245256, \quad 0.306570, \quad -0.919709 \\ z' &= -0.718415, \quad 0.694468, \quad 0.039912 \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta_{x'x} & \cos\theta_{x'y} & \cos\theta_{x'z} \\ \cos\theta_{y'x} & \cos\theta_{y'y} & \cos\theta_{y'z} \\ \cos\theta_{z'x} & \cos\theta_{z'y} & \cos\theta_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The angle between the new coordinate system and the standard [100],[010],[001] coordinate system was obtained. Since the magnitude of x', y', and z' are 1, the following angles were determined. Thus, the transformation matrix can be seen as,

$$= \begin{bmatrix} \cos(49.39) & \cos(49.39) & \cos(67.01) \\ \cos(79.80) & \cos(72.15) & \cos(156.88) \\ \cos(135.92) & \cos(46.02) & \cos(97.71) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$