UNIVERSITY OF MANITOBA

DATE: April 23 , 2015 FINAL EXAMINATION
TITLE PAGE
EXAMINATION: Vector Geometry and Linear Algebra TIME: 2 hours
COURSE: MATH 1300 EXAMINER: Kalajdzievski, Moghaddam, Zhao

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________________
(I understand that cheating is a serious offense)

Please place a check mark (✓) for your section.

❑	A01	10:30–11:20 AM	MWF (204 Armes)	Xiangui Zhao
❑	A02	10:00–11:15 AM	TR (208 Armes)	Sasho Kalajdzievski
❑	A03	1:30–2:20 PM	MWF (204 Armes)	G. I. Moghaddam

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	9	
2	13	
3	8	
4	14	
5	9	
6	13	
7	5	
8	9	
9	10	
10	10	
Total:	100	

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1. The augmented matrix of a linear system is $A = \left[\begin{array}{ccc|c} 1 & 2 & a & 4 \\ 0 & 1 & b & 5 \\ 0 & 0 & a+2b & a^2-4b^2 \end{array}\right]$.

Find **all** values of a and b (if any) such that the system has

[3] (a) infinitely many solutions;

[3] (b) exactly one solution;

[3] (c) no solution.

2. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$.

[5] (a) Find the matrix B such that $A^2 - \left(\det(A^T)\right) B = 4I$.

[5] (b) Find A^{-1}.

[3] (c) Evaluate $|A + A^{-1}|$.

3. Let $A = \begin{bmatrix} 3 & -2 \\ 7 & -5 \end{bmatrix}$.

[4] (a) Find $\det(A)$ and $\det(A^{2015})$.

[4] (b) Find $(\text{adj}(A))^{-1}$.

4. Let $\vec{u} = (-1, 1, 2)$ and $\vec{v} = (0, 2, 4)$ be vectors in $\mathbf{R}^3$.

[5] (a) Evaluate $\|\vec{u}\|(\vec{v} - 2\vec{u})$ where $\|\vec{u}\|$ stands for the length of $\vec{u}$.

[5] (b) Find $\text{proj}_{\vec{v}}\,\vec{u}$.

[4] (c) Find a unit vector that is perpendicular to both $\vec{u}$ and $\vec{v}$.

5. Let $P_1 = (1, -1, 2)$, $P_2 = (2, 0, 4)$ and $P_3 = (4, 0, 3)$ be three points in $\mathbf{R}^3$.

[5] (a) Find the area of the triangle determined by these three points.

[4] (b) Is the triangle determined by these points a right-angled triangle? Explain your answer.

6. Let $\Pi_1 : x + 3y - z = 1$ and $\Pi_2 : x + 4y + 3z = 2$ be two planes, $L : (x, y, z) = (1, 0, 2) + t(1, 2, 3)$, $t \in \mathbf{R}$, be a line and $A(-2, 3, -1)$ be a point in $\mathbf{R}^3$.

[5] (a) Find the distance between the point A and the plane Π_1.

[4] (b) Find the parametric equations of the line of the intersection of the planes Π_1 and Π_2.

[4] (c) Find the point of intersection of the line L and the plane Π_2.

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[5] 7. Consider the vectors $\vec{OP} = (3, -2, 3)$, $\vec{OQ} = (1, 1, 1)$ and $\vec{OR} = (0, 5, 0)$ in $\mathbb{R}^3$. Show that the points O, P, Q and R lie in the same plane.

8. Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation that sends each vector v (starting at the origin) to its reflection with respect to the xy-plane.

[5] (a) Find the standard matrix A of the linear transformation T.

[4] (b) Let $B = A^3$ where A is the matrix found in part (a). Find $T_B(1, 2, 3)$ if T_B is the matrix transformation for the matrix B.

9. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix}$.

[3] (a) Find the characteristic polynomial for the matrix A.

[3] (b) Two eigenvalues for the matrix A are $\lambda_1 = 1$ and $\lambda_2 = 2$. Find the third eigenvalue.

[4] (c) Find one specific eigenvector associated to the eigenvalue $\lambda = 2$.

10. The eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ are $\lambda_1 = 1$ and $\lambda_2 = 2$, and $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are two eigenvectors corresponding to λ_1 and λ_2 respectively.

[6] (a) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix D, and find the matrix D.

[4] (b) Find A^{10}.

BLANK PAGE FOR ROUGH WORK