ME 478 FINITE ELEMENT METHOD

Chapter 6.25 Flowchart to Compute K

Step 1 Decide what Gaussian Rule you want to use

(i.e. s_i, t_i and W_i)

Step 2 Calculate the shape functions and their derivatives

$$N_a(s_{i,}t_i), \quad \frac{\partial N_a(s_{i,}t_i)}{\partial s}, \text{ and } \frac{\partial N_a(s_{i,}t_i)}{\partial s} \qquad \begin{matrix} i = 1,2 \ldots n_{\text{int}} \\ a = 1,2 \ldots n_{node} \end{matrix}$$

Step 3 Calculate the Jacobian matrix and the Jacobian

$$\frac{\partial x}{\partial s} = \sum_{a=1}^{n_{node}} \frac{\partial N_a(s_{i,}t_i)}{\partial s} x_a, \qquad \frac{\partial y}{\partial s} = \sum_{a=1}^{n_{node}} \frac{\partial N_a(s_{i,}t_i)}{\partial s} y_a$$

$$\frac{\partial x}{\partial t} = \sum_{a=1}^{n_{node}} \frac{\partial N_a(s_{i,}t_i)}{\partial t} x_a, \qquad \frac{\partial y}{\partial t} = \sum_{a=1}^{n_{node}} \frac{\partial N_a(s_{i,}t_i)}{\partial t} y_a$$

$$\left|J(s_{i,}t_i)\right| = \det\begin{bmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{bmatrix} = \frac{\partial x}{\partial s}(s_{i,}t_i)\,\frac{\partial y}{\partial t}(s_{i,}t_i) - \frac{\partial x}{\partial t}(s_{i,}t_i)\,\frac{\partial y}{\partial s}(s_{i,}t_i)$$

Step 4 Calculate the derivatives of the shape functions w.r.t. the global coordinate system at the Gauss points

$$\frac{\partial N_a(s_i,t_i)}{\partial x} = \frac{1}{\left|J(s_i,t_i)\right|}\left(\frac{\partial N_a(s_i,t_i)}{\partial s}\frac{\partial y}{\partial t}(s_i,t_i) - \frac{\partial N_a(s_i,t_i)}{\partial t}\frac{\partial y}{\partial s}(s_i,t_i)\right)$$

$$\frac{\partial N_a(s_i,t_i)}{\partial y} = \frac{1}{\left|J(s_i,t_i)\right|}\left(-\frac{\partial N_a(s_i,t_i)}{\partial s}\frac{\partial x}{\partial t}(s_i,t_i) + \frac{\partial N_a(s_i,t_i)}{\partial t}\frac{\partial x}{\partial s}(s_i,t_i)\right)$$

Step 5 Evaluate the stiffness matrix

$$\mathbf{k}^{(e)} = t\iint_A \mathbf{B}^T\mathbf{D}\mathbf{B}\left|\mathbf{J}\right|\, dA$$

$$\mathbf{k}^{(e)} = t\sum_{n=1}^{n_{\text{int}}} W_i \mathbf{B}^T(s_i,t_i)\mathbf{D}(s_i,t_i)\mathbf{B}(s_i,t_i)\left|\mathbf{J}(s_i,t_i)\right|$$