UNIVERSITY OF MANITOBA

DATE: January 9, 2017 | **FINAL EXAM**

TITLE PAGE

EXAMINATION: Differential Calculus | **TIME**: 3 hours

COURSE: MATH 1230 | **EXAMINER**: A. Clay

FAMILY NAME: (Print) ______________________

GIVEN NAME(S):______________________

STUDENT NUMBER: ______________________

SIGNATURE: ______________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions, a table of Laplace transforms and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 140 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	18	
2	17	
3	10	
4	10	
5	10	
6	15	
7	10	
8	10	
9	30	
10	10	
Total:	140	

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1. State the following definitions:

[3] (a) The formal definition of $\lim_{x\to a} f(x) = L$.

[3] (b) The formal definition of $\lim_{x\to -\infty} f(x) = -\infty$.

[3] (c) The definition of the *derivative* $f'(x)$ of a function $f(x)$.

[3] (d) The definition of the *inverse function* $f^{-1}(x)$ of a function $f(x)$.

[3] (e) The definition of a *horizontal asymptote* of the function $f(x)$.

[3] (f) The definition of an *inflection point* of $f(x)$.

2. Find the derivative of each of the following. Do not simplify your answers.

[5] (a) $f(x) = \log_2(3^x) - \ln(\cos(x^2))$

[7] (b) $f(x) = (x^2 + 2x + 1)^{x^3}$

[5] (c) $f(x) = e^{\sin^{-1}(3x^2-1)}$

[10] 3. Use the formal definition of the limit to prove that

$$\lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}} = 0.$$

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[10] 4. Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x \geq 0 \\ \ln(x+1) & \text{if } x < 0. \end{cases}$$

Use the definition of the derivative to verify that $f'(0) = 1$. (Hint: L'Hôpital's rule may be useful).

[10] 5. A lighthouse is located on an island that is 2km from the nearest point P on the shore. The light in the lighthouse rotates at a constant rate of 4 revolutions per minute as its beam of light shines on the nearby shore. How fast is the beam of light moving along the shoreline when it is 1km from the point P? (Assume the shoreline is perfectly straight)

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[15] 6. Find the point (x, y) on the graph of the function $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

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[10] 7. Use implicit differentiation to find the equation of the line tangent to the curve $e^{xy} = y^3 + x^3$ passing through the point $(0, 1)$.

[10] 8. Find the antiderivative of $y' = x^{-2} - x^{-3}$ satisfying $y(-1) = 0$.

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9. Let $f(x) = \dfrac{3x^2}{x^2-4}$. You may use, without checking, that $f'(x) = \dfrac{-24x}{(x^2-4)^2}$ and $f''(x) = \dfrac{24(3x^2+4)}{(x^2-4)^3}$ in the questions that follow.

[5] (a) Find the x and y intercepts, if any, and state the domain of $f(x)$.

[2] (b) Is $f(x)$ an even function, odd function, or neither? Prove your claim.

[8] (c) Calculate the limits associated with finding the vertical and horizontal asymptotes of the function $f(x)$. Give the equations of these asymptotes, if any.

[5] (d) Find the critical points and singular points of $f(x)$. Find the intervals where $f(x)$ is increasing/decreasing. State the x-values corresponding to any local maxima and/or minima, if there are any.

[5] (e) Find where $f(x)$ is concave up, and where $f(x)$ is concave down. Find any inflection points.

[5] (f) Sketch the curve, and label on your sketch all of the information found in parts (a)–(e).

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[10] 10. Compute the Taylor polynomials $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$ about $\pi/4$ of $f(x) = \sin(x)+1$.

THIS PAGE IS INTENTIONALLY LEFT BLANK FOR ROUGH WORK