UNIVERSITY OF MANITOBA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

## 24.781 COMPUTATIONAL ELECTROMAGNETICS

# ASSIGNMENT 2 Part A
# Time Domain Electromagnetics in 1-D by Finite Difference Methods

**October 1, 2003** **Due Date: Wednesday, October 22, 2003**

As was discussed in class, Maxwell's equations for the one-dimensional time-domain case can be written as

$$\partial_t \boldsymbol{u}(x, t) + A\partial_x \boldsymbol{u}(x, t) = \mathbf{0}$$

where the solution vector $\boldsymbol{u}$ is given by

$$\boldsymbol{u} = \begin{bmatrix} E_y \\ H_z \end{bmatrix}, \text{ and } A = \begin{bmatrix} 0 & e \\ m & 0 \end{bmatrix}, \; e = \frac{1}{\varepsilon}, \; m = \frac{1}{\mu}, \; c^2 = me.$$

For the case of the grid function given by $\boldsymbol{u}_i^n \cong \boldsymbol{u}(i\Delta x, n\Delta t)$ and shown in Figure 1, write a program to calculate the value of $\boldsymbol{u}_i^{100}$ using the discretized analytic solution <u>and</u> the Yee version of the Leap-Frog scheme. Comment the code and include it as an appendix to your assignment. The initial conditions, $\boldsymbol{u}_i^0$, are given by the function shown in Figure 2.

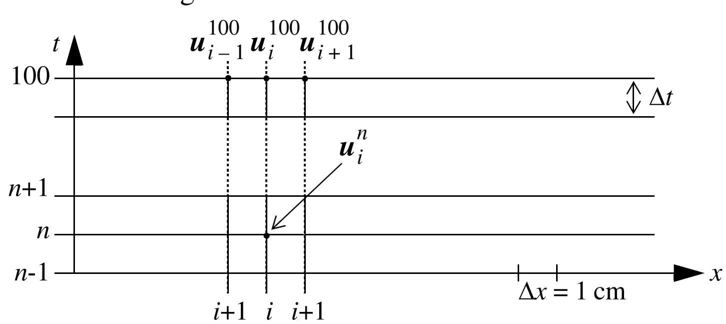


Figure 1. The 1-D FDTD Grid

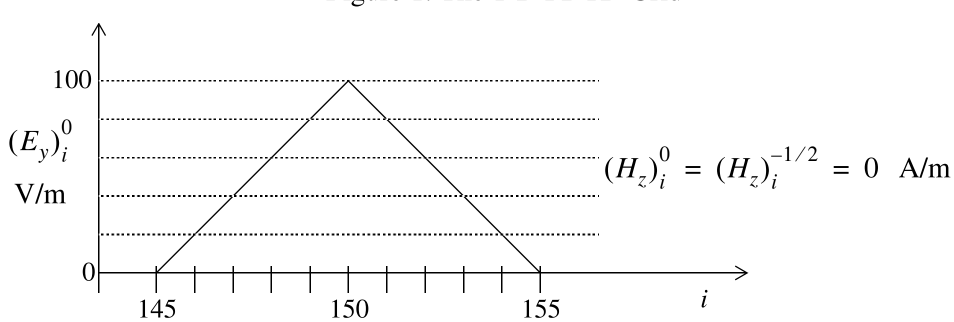


Figure 2. Initial Conditions: $(E_y)_i^0$ and $(H_z)_i^0$.

Plot the spatial distribution of the solution vector, *i.e.* both $E_y$ and $H_z$, at $t = 100\,\Delta t$ for both numerical methods (Note that in the Yee algorithm $n$ = 100 means $(E_y)_i^{100}$ and $(H_z)_i^{100.5}$). For the Yee algorithm use a time step of $\Delta t = \Delta x/c$. What happens if in the Yee algorithm if you set $\Delta t = 1.001(\Delta x/c)$? Plot the solution vector using the Yee algorithm at $t = 100\,\Delta t$ if $\Delta t = 0.8(\Delta x/c)$. Comment on the results.

Using the initial Gaussian electric field distribution shown in figure 3 as your initial conditions, modify your programs to include perfectly conducting walls at the points $i$ = 300 and $i$ = 0, a dielectric slab of relative permittivity $\varepsilon_r$ = 3 between the points $i$ = 200 and $i$ = 250 inclusive, and a lossy slab of conductivity $\sigma$ = .01 [S/m] between the points $i$ = 1 and $i$ = 50 inclusive (see figure 4).

Plot the electric and magnetic fields at times $n$ = {25, 50, 75, 100, 125, 150, 175, 200} across the whole x axis.

