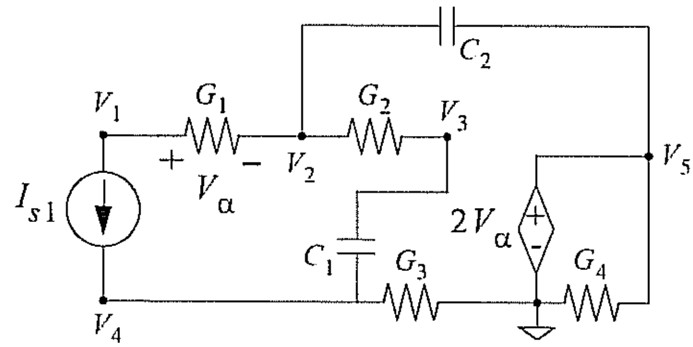
ECE4390 Lab 2 Quiz

Determine the node voltages for the circuit below in the frequency-domain at 30kHz. This question may only be done using MATLAB, your MNA program, by hand or any combination of the above. Full marks will be given for a correct final answer. If your answer is incorrect then part-marks will be awarded for any relevant work shown.



$G_1 = 40S$, $G_2 = 20S$, $G_3 = 20S$, $G_4 = 10S$, $C_1 = 0.5F$, $C_2 = 0.6F$, $I_{s1} = 1A$

$(\Delta S + T)\bar{v} = \bar{w}$, $\Delta = j\omega$, $\bar{v} = [V_1, V_2, V_3, V_4, V_5, I_1]^T$

From Ideal Element MNA Table:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 & -C_2 & 0 \\ 0 & 0 & C_1 & -C_1 & 0 & 0 \\ 0 & 0 & -C_1 & C_1 & 0 & 0 \\ 0 & -C_2 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad T = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2 & -G_2 & 0 & 0 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_4 & 1 \\ -2 & 2 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{w} = \begin{bmatrix} -I_{s_1} \\ 0 \\ 0 \\ I_{s_1} \\ 0 \\ 0 \end{bmatrix}$$

Plug-in Values and Solve using Matlab to get:

$V_1 = -0.075\,V \angle 0^\circ$

$V_2 = -0.05\,V \angle 0^\circ$

$V_3 = 0\,V$

$V_4 = 0\,V$

$V_5 = -0.05\,V \angle 0^\circ$

$(I_1 = 0.5A)$

ECE4390 Lab 3 Quiz

Referring to class notes and using Matlab is permitted during this quiz.

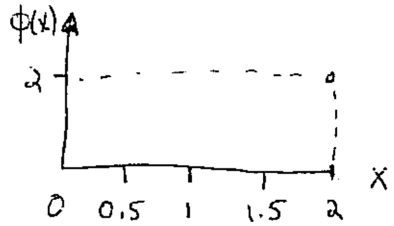
Consider the 1-D Laplace equation $\nabla^2\Phi(x) = 0$ with purely Dirichlet boundary conditions.

a) Draw the domain with finite difference grid points for $x \in [0...2]$, $\Delta x = 0.5$ and with boundary conditions $\phi(0) = 0$ and $\phi(2) = 2$. Label any axes depicted. Using a central difference approximation to Laplace's equation, give the matrix equation representing this boundary value problem in the form $A\underline{\phi} = \underline{b}$.

b) Assume that the SOR method is used to solve the matrix equation determined in (a). Show that this method will converge to a solution for an over-relaxation factor of $\omega = 1.2$.

c) Now assume that the true solution to the matrix equation from (a) is $\underline{\phi} = [0.5\ 1.0\ 1.5]^T$. If the initial guess was chosen to be $\underline{\phi}^0 = [1\ 1\ 1]^T$, what is the error vector for the computed solution using the Gauss-Seidel method at iteration $k = 20$? (The error vector is defined as $\underline{\epsilon}^k = \underline{\phi} - \underline{\phi}^k$)

a)



$$\frac{\phi_0 - 2\phi_1 + \phi_2}{\Delta x^2} = 0$$
$$\frac{\phi_1 - 2\phi_2 + \phi_3}{\Delta x^2} = 0 \quad \Rightarrow$$
$$\frac{\phi_2 - 2\phi_3 + \phi_4}{\Delta x^2} = 0$$

$$-2\phi_1 + \phi_2 = 0$$
$$\phi_1 - 2\phi_2 + \phi_3 = 0 \quad \rightarrow \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} =$$
$$\phi_2 - 2\phi_3 = -2$$

b) From notes 5 (p.14), SOR update eq is: $Mx^{(K+1)} = (M-A)x^{(K)} + b \iff$
$x^{(K+1)} = M^{-1}(M-A)x^{(K)} + M^{-1}b$

where $A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$, $M = \begin{bmatrix} \frac{-2}{\omega} & 0 & 0 \\ 1 & \frac{-2}{\omega} & 0 \\ 0 & 1 & \frac{-2}{\omega} \end{bmatrix}$

