Generalized Medical Instrumentation System

The sensors convert energy or information from the measurand to another form of signal (usually electric). This signal is then processed and displayed so that humans can perceive this information.

Characteristics of an instrument performance are usually subdivided into 2 classes on the basis of the frequency of the input signals: **1- Static characteristics**, which describe the performance of instrument for dc or very low frequency inputs; **2- Dynamic characteristics**, which requires the use of differential and/or integral equations to describe the quality of the measurements.

1. <u>Static characteristics</u>

- 1.1 **Accuracy** is the difference between the true value and the measured value/ divided by the true value. Since the true value is seldom available, the accepted true value or reference value should be treacle to the nation Instrumentation of Standards and Technology. Accuracy is a measure of the total error without regard to the type of the error.
- 1.2 **Precision** of a measurement expresses the number of distinguishable alternations from which a result is selected. For example, a meter that can read 2.432 V is more precise than 2.43 V. However, high precision doesn't mean high accuracy!
- 1.3 *Resolution* is the smallest incremental quantity that can be measured.
- 1.4 *Reproducibility* is the ability of an instrument to give the same output for equal inputs applied over some period of time.
- 1.5 **Statistical Control-** The accuracy of an instrument is not meaningful unless all factors, such as the environment and the method of use, are considered. Statistical control ensures that random variations in measured quantities that result from all factors that influence the measurement process are tolerable. Any systematic error or bias can be removed by calibration and correction factors, but random variation are more of concern. If the source of this variability cannot be determined, then statistical analysis must be used to determine the error variation.
- 1.6 **Sensitivity** is the ratio of the incremental output to the incremental input quantity. The static sensitivity might be constant for only a part of operation.
- 1.7 **Zero Drift** occurs when all the output values increase or decrease by the same absolute amount. The slope of sensitivity doesn't change. It might be caused by temperature variation, hysteresis variation, shock or undesired forces.
- 1.8 **Sensitivity Drift** is the change of the slope of the calibration curve as a result of an interfering and/or modifying input. It can result from manufacturing tolerances, variation in power supply, nonlinearities, and changes in ambient temperature and pressure.
- 1.9 *Linearity* Independent nonlinearity expresses the maximal deviation of points from the least squares fitted line as A% of the reading or B% of full scale, whichever is greater.

- 1.10 *Input Ranges* is the maximal operating range of input that doesn't damage the instrument.
- 1.11 *Input Impedance* in general is the ratio of the input effort (voltage, force, pressure) by the flow (current, velocity, flow). Biological source impedances are usually unknown, variable and difficult to measure and control.

2. Dynamic Characteristics

Most of the biological signals time-varying in nature and therefore we should make sure that the instrument is a time-invariant system for an accurate measurement. The dynamic characteristics of an instrument include its transfer function, its frequency response, and its phase or time delay.

small percent-of-reading deviations near zero. All data points must fall inside the "funnel" shown in Figure 1.4(b). For most instruments that are essentially linear, if other sources of error are minimal, the accuracy is equal to the nonlinearity.

INPUT RANGES

Several maximal ranges of allowed input quantities are applicable for various conditions. Minimal resolvable inputs impose a lower bound on the quantity to be measured. The normal linear operating range specifies the maximal or near-maximal inputs that give linear outputs.

The static linear range and the dynamic linear range may be different. The maximal operating range is the largest input that does not damage the instrument. Operation in the upper part of this range is more likely to be nonlinear. Finally, storage conditions specify environmental and interfering input limits that should not be exceeded when the instrument is not being used. These ranges are not always symmetric with respect to zero input, particularly for storage conditions. Typical operating ranges for blood-pressure sensors have a positive bias, such as +200 mm Hg to -60 mm Hg (+26.6 to -8.0 kPa).