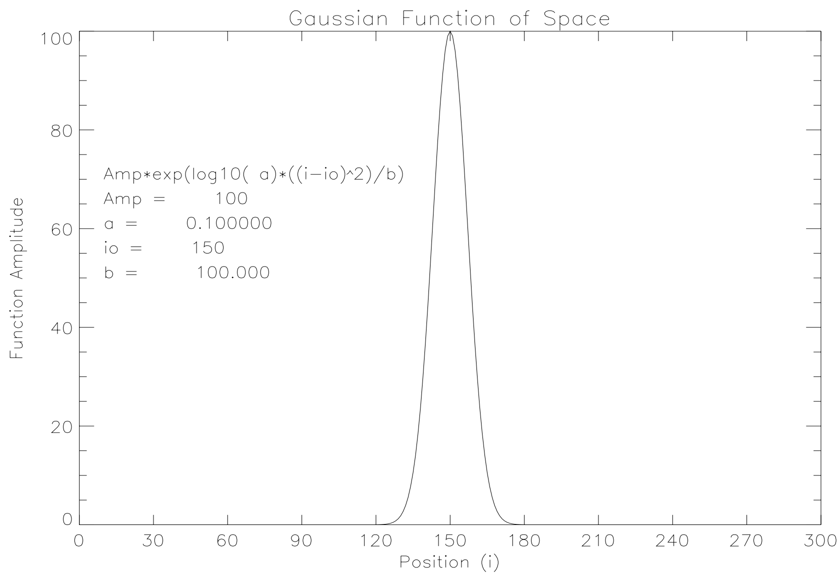


Figure 3. Initial conditions on electric field: $E_i^0 = 100\,exp\left[-\left(\frac{(i-150)^2}{100}\right)\right]$

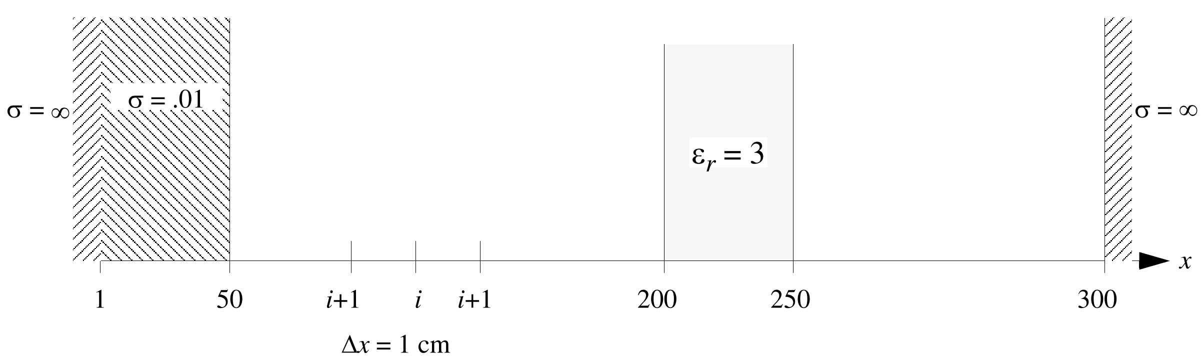


Figure 4. Dielectric slab and perfectly conducting boundaries.

**Pseudo Code (Discretized Analytic Solution)**

```
declare E(300), H(300), Enew(300), Hnew(300)
set eps = 8.854e-12, mu = 4πe-7, Z = sqrt(mu/eps), Y = 1/Z
for i = 1(1)300                  / initialize all fields to zero
        set E(i) = 0.0, H(i) = 0.0
end
for i = 146(1)150                / set triangular wave as initial conditions
        set E(i) = (i-145)*20, E(i+5) = (150-i)*20
end
for n=1(1)100                    / time steps
        for i = 2(1)299          / calculate new field values from old
                set Enew(i) = 0.5*[E(i+1)+E(i-1)+Z*(H(i-1)-H(i+1))]
                    Hnew(i) = 0.5*[H(i+1)+H(i-1)+Y*(E(i-1)-E(i+1))]
        end
        for i = 2(1)299          / replace old field values with new current ones
                set E(i) = Enew(i)
                    H(i) = Hnew(i)
        end
end
output E(i), H(i), i=1(1)300     / output the field values after 100 time steps
```

**Pseudo Code (Yee version of Leap-Frog Scheme)**

```
declare E(300), H(300)
input Δx, Courant                / input spatial step size and Courant number
set eps = 8.854e-12, mu = 4πe-7, m = 1/mu, e = 1/eps
coef = 0.5                       / coef of 0.5 is used in the first time step
C = sqrt(m*e)                    / maximum speed of propagation
Δt = Δx*Courant/C                / set time step
for i = 1(1)300                  / initialize all fields to zero
        set E(i) = 0.0, H(i) = 0.0
end
for i = 146(1)150                / set triangular wave as initial conditions
        set E(i) = (i-145)*20, E(i+5) = (150-i)*20
end
for n=1(1)100
        if n=2 set coef = 1.0
        for i = 2(1)299          / be careful here: should be two loops later
                set H(i) = H(i) - coef*(Δt/Δx)*m*[E(i+1)-E(i)]
                    E(i) = E(i) - (Δt/Δx)*e*[H(i)-H(i-1)]
        end
end
output E(i), H(i), i=1(1)300     / output field values after 100 time steps
```