

## 1] Lagrangian Infinitesimal Strain Theory:

$$U_x = 5yX^2 + X^4y^2 + 5yz^3 - 2xy^4$$

$$U_y = 8x^4 + 5xy^3 + 2yz^2 - xy^4$$

$$U_z = 2y^3x - 2x^3yz^2 - 5yz^3 - xy^2$$

$$\epsilon_x = \frac{\partial U_x}{\partial x} = 4x^3y^2 - 2y^4 + 10xy \quad | \quad \epsilon_{xy} = \frac{1}{2} \left( \frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x} \right)$$

$$\epsilon_y = \frac{\partial U_y}{\partial y} = -4xy^3 + 15xy^2 + 2z^2 \quad | \quad = \frac{1}{2} (2x^4y - 8xy^3 - y^4 + 32x^3 + 5y^3 + 5z^3 + 5x^2)$$

$$\epsilon_z = \frac{\partial U_z}{\partial z} = -4x^3yz - 15yz^2 \quad | \quad \epsilon_{xz} = \frac{1}{2} \left( \frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x} \right)$$

$$\epsilon_{yz} = \frac{1}{2} \left( \frac{\partial U_y}{\partial z} + \frac{\partial U_z}{\partial y} \right) = \frac{1}{2} (-6x^2yz^2 + 2y^3 + 15yz^2 - y^2) \\ = \frac{1}{2} (-2x^3z^2 + 6xy^2 - 5z^3 - 2xy + 4yz)$$

\*All derivatives calculated using Maple software.

## 2] Compatibility Check:

$$0 \stackrel{?}{=} 2 \frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} - \frac{\partial^2 \epsilon_y}{\partial z^2} - \frac{\partial^2 \epsilon_z}{\partial y^2} = 0 \checkmark$$

$$0 \stackrel{?}{=} 2 \frac{\partial^2 \epsilon_{xz}}{\partial x \partial z} - \frac{\partial^2 \epsilon_z}{\partial x^2} - \frac{\partial^2 \epsilon_x}{\partial z^2} = 0 \checkmark$$

$$0 \stackrel{?}{=} 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} - \frac{\partial^2 \epsilon_x}{\partial y^2} - \frac{\partial^2 \epsilon_y}{\partial x^2} = 0 \checkmark$$

$$0 \stackrel{?}{=} \frac{\partial}{\partial x} \left( - \frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} \right) - \frac{\partial^2 \epsilon_x}{\partial y \partial z} = 0 \checkmark$$

$$0 \stackrel{?}{=} \frac{\partial}{\partial y} \left( - \frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} \right) - \frac{\partial^2 \epsilon_y}{\partial x \partial z} = 0 \checkmark$$

$$0 \stackrel{?}{=} \frac{\partial}{\partial z} \left( - \frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} \right) - \frac{\partial^2 \epsilon_z}{\partial x \partial y} = 0 \checkmark$$

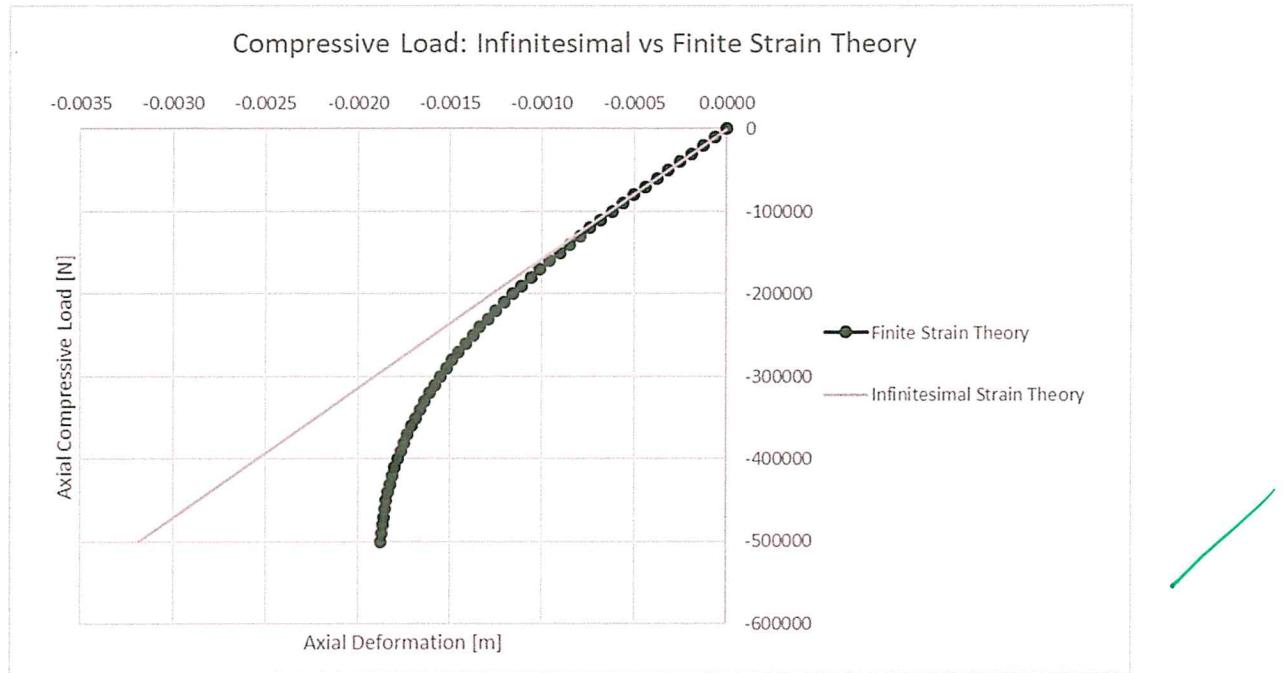
∴ Compatibility of strain field is satisfied.

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SAMPLE

**Problem 3:**

In the plot below of the Compressive load as a function of axial deformation, it is shown that when large displacements are considered (finite strain theory) the required load is higher in magnitude than that predicted using infinitesimal strain theory. This is because finite strain theory considers the increase in cross-sectional area which exponentially resists deformation.

**Problem 4 and 5:**

In the plot below of the Tensile load as a function of axial deformation, it is shown that when large displacements are considered (finite strain theory) the required load is lower in magnitude than that predicted using infinitesimal strain theory. This is because finite strain theory considers the decrease in cross-sectional area which required less force to stretch. When comparing curves for  $\nu=0.3$  and  $\nu=0.5$ , it can be seen less force is required when  $\nu=0.5$  since the lateral strain is higher and thus cross-section area decreases quicker than if  $\nu=0.3$ .

