UNIVERSITY OF MANITOBA

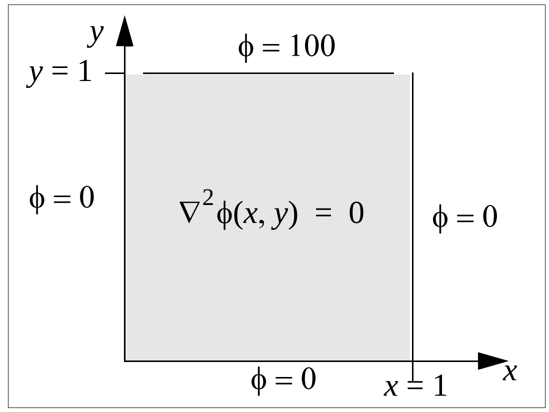
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

24.781 COMPUTATIONAL ELECTROMAGNETICS

ASSIGNMENT 1
Solution of Electrostatic Problems by Finite Difference Methods

September 24, 2003 **Due Date: Wednesday, October 8, 2003**

A) For the two dimensional rectangular region shown in the figure write a program (any language) to solve for the scalar potential $\phi(x,y)$ on a rectangular grid using Successive Over-Relaxation (SOR).



Use the relative displacement norm, ε, to stop the iterative loop. The relative displacement norm at the $(m+1)^{th}$ iteration is derived from the displacement norm δ as:

$$\delta = \sum_{i=1}^{N} \left|\phi_i^{(m+1)} - \phi_i^{(m)}\right| = \|\Delta\phi\| \qquad \varepsilon = \frac{\delta}{\|\phi\|} = \delta \Big/ \sum_{i=1}^{N} \left|\phi_i^{(m+1)}\right|$$

where N is the total number of grid points in the two dimensional grid.

B) Using $h = \Delta x = \Delta y = 0.1$ determine the optimum over-relaxation constant ω_{opt}. Choose $\varepsilon = 10^{-6}$ as the stopping condition. Plot number of iterations vs. ω.

C) Using $h = \Delta x = \Delta y = 0.1$ and the optimum over-relaxation constant ω_{opt} found in part B, plot the required iteration number with respect to relative displacement norm ε. Choose $\varepsilon = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}\}$. What happens when ε is set too low?

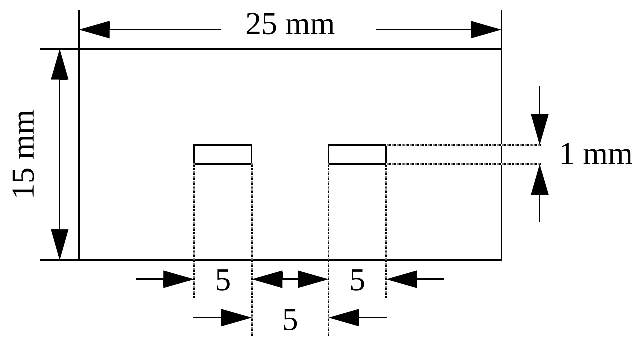
D) Decrease h by a factor of 2 three times to $h = \Delta x = \Delta y = \{0.05, 0.025, 0.0125\}$ and set $\varepsilon = 10^{-6}$. Compare the required number of iterations for each h using the optimum over-relaxation constant ω_{opt} found in part B.

E) Check the total flux emanating from the enclosed region by numerically integrating the normal component of the electric field around the boundary. Show that it is equal to zero and discuss why this should be so.

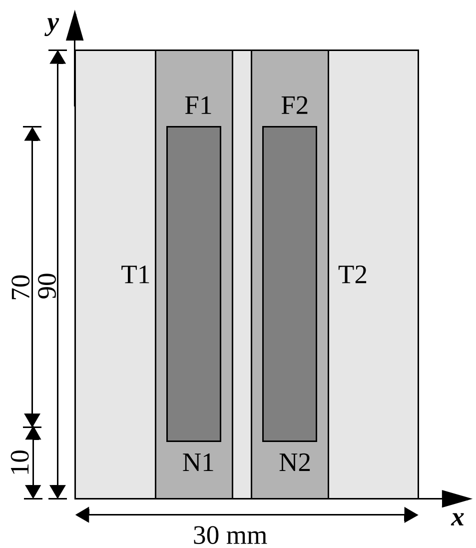
F) Reformulate the problem taking advantage of symmetry across the line defined by $x = 0.5$. What boundary conditions are required for the new problem? Modify your computer program for this new problem and check that the results are the same.

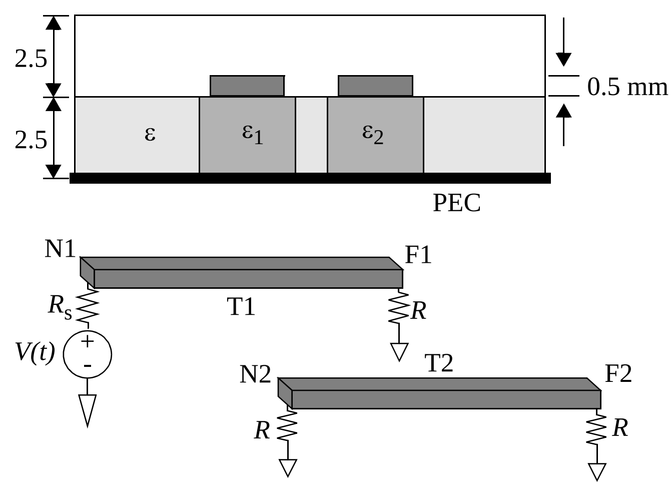
G) Derive the analytic solution to this problem by eigenfunction expansion and check the finite difference solution at the point (x = 0.5, y = 0.5) using as many terms as required to justify a valid comparison.

H) Modify your program to solve for the Capacitance matrix of the transmission line shown in the figure.



I) Modify one of your programs to solve for the capaciatnce matrix of the transmission line shown in the figure. The width of the striplines is 3 mm and they are spaced 4 mm apart. The dielectric strips are centered under the stripline conductors and are 4 mm wide. Assume that a perfect electric conductor surrounds the striplines completely.





The substrate is made of alumina (*i.e.* a ceramic ε = 10) and the dielectric strips have the following dielectric constant values:

Table 1: Nine Cases of Ceramic Substrate Compensation (ε = 10.0)

Case	1	2	3	4	5	6	7	8	9
ε_1	10.0	10.0	4.7	2.2	2.2	2.2	4.7	4.7	10.0
ε_2	4.7	2.2	10.0	10.0	2.2	4.7	4.7	2.2	10.0