

## 1.2. GAUSSIAN ELIMINATION

RECALL FROM 1.1., EXAMPLE (