THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1230 MIDTERM TEST
17 December 2015   1:30–3:30 PM
Examiner: D. Krepski

Name (print): _____

Signature: _____

ID number: _____

Please circle your Tutorial/Lab Section below:

> B01   Fridays 1:30
> B02   Fridays 8:30
> B03   Fridays 9:30

INSTRUCTIONS:

1. This exam is 11 pages long. It is printed double-sided. There are 9 questions, including one bonus question. It is your responsibility to check that you have a complete exam booklet.

2. All questions must be answered in the space provided. Indicate your responses clearly, showing all of your work with complete explanations and full justification.

3. Do **not** unstaple the exam booklet.

4. Should you need extra space, a blank page is provided at the end of the booklet. Please clearly indicate if your response continues on this blank page (e.g. "continued on p. 11") to ensure your complete response is examined.

5. **No aids of any kind are permitted: electronic devices, notes, abacuses, etc. are not permitted.**

Question:	1	2	3	4	5	6	7	8	BONUS	Total
Marks:	15	14	4	8	6	8	18	9	0	82
Score:										

15   1. **MULTIPLE-CHOICE QUESTIONS**

Each of the following multiple-choice questions has exactly ONE correct answer. Clearly indicate your answer to each question by circling your response.

Marking scheme: 2.5 marks for selecting the correct choice; 0 marks for selecting a wrong choice, or not selecting a choice, or selecting more than one choice.

(a) Find $\lim\limits_{x\to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$.

     A. 0

     B. does not exist

     C. 5/4

     D. $-1/4$

     E. 3/2

(b) Find $\lim\limits_{x\to -\infty} \dfrac{2x^3 + 3}{7 - 3x^2 - 9x^3}$.

     A. 2/9

     B. 3/7

     C. $-2/9$

     D. $-\infty$

     E. ∞

(c) Find $\lim\limits_{x\to 0} \dfrac{\sin(2x)\cos(5x)}{\cos(3x)\sin(4x)}$.

     A. 2/3

     B. 5/3

     C. 1/2

     D. 5/4

     E. 0

(d) If $xy^3 + y - x = 23$, find the equation of the tangent line at the point $P(3, 2)$.

    A. $y - 2 = \frac{4}{25}(x - 3)$
    B. $y - 2 = \frac{3}{38}(x - 3)$
    C. $y - 2 = \frac{5}{37}(x - 3)$
    D. $y - 2 = -\frac{6}{35}(x - 3)$
    E. $y - 2 = -\frac{7}{37}(x - 3)$

(e) The product of two positive numbers is 100. What is the smallest possible value for their sum?

    A. 18
    B. 19
    C. 20
    D. 21
    E. 22

(f) The n-th order Taylor polynomial of a function f about $x = 0$ is

$$P_n(x) = 1 + 2x + 3x^2 + 4x^3 + \cdots + (n + 1)x^n.$$

What is $f^{(1230)}(0)$?

    A. 1230!
    B. 1/1230!
    C. 1231/1230!
    D. 1231!
    E. 1231

2. Differentiate the following functions. (You do not need to simplify your answer.)

4   (a) $f(x) = (\ln x + \sin x)(\tan x + 1)$

5   (b) $f(x) = \dfrac{e^x \sin(2x)}{\sqrt{1 - 3x^2}}$

5   (c) $f(x) = x^{(e^x)}$

4   3. Circle ONE of the following theorems, and give the full precise statement of that theorem:

- The Intermediate Value Theorem
- The Mean Value Theorem

3   4. (a) State the precise definition of $\lim\limits_{x\to\infty} f(x) = L$.

5   (b) Using the precise definition of limit, show that $\lim\limits_{x\to\infty} \dfrac{1}{\sqrt{x-1}} = 0$.

6 5. A spotlight on the ground shines on a wall 1000 cm away. A person walks from the spotlight toward the wall at a speed of 120 cm/s. At the moment when the person is 400 cm from the wall, the length of their shadow **on the wall** is decreasing at 46 cm/s. How tall is the person?

8   6. Find the absolute maximum and minimum values (if any) of the function

$$f(x) = x(21 - x^2)^{2/3}, \quad 0 \le x \le 6.$$

7. Let $f(x) = \dfrac{4x}{x^2 + 1}$. Then $f'(x) = \dfrac{4(1 - x^2)}{(x^2 + 1)^2}$, and $f''(x) = \dfrac{8x(x^2 - 3)}{(x^2 + 1)^3}$.

2   (a) Show that f is an odd function.

4   (b) Determine the intervals of increase/decrease for f.

4   (c) Identify all critical and singular points (if any) of f and classify them as local maxima/minima or neither.

4   (d) Determine (if any) the interval(s) on which f is concave up and the interval(s) on which f is concave down.

4   (e) Provide a sketch of the graph of f that exhibits all the information you found above.

$\boxed{4}$ 8.   (a) Calculate the Taylor polynomial $P_3(x)$ of order 3 about $x = 0$ for $f(x) = \ln(1 + x)$.

$\boxed{1}$ (b) Use P_3 to write down an approximation for $\ln(2)$.

$\boxed{4}$ (c) Use Taylor's formula (i.e. Lagrange formula for the error) to estimate the error in your approximation above.

5 (bonus) 9. (Bonus!) Suppose f is a differentiable function such that $f'(8) = 2$. Find the value of the limit
$$\lim_{x \to 8} \frac{f(x) - f(8)}{x^{1/3} - 2},$$
WITHOUT using L'Hopital's Rule. (If you do not know L'Hopital's Rule, do not be concerned. Solution attempts that use L'Hopital's Rule will receive no credit.)

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?
Did you give full explanations and show all of your work?