From theorem notes 5 (p.17), this method is convergent if eigen values of $M^{-1}(M-A)$ (denoted $\underline{\lambda}$) lie within complex unit circle

using Matlab command: abs(eig(inv(M) * (M-A)));

we get $\underline{\lambda} = [0.2, 0.2, 0.2]^T < 1$ ✓

c) From notes 5 (p 13) for Gauss-Seidel Method:

$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$, $M = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix}$

From notes 5 (p16) error at step K is $\underline{e}^{(K)} = (M^{-1}(M-A))^{(K)}\underline{e}^{(0)}$

where $\underline{e}^{(0)}$ is error vector for initial guess: $\underline{e}^{(0)} = \underline{\phi} - \underline{\phi}^{(0)}$

using Matlab; (inv(M) * (M-A))^20 * [-0.5; 0; 0.5] gives:

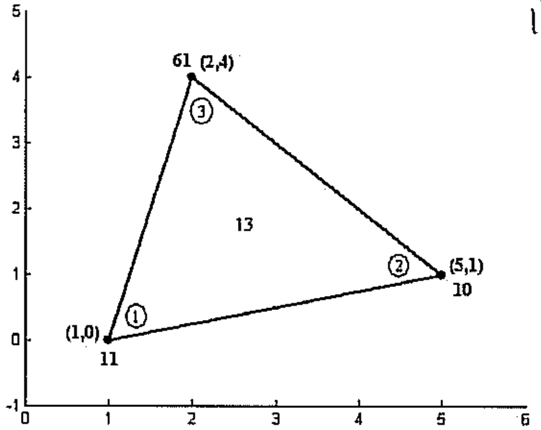
$e^{(20)} = [0.477e-6, 0.477e-6, 0.238e-6]^T$

ECE4390 Lab 4 Quiz

Suppose that the solution to a 2D Poisson equation ($\nabla^2\Phi(x,y) = f(x,y)$) is to be computed using the Finite Element Method for a particular domain discretization. The triangle displayed below is element number 13 from the domain mesh. Its global node numbers are in boldface, its local node numbers are circled, its node coordinates are in round brackets and it's area is $7.5cm^2$.

1. Suppose that the local matrix equation for this element is of the form $S^{(13)}\underline{\phi} = \underline{b}^{(13)}$. If the right-hand side of the Poisson equation is set to $f(x,y) = 3$, give the local stiffness matrix, $S^{(13)}$, and local right-hand side, $\underline{b}^{(13)}$, for this element.

2. Where will the values of $S^{(13)}$ and $\underline{b}^{(13)}$ be added into the global stiffness matrix and global right-hand side? Give your answer in [row,column] format.



1) from notes 8:

$$S^{(t)} = \frac{1}{4A_e^{(t)}}\begin{bmatrix} b_ib_i + c_ic_i & b_ib_j + c_ic_j & b_ib_m + c_ic_m \\ b_jb_i + c_jc_i & b_jb_j + c_jc_j & b_jb_m + c_jc_m \\ b_mb_i + c_mc_i & b_mb_j + c_mc_j & b_mb_m + c_mc_m \end{bmatrix}$$

where $b_i = y_j - y_m$, $c_i = x_m - x_j$,

and $i \rightarrow j \rightarrow m \rightarrow i$ and $A_e^{(13)} = 7.5cm^2$

$\Rightarrow b_i = -3 \quad c_i = -3$

$b_j = 4 \quad c_j = -1$

$b_m = -1 \quad c_m = 4$

$$\therefore S^{(13)} = \frac{1}{4(7.5)}\begin{bmatrix} 18 & -9 & -9 \\ -9 & 17 & -8 \\ -9 & -8 & 17 \end{bmatrix}$$

