UNIVERSITY OF MANITOBA

DATE: February 26, 2015 MIDTERM

TITLE PAGE

EXAMINATION: Vector Geometry and Linear Algebra TIME: 1 hour

COURSE: MATH 1300 EXAMINER: Kalajdzievski, Moghaddam, Zhao

NAME: (Print in ink) ______________________

STUDENT NUMBER: ____________

SEAT NUMBER: ____________

SIGNATURE: (in ink) ______________________

(I understand that cheating is a serious offense)

Please place a check mark (✓) for your section.

❏	A01	10:30–11:20 AM	MWF (204 Armes)	Xiangui Zhao
❏	A02	10:00–11:15 AM	TR (208 Armes)	Sasho Kalajdzievski
❏	A03	1:30–2:20 PM	MWF (204 Armes)	G. I. Moghaddam

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 70 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	10	
2	11	
3	4	
4	6	
5	9	
6	16	
7	6	
8	8	
Total:	70	

[10] 1. Consider the linear system

$$\begin{array}{rrrrcr} & & x_3 & -3x_4 & = & 4 \\ 3x_1 & +6x_2 & -3x_3 & & = & 9\,. \\ 3x_1 & +6x_2 & -2x_3 & -3x_4 & = & 13 \end{array}$$

(a) Find the augmented matrix of this system.

(b) Find the reduced row echelon form of the augmented matrix.

(c) Write **all** of the solutions, if there are any.

[11] 2. Let $A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 1 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 0 \\ 1 \\ 9 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & -1 \\ 6 & 1 \end{bmatrix}$.

(a) Indicate if the expression is defined or undefined by placing a check mark (✓) in the appropriate column. If it is defined, then indicate the size of the resulting matrix.

EXPRESSION	UNDEFINED	DEFINED	SIZE
$C(2A+3B)$			
$A^T(C-4C^T)$			
$A(BB^T)$			

(b) Evaluate $AA^T + C^2$.

[4] 3. Let $A = \begin{bmatrix} a-4 & 0 \\ 1 & a+4 \end{bmatrix}$. Find all values of a for which the matrix A is invertible.

4. Calculate the following determinants.

[3] (a) $\begin{vmatrix} 4 & 5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & -1 \end{vmatrix}$

[3] (b) $\begin{vmatrix} a & a & b \\ 1 & 1 & 1 \\ a+b & a+b & 2b \end{vmatrix}$

5. Let A and B be 2×2 matrices such that $\det(A) = 3$ and $AB^T = A^2$.

[4] (a) Find $\det(B)$.

[5] (b) Find $\det\big(A(A + B^T)\big)$.

6. Let $A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$.

[6] (a) Given that $adj(A) = \begin{bmatrix} 1 & -3 & a \\ -2 & 7 & 7 \\ -2 & b & 7 \end{bmatrix}$; find the values of a and b.

[6] (b) Find A^{-1} by using the $adj(A)$.

[4] (c) Use A^{-1} to solve the linear system $\begin{array}{rrrcr} 7x & +3y & & = & -1 \\ & y & -z & = & 1 \\ 2x & & +z & = & 0 \end{array}$.

No mark will be given for any other method.

[6] 7. Consider the linear system $\begin{array}{rrrcl} ax & +2y & +z & = & 2 \\ bx & +y & -z & = & 3 \\ cx & +2y & -3z & = & 7 \end{array}$. It is known that $det(A) = 3$, where A is the coefficient matrix of the system. Find the value of x **only**.

[8] 8. Let $A = \begin{bmatrix} 2 & -8 \\ 0 & 1 \end{bmatrix}$. First find elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$, and then express A as a product of elementary matrices.