INPUT IMPEDANCE

Because biomedical sensors and instruments usually convert nonelectric quantities into voltage or current, we introduce a generalized concept of input impedance. This is necessary so that we can properly evaluate the degree to which instruments disturb the quantity being measured. For every desired input X_{d1} that we seek to measure, there is another implicit input quantity X_{d2} such that the product $X_{d1} \cdot X_{d2}$ has the dimensions of power. This product represents the instantaneous rate at which energy is transferred across the tissue–sensor interface. The generalized input impedance Z_x is the ratio of the phasor equivalent of a steady-state sinusoidal *effort* input variable (voltage, force, pressure) to the phasor equivalent of a steady-state sinusoidal *flow* input variable (current, velocity, flow).

$$Z_x = \frac{X_{d1}}{X_{d2}} = \frac{\text{effort variable}}{\text{flow variable}}$$
(1.12)

The power P is the time rate of energy transfer from the measurement medium.

$$P = X_{d1} \cdot X_{d2} = \frac{X_{d1}^2}{Z_x} = Z_x X_{d2}^2$$
(1.13)

To minimize P, when measuring effort variables X_{d1} , we should make the generalized input impedance as large as possible. This is usually achieved by

minimizing the flow variable. However, most instruments function by measuring minute values of the flow variable, so the flow variable cannot be reduced to zero. On the other hand, when we are measuring flow variables X_{d2} , small input impedance is needed to minimize P. The loading caused by measuring devices depends on the magnitude of the input impedance $|Z_x|$ compared with the magnitude of the source impedance $|Z_s|$ for the desired input. Unfortunately, biological source impedances are usually unknown, variable, and difficult to measure and control. Thus the instrument designer must usually focus on maximizing the input impedance Z_x for effort-variable measurement. When the measurand is a flow variable instead of an effort variable, it is more convenient to use the admittance $Y_x = 1/Z_x$ than the impedance.

1.10 GENERALIZED DYNAMIC CHARACTERISTICS

Only a few medical measurements, such as body temperature, are constant or slowly varying quantities. Most medical instruments must process signals that are functions of time. It is this time-varying property of medical signals that requires us to consider dynamic instrument characteristics. Differential or integral equations are required to relate dynamic inputs to dynamic outputs for continuous systems. Fortunately, many engineering instruments can be described by ordinary linear differential equations with constant coefficients. The input x(t) is related to the output y(t) according to the following equation:

$$a_n \frac{d^n y}{dt^n} + \dots + a_1 \frac{dy}{dt} + a_0 y(t) = b_m \frac{d^m x}{dt^m} + \dots + b_1 \frac{dx}{dt} + b_0 x(t)$$
(1.14)

where the constants a_i (i = 0, 1, ..., n) and b_j (j = 0, 1, ..., m) depend on the physical and electric parameters of the system. By introducing the differential operator $D^k \equiv d^k()/dt^k$, we can write this equation as

 $(a_n D^n + \dots + a_1 D + a_0)y(t) = (b_m D^m + \dots + b_1 D + b_0)x(t)$ (1.15)

Readers familiar with Laplace transforms may recognize that D can be replaced by the Laplace parameter s to obtain the equation relating the transforms Y(s) and X(s). This is a *linear* differential equation, because the linear properties stated in Figure 1.4(a) are assumed and the coefficients a_i and b_j are not functions of time or the input x(t). The equation is ordinary, because there is only one independent variable y. Essentially such properties mean that the instrument's methods of acquiring and analyzing the signals do not change as a function of time or the quantity of input. For example, an autoranging instrument may violate these conditions.

Most practical instruments are described by differential equations of zero, first, or second order; thus n = 0, 1, 2, and derivatives of the input are usually absent, so m = 0.

The input x(t) can be classified as transient, periodic, or random. No general restrictions are placed on x(t), although, for particular applications, bounds on amplitude and frequency content are usually assumed. Solutions for the differential equation depend on the input classifications. The step function is the most common transient input for instrumentation. Sinusoids are the most common periodic function to use because, through the Fourier-series expansion, any periodic function can be approximated by a sum of sinusoids. Band-limited white noise (uniform-power spectral content) is a common random input because one can test instrument performance for all frequencies in a particular bandwidth.