From notes 8 (p.3), since $f(x,y) = \rho/\varepsilon = 3$ and $\underline{b}^{(t)} = \frac{1}{3}\begin{pmatrix} \rho/\varepsilon \, A_e^{(t)} \\ \rho/\varepsilon \, A_e^{(t)} \\ \rho/\varepsilon \, A_e^{(t)} \end{pmatrix}$

$$\Rightarrow \underline{b}^{(13)} = \begin{pmatrix} 7.5 \\ 7.5 \\ 7.5 \end{pmatrix}$$

2) from notes 8 (p.4) and global numbers of element 13:

$S^{(13)}(1,1) \Rightarrow S(11,11)$ $\quad S^{(13)}(2,1) \rightarrow S(10,11)$ $\quad S^{(13)}(3,1) \rightarrow S(61,11)$

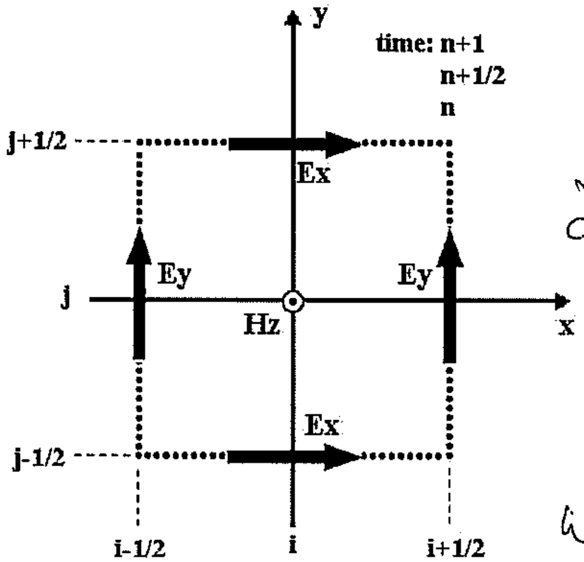
$S^{(13)}(1,2) \rightarrow S(11,10)$ $\quad S^{(13)}(2,2) \rightarrow S(10,10)$ $\quad S^{(13)}(3,2) \rightarrow S(61,10)$

$S^{(13)}(1,3) \rightarrow S(11,61)$ $\quad S^{(13)}(2,3) \rightarrow S(10,61)$ $\quad S^{(13)}(3,3) \rightarrow S(61,61)$

$\underline{b}^{(13)}(1) \rightarrow \underline{b}(11)$ $\quad \underline{b}^{(13)}(2) \rightarrow \underline{b}(10)$ $\quad \underline{b}^{(13)}(3) \rightarrow \underline{b}(61)$

ECE4390 Lab 5 Quiz

Write the update equation for the H_z component of the leap-frog scheme for EM-field propagation in two dimensions (Transverse Electric to z case). Use the given computational molecule below and use a center-difference approximation for the derivatives. It can be assumed that the propagation medium is free-space.



From notes 9 (p. 12) for TE case:

$$\partial_t \begin{pmatrix} E_x \\ E_y \\ H_z \end{pmatrix} + \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & m \end{bmatrix} \partial_x \begin{pmatrix} 0 \\ H_z \\ E_y \end{pmatrix} + \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & m \end{bmatrix} \partial_y \begin{pmatrix} -H_z \\ 0 \\ -E_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where $e = 1/\varepsilon_0$, $m = 1/\mu_0$

from the above system we are only interested in:

$$\partial_t H_z + m\partial_x E_y - m\partial_y E_x = 0$$

using central differencing leap-frog for given molecule:

$$\frac{H_{z_{ij}}^{n+1} - H_{z_{ij}}^{n}}{K} + m\,\frac{E_{y_{i+\frac{1}{2}j}}^{n+\frac{1}{2}} - E_{y_{i-\frac{1}{2}j}}^{n+\frac{1}{2}}}{h} - m\,\frac{E_{x_{ij+\frac{1}{2}}}^{n+\frac{1}{2}} - E_{x_{ij-\frac{1}{2}}}^{n+\frac{1}{2}}}{h} = 0$$

where $K = \Delta t$, $h = \Delta x = \Delta y$, and n is time step

re arrange to obtain:

$$H_{z_{ij}}^{n+1} = H_{z_{ij}}^{n} - \frac{mK}{h}\left(E_{y_{i+\frac{1}{2}j}}^{n+\frac{1}{2}} - E_{y_{i-\frac{1}{2}j}}^{n+\frac{1}{2}} - E_{x_{ij+\frac{1}{2}}}^{n+\frac{1}{2}} + E_{x_{ij-\frac{1}{2}}}^{n+\frac{1}{2}} \right)$$