Last Name (Print)______________________________

First Name (Print)______________________________

I understand that cheating is a serious offense.

Signature:______________________________

Student Number______________________________

Room _______________ Seat Number_______________

THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

136.130 Vector Geometry and Linear Algebra

Final Exam

Paper No: 411
Date: Monday, April 17, 2006
Time: 6:00–8:00 PM

Identify your section

	Section	Instructor	Slot	Time	Room
☐	L05	K. Kopotun	5	TTh 10:00–11:15am	208 Armes
☐	L06	G. I. Moghaddam	8	MWF 1:30–2:20pm	204 Armes
☐	L07	G. I. Moghaddam	12	MWF 3:30–4:20pm	208 Armes
☐	L08	C. Platt	15	TTh 4:00–5:15pm	200 Armes
☐	L09	J. Sichler	E2	T 7:00–10:00pm	204 Armes
☐	Other	(challenge, deferred, etc.)			

DO NOT WRITE IN THIS COLUMN

Question	Marks
1	/8
2	/8
3	/6
4	/10
5	/8
6	/12
7	/9
8	/10
9	/11
10	/12
11	/6

Instructions

Fill in ***all*** *the information above.*

This is a two-hour exam.

No *calculators, texts, notes, or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has 11 questions on 11 numbered pages, for a total of 100 points. ***Check now*** *that you have a complete exam.*

Answer all questions on the exam paper in the space provided. If you need

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[Values]

1. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & -3 & 0 \\ 2 & 0 & 4 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 12 \\ 7 \end{bmatrix}$.

[5] **(a)** Find the reduced row echelon form (RREF) of the augmented matrix $\left[A \mid \mathbf{b}\right]$.

[3] **(b)** Find all solutions of the linear system $A\mathbf{x} = \mathbf{b}$.

[Values]

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$, $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}$, $F = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$.

For each case, determine, *without actually performing any matrix algebra*, whether the given expression exists. If it does not exist, give a reason why not. If it exists, evaluate the expression.

[2] **(a)** $2AC^T - B^2$

[2] **(b)** $(A^T - E)D$

[2] **(c)** $AC - 3D$

[2] **(d)** $E^T A + 2F$

[Values]

3. Assume A, B, C are $n \times n$ matrices.

[3] **(a)** If A is an *invertible* matrix, show that $AB = AC$ implies $B = C$.

[3] **(b)** Let $D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $E = \begin{bmatrix} 2 & -3 \\ -2 & 3 \end{bmatrix}$. Calculate DE, and use that information to find a matrix F such that $DE = DF$, but $E \neq F$.

[Values]

4. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 5 & -2 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

[2] **(a)** Find $\det(A)$.

[6] **(b)** Find the missing entries, a and b in the adjoint : $\operatorname{adj}(A) = \begin{bmatrix} -18 & 0 & 0 & 0 \\ 0 & -18 & \boxed{b} & -12 \\ 0 & 0 & 12 & 0 \\ \boxed{a} & 0 & 0 & -12 \end{bmatrix}$

[2] **(c)** Find A^{-1} using the results of (a) and (b).

[Values]

[8] **5.** Use Cramer's rule to find z, where

$$\begin{aligned} 3x - 2z &= 1 \\ 2x - y + 4z &= 2 \\ x + y - z &= 0 \end{aligned}$$

Note: There is no need to find x or y. No marks for any other method.

[Values]

6. In $\mathbb{R}^3$ let L be the line through points $P(5,0,3)$ and $Q(6,5,2)$.

[4] **(a)** Find equations, in both vector form and parametric form, of the line L.

[4] **(b)** Find the point of intersection of the line L and the plane with equation $x+y+z+2=0$.

[4] **(c)** Find the distance from the point $(3,-8,-1)$ to the plane with equation $x+y+z+2=0$.

[Values]

[9] **7.** Find an equation of the plane containing the points $P(1,1,1)$, $Q(2,2,0)$, and $R(3,0,0)$. Express your answer in the general form $ax+by+cz+d=0$.

[Values]

8. P_3 is the vector space of polynomials of the form $a + bx + cx^2 + dx^3$.

Let $\mathbf{p}_1 = 1 + x + x^3$, $\mathbf{p}_2 = x - x^2$, and $\mathbf{p}_3 = 1 - x + x^2 - x^3$.

[5] **(a)** Show that $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are linearly independent.

[3] **(b)** Explain why $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ *cannot* span P_3.

[2] **(c)** Let $W = \text{span}\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$. Find the dimension of W, and justify your answer.

[Values]

9. In each question, determine whether the given set W is a subspace of the vector space V, and justify your answer.

[3] **(a)** $V = \mathbb{R}^2$ and W is the set of all vectors $\mathbf{v} = (a, b)$ such that $ab \leq 0$.

[3] **(b)** V is the space of all 2×2 matrices and W is the set of all *invertible* 2×2 matrices.

[5] **(c)** V is the space of all 3×3 matrices and W consists of all matrices of the form $\begin{bmatrix} a & a & a \\ b & b & b \\ 0 & 0 & 0 \end{bmatrix}$.

[Values]

10. Given (you don't have to show this!):

The reduced row echelon form of $A = \begin{bmatrix} 2 & 0 & 4 & 0 & 8 \\ 3 & 1 & 9 & 0 & 14 \\ -1 & 1 & 1 & 0 & -2 \\ -2 & 0 & -4 & 1 & -8 \end{bmatrix}$ is $R = \begin{bmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Find a basis for each subspace below.

[1] **(a)** The row space of R.

[2] **(b)** The row space of A.

[1] **(c)** The column space of R.

[2] **(d)** The column space of A.

[4] **(e)** The nullspace of R.

[2] **(f)** The nullspace of A.

[Values]

[6] **11.** Let V be a vector space, and let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in V.

Show that $\mathbf{x}_1 = \mathbf{u} - \mathbf{v},\ \mathbf{x}_2 = \mathbf{v} - \mathbf{w},$ and $\mathbf{x}_3 = \mathbf{w} - \mathbf{u}$ form a linearly dependent set.