TRANSFER FUNCTIONS

The transfer function for a linear instrument or system expresses the relationship between the input signal and the output signal mathematically. If the transfer function is known, the output can be predicted for any input. The *operational transfer function* is the ratio y(D)/x(D) as a function of the differential operator D.

$$\frac{y(D)}{x(D)} = \frac{b_m D^m + \dots + b_1 D + b_0}{a_n D^n + \dots + a_1 D + a_0}$$
(1.16)

This form of the transfer function is particularly useful for transient inputs. For linear systems, the output for transient inputs, which occur only once and do not repeat, is usually expressed directly as a function of time, y(t), which is the solution to the differential equation.

The frequency transfer function for a linear system is obtained by substituting $j\omega$ for D in (1.16).

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{b_m(j\omega)^m + \dots + b_1(j\omega) + b_0}{a_n(j\omega)^n + \dots + a_1(j\omega) + a_0}$$
(1.17)

where $j = +\sqrt{-1}$ and ω is the angular frequency in radians per second. The input is usually given as $x(t) = A_x \sin(\omega t)$, and all transients are assumed to have died out. The output y(t) is a sinusoid with the same frequency, but the amplitude and phase depend on ω ; that is, $y(t) = B(\omega) \sin[\omega t + \phi(\omega)]$. The frequency transfer function is a complex quantity having a magnitude that is the ratio of the magnitude of the output to the magnitude of the input and a phase angle ϕ that is the phase of the output y(t) minus the phase of the input x(t). The phase angle for most instruments is negative. We do not usually express the output of the system as y(t) for each frequency, because we know that it is just a sinusoid with a particular magnitude and phase. Instead, the amplitude ratio and the phase angle are given separately as functions The dynamic characteristics of instruments are illustrated below by examples of zero-, first-, and second-order linear instruments for step and sinusoidal inputs.

ZERO-ORDER INSTRUMENT

The simplest nontrivial form of the differential equation results when all the *a*'s and *b*'s are zero except a_0 and b_0 .

$$a_0 y(t) = b_0 x(t) \tag{1.18}$$

This is an algebraic equation, so

$$\frac{y(D)}{x(D)} = \frac{Y(j\omega)}{X(j\omega)} = \frac{b_0}{a_0} = K = \text{static sensitivity}$$
(1.19)

where the single constant K replaces the two constants a_0 and b_0 . This zeroorder instrument has ideal dynamic performance, because the output is proportional to the input for all frequencies and there is no amplitude or phase distortion.

A linear potentiometer is a good example of a zero-order instrument. Figure 1.5 shows that if the potentiometer has pure uniform resistance, then the output voltage y(t) is directly proportional to the input displacement x(t), with no time delay for any frequency of input. In practice, at high frequencies, some parasitic capacitance and inductance might cause slight distortion. Also, low-resistance circuits connected to the output can load this simple zero-order instrument.

FIRST-ORDER INSTRUMENT

If the instrument contains a single energy-storage element, then a first-order derivative of y(t) is required in the differential equation.

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$
(1.20)

This equation can be written in terms of the differential operator D as

$$(\tau D + 1)y(t) = Kx(t)$$
 (1.21)

where $K = b_0/a_0$ = static sensitivity, and $\tau = a_1/a_0$ = time constant.

Exponential functions offer solutions to this equation when appropriate constants are chosen. The operational transfer function is



Figure 1.5 (a) A linear potentiometer, an example of a zero-order system. (b) Linear static characteristic for this system. (c) Step response is proportional to input. (d) Sinusoidal frequency response is constant with zero phase shift.

$$\frac{y(D)}{x(D)} = \frac{K}{1+\tau D} \tag{1.22}$$

and the frequency transfer function is

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{1+j\omega\tau} = \frac{K}{\sqrt{1+\omega^2\tau^2}} / \phi = \arctan\left(-\omega\tau/1\right)$$
(1.23)

The *RC* low-pass filter circuit shown in Figure 1.6(a) is an example of a first-order instrument. The input is the voltage x(t), and the output is the voltage y(t) across the capacitor. The first-order differential equation for this circuit is RC[dy(t)/dt] + y(t) = Kx(t). The static-sensitivity curve given in Figure 1.6(b) shows that static outputs are equal to static inputs. This is verified by the differential equation because, for static conditions, dy/dt = 0. The step response in Figure 1.6(c) is exponential with a time constant $\tau = RC$.

$$y(t) = K(1 - e^{-t/\tau})$$
(1.24)



Figure 1.6 (a) A low-pass *RC* filter, an example of a first-order instrument. (b) Static sensitivity for constant inputs. (c) Step response for large time constants (τ_L) and small time constants (τ_S). (d) Sinusoidal frequency response for large and small time constants.

