Midterm Test 2 hours

Q1]... [9 points]
Let $f(x)$ be a function. Give the formal definition of the limit statements:
(a) $\lim_{x\to a} f(x) = L$
(b) $\lim_{x\to -\infty} f(x) = L$
(c) $\lim_{x\to a} f(x)$ is NOT equal to L (negate the statement in part (a))

Q2]... [6 points]
Let $f(x)$ be a real-valued function.
(a) State the meaning of the phrase "$f(x)$ is continuous at $x = a$".
(b) State the definition of the derivative $f'(a)$.

Q3]... [10 points] Use the ϵ-δ definition of a limit to show that $\lim_{x\to 2} \dfrac{x-2}{1+x^2} = 0$.

Q4]... [15 points] Calculate the given limits, or state that they do not exist. You do NOT need to use ϵ-δ reasoning, and you may use the limit laws without further justification.
(a) $\lim_{x\to 1^-} \dfrac{1}{|x-1|}$
(b) $\lim_{x\to\infty} \left(\dfrac{x^2}{x+1} - \dfrac{x^2}{x-1} \right)$
(c) $\lim_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$

Q5]... [10 points] Use the the definition of the derivative to calculate $f'(1)$ if $f(x) = \dfrac{1}{x^2}$.

Q6]... [10 points] Use differentiation rules to complete the following calculations:
(a) Calculate y' and y'' if $y = \dfrac{\sin(x)}{x}$.
(b) Compute the derivative of $f(x) = \tan\left(\dfrac{x^2}{x^3-1} \right)$.

Q7]... [10 points] Use the intermediate value theorem to prove that $x^3 - 3x + 1 = 0$ has at least three solutions in the interval $[-2, 2]$.