MATH1230, Assignment No. 1

September 26, 2018

The assignment is due Wednesday, October 3, in class. Late assignments receive a mark zero.

Solve the following questions by **showing and explaining all the details** of you work.

1. Solve the following inequalities. Give the solution set in terms of intervals.

   a) $\frac{3}{x-1} \le \frac{2}{x+1}$.

   b) $|x-4| > 2|1+x|$.

2. a) Find the y coordinates of the points of intersection of the circle $x^2+y^2=4$ and the ellipse $x^2+2y^2+8y+4=0$. Draw a picture.

   b) What is the length (no decimal numbers, please) of the line segment having one end on the x axis, the other end on the y axis, passing through the point $(1, 2-\frac{2}{\sqrt{3}})$ and making an angle of 150^o with the positive part of the x axis. (Draw a picture.)

3. Let $f(x) = x^2 + 2x - 1.5$.

   a) Knowing that the limit of f exists as x approaches –2, use tables to guess the value of the limit.

   b) Using the formal definition of the limit, verify your guess from a) for the value of $\lim_{x\to -2}(x^2 + 2x - 1.5)$.

4. a) Write the definition of $\lim_{x\to -1} f(x) = 0$.

   b) Let $|g(x)| \le 3$, for all x in $\mathbb{R}$. Show by using the definition of limit that if $\lim_{x\to -1} f(x) = 0$, then $\lim_{x\to -1} f(x)g(x) = 0$.

   c) Give examples of f and g such that $\lim_{x\to -1} f(x) = 0$, but $\lim_{x\to -1} f(x)g(x)$ does not exist.

   d) Give examples of f and g such that $\lim_{x\to -1} f(x) = 0$, but $\lim_{x\to -1} f(x)g(x) = L$ with $L \ne 0$.

5. Use the Limit Rules Theorem to find the following limits. State the rules you are using, and explain why you can use them. You can also use that $\lim_{x\to a} x = a$, $\lim_{x\to a} c = c$, and that for $a > 0$, $\lim_{x\to a} \sqrt{x} = \sqrt{a}$.

   a) $\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$.

   b) Given that $\lim_{x\to 5}\left(3 + \frac{f(x)-2}{1-x}\right) = 2$, show that $\lim_{x\to 5} f(x)$ exists, and find its value.

Each question is worth 8 marks, for a total of 40.