The smaller the time constant, the faster the output approaches the input. For sinusoids, (1.23) and Figure 1.6(d) show that the magnitude of the output decreases as frequency increases. For larger time constants, this decrease occurs at lower frequency.

When $\omega = 1/\tau$, the magnitude is $1/\sqrt{2} = 0.707$ times smaller, and the phase angle is -45° . This particular frequency ω is known as the *corner, cutoff*, or *break* frequency. Figure 1.6(d) verifies that this is a low-pass filter; low-frequency sinusoids are not severely attenuated, whereas high-frequency sinusoids produce very little output voltage. The ordinate of the frequency-response magnitude in Figure 1.6(d) is usually plotted on a log scale and may be given in units of decibels (dB), which are defined as dB = $20 \log_{10} |Y(j\omega)|$

 $X(j\omega)$ |. A mercury-in-glass thermometer is another example of a low-pass first-order instrument.

EXAMPLE 1.1 A first-order low-pass instrument has a time constant of 20 ms. Find the maximal sinusoidal input frequency that will keep output error due to frequency response less than 5%. Find the phase angle at this frequency.

ANSWER

$$\frac{T(j\omega)}{X(j\omega)} = \frac{K}{1+j\omega\tau}$$
$$\left|\frac{K}{1+j\omega\tau}\right| = \frac{K}{\sqrt{1+\omega^2\tau^2}} = 0.95K$$
$$(\omega^2\tau^2+1)(0.95)^2 = 1$$
$$\omega^2 = \frac{1-(0.95)^2}{(0.95)^2(20\times10^{-3})^2}$$
$$\omega = 16.4 \text{ rad/s}$$
$$f = \frac{\omega}{2\pi} = 2.62 \text{ Hz}$$
$$\phi = \tan^{-1}\left(\frac{-\omega\tau}{1}\right) = -18.2^4$$

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If *R* and *C* in Figure 1.6(a) are interchanged, the circuit becomes another first-order instrument known as a *high-pass filter*. The static characteristic is zero for all values of input, and the step response jumps immediately to the step voltage but decays exponentially toward zero as time increases. Thus $y(t) = Ke^{-t/\tau}$. Low-frequency sinusoids are severely attenuated, whereas high-frequency sinusoids are little attenuated. The sinusoidal transfer function is $Y(j\omega)/X(j\omega) = j\omega\tau/(1 + j\omega\tau)$.

SECOND-ORDER INSTRUMENT

An instrument is second order if a second-order differential equation is required to describe its dynamic response.

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$
(1.25)

Many medical instruments are second order or higher, and low pass. Furthermore, many higher-order instruments can be approximated by second-order characteristics if some simplifying assumptions can be made. The four constants in (1.25) can be reduced to three new ones that have physical significance:

$$\left[\frac{D^2}{\omega_n^2} + \frac{2\zeta D}{\omega_n} + 1\right] y(t) = Kx(t)$$
(1.26)

where

$$K = \frac{b_0}{a_0} = \text{static sensitivity, output units divided by input units}$$
$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \text{undamped natural frequency, rad/s}$$
$$\zeta = \frac{a_1}{2\sqrt{a_0a_2}} = \text{damping ratio, dimensionless}$$

Again exponential functions offer solutions to this equation, although the exact form of the solution varies as the damping ratio becomes greater than, equal to, or less than unity. The operational transfer function is

$$\frac{y(D)}{x(D)} = \frac{K}{\frac{D^2}{\omega_n^2} + \frac{2\zeta D}{\omega_n} + 1}$$
(1.27)

and the frequency transfer function is

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{(j\omega/\omega_n)^2 + (2\zeta j\omega/\omega_n) + 1}$$
$$= \frac{K}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + 4\zeta^2 \omega^2/\omega_n^2}} / \phi = \arctan\frac{2\zeta}{\omega/\omega_n - \omega_n/\omega}$$
(1.28)

A mechanical force-measuring instrument illustrates the properties of a second-order instrument (Doebelin, 1990). Mass, spring, and viscous-damping elements oppose the applied input force x(t), and the output is the resulting displacement y(t) of the movable mass attached to the spring [Figure 1.7(a)]. If the natural frequency of the spring is much greater than the frequency components in the input, the dynamic effect of the spring can be included by adding one-third of the spring's mass to the mass of the moving elements to obtain the equivalent total mass M.

Hooke's law for linear springs is assumed, so the spring constant is K_s . Dry friction is neglected and perfect viscous friction is assumed, with constant *B*.



Figure 1.7 (a) Force-measuring spring scale, an example of a second-order instrument. (b) Static sensitivity. (c) Step response for overdamped case $\zeta = 2$, critically damped case $\zeta = 1$, underdamped case $\zeta = 0.5$. (d) Sinusoidal steady-state frequency response, $\zeta = 2$, $\zeta = 1$, $\zeta = 0.5$. [Part (a) modified from *Measurement Systems: Application and Design*, by E. O. Doebelin. Copyright © 1990 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.]

To eliminate gravitational force from the equation, we adjust the scale until y = 0 when x = 0. Then the sum of the forces equals the product of mass and acceleration.

$$x(t) - B\frac{dy(t)}{dt} - K_{s}y(t) = M\frac{d^{2}y(t)}{dt^{2}}$$
(1.29)

This equation has the same form as (1.26) when the static sensitivity, undamped natural frequency, and damping ratio are defined in terms of K_s , B, and M, as follows:

 $K = 1/K_s \tag{1.30}$

$$\omega_{\rm n} = \sqrt{K_{\rm s}/M} \tag{1.31}$$

$$\zeta = \frac{B}{2\sqrt{K_{\rm m}M}}\tag{1.32}$$

The static response is y(t) = Kx(t), as shown in Figure 1.7(b). The step response can have three forms, depending on the damping ratio. For a unit-step input, these three forms are

Overdamped, $\zeta > 1$:

$$y(t) = -\frac{\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} K e^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + \frac{\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} K e^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t} + K$$
(1.33)

Critically damped, $\zeta = 1$:

$$v(t) = -(1 + \omega_{\rm n} t) K e^{-\omega_{\rm n} t} + K$$
(1.34)

Underdamped, $\zeta < 1$:

$$y(t) = -\frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} K \sin\left(\sqrt{1-\zeta^2}\omega_n t + \phi\right) + K$$

$$\phi = \arcsin\sqrt{1-\zeta^2}$$
(1.35)

Examples of these three step responses are represented in Figure 1.7(c). Only for damping ratios less than unity does the step response overshoot the final value. Equation (1.35) shows that the frequency of the oscillations in the underdamped response in Figure 1.7(c) is the damped natural frequency $\omega_d = \omega_n \sqrt{1 - \zeta^2}$. A practical compromise between rapid rise time and minimal overshoot is a damping ratio of about 0.7.

EXAMPLE 1.2 For underdamped second-order instruments, find the damping ratio ζ from the step response.

ANSWER To obtain the maximums for the underdamped response, we take the derivative of (1.35) and set it to zero. For $\zeta < 0.3$ we approximate positive maximums when the sine argument equals $3\pi/2$, $7\pi/2$, and so forth. This occurs at

$$t_n = \frac{3\pi/2 - \phi}{\omega_n \sqrt{1 - \zeta^2}}$$
 and $t_{n+1} = \frac{7\pi/2 - \phi}{\omega_n \sqrt{1 - \zeta^2}}$ (1.36)

The ratio of the first positive overshoot y_n to the second positive overshoot y_{n+1} [Figure 1.7(c)] is

$$\frac{y_n}{y_{n+1}} = \frac{\left(\frac{K}{\sqrt{1-\zeta^2}}\right) \left(\exp\left\{-\zeta\omega_n \left[\frac{(3\pi/2-\phi)}{\omega_n\sqrt{1-\zeta^2}}\right]\right\}\right)}{\left(\frac{K}{\sqrt{1-\zeta^2}}\right) \left(\exp\left\{-\zeta\omega_n \left[\frac{(7\pi/2-\phi)}{\omega_n\sqrt{1-\zeta^2}}\right]\right\}\right)}$$
$$= \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$
(1.37)
$$\ln\left(\frac{y_n}{y_{n+1}}\right) = \Lambda = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

where Λ is defined as *logarithmic decrement*. Solving for ζ yields

$$\zeta = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}} \tag{1.38}$$

For sinusoidal steady-state responses, the frequency transfer function (1.28) and Figure 1.7(d) show that low-pass frequency responses result. The rate of decline in the amplitude frequency response is twice the rate of that decline for first-order instruments. Note that resonance phenomena can occur if the damping ratio is too small. Also note that the output phase lag can be as much as 180° , whereas for single-order instruments, the maximal phase lag is 90° .

TIME DELAY

Instrument elements that give an output that is exactly the same as the input, except that it is delayed in time by τ_d , are defined as *time-delay elements*. The mathematical expression for these elements is

$$y(t) = Kx(t - \tau_d), \quad t > \tau_d$$
 (1.39)

These elements may also be called analog delay lines, transport lags, or dead times. Although first-order and second-order instruments have negative phase angles that imply time delays, the phase angle varies with frequency, so the delay is not constant for all frequencies. For time delays, the static characteristic is the constant K, the step response is specified by (1.39), and the sinusoidal frequency response for magnitude and phase is

$$\frac{Y(j\omega)}{X(j\omega)} = Ke^{-j\omega\tau_d}$$
(1.40)

Time delays are present in transmission lines (electric, mechanical, hydraulic blood vessels, and pneumatic respiratory tubing), magnetic tape recorders, and some digital signal-processing schemes. Usually these time delays are to be avoided, especially in instruments or systems that involve feedback, because undesired oscillations may result.

If the instrument is used strictly for measurement and is not part of a feedback-control system, then some time delay is usually acceptable. The transfer function for undistorted signal reproduction with time delay becomes $Y(j\omega)/X(j\omega) = K / - \omega \tau_d$. Our previous study of time-delay elements shows that the output magnitude is K times the input magnitude for all frequencies and that the phase lag increases linearly with frequency.

The transfer-function requirements concern the *overall* instrument transfer function. The overall transfer function of linear elements connected in series is the product of the transfer functions for the individual elements. Many combinations of nonlinear elements can produce the overall linear transfer function required. Various forms of modulation and demodulation are used, and unavoidable sensor nonlinearities can sometimes be compensated for by other instrument elements.

1.11 DESIGN CRITERIA

As shown, many factors affect the design of biomedical instruments. The factors that impose constraints on the design are of course different for each type of instrument. However, some of the general requirements can be categorized as signal, environmental, medical, and economic factors. Figure 1.8 shows how these factors are incorporated into the initial design and development of an instrument.

Note that the type of sensor selected usually determines the signal-processing equipment needed, so an instrument specification includes more than just what type of sensor to use. To obtain a final design, some compromises in specifications are usually required. Actual tests on a prototype are always needed before final design decisions can be made. Changes in performance and interaction of the elements in a complex instrument often dictate design modifications. Good designs are frequently the result of many compromises throughout the development of the instrument. In Chapter 2, we shall examine basic methods of sensing biomedical quantities to ensure that many sensor design alternatives are considered.

1.12 COMMERCIAL MEDICAL INSTRUMENTATION DEVELOPMENT PROCESS

A commercial medical instrument project has many phases, and the size of the team needed grows as the project progresses. Ideas often come from people working where health care is delivered, because clinical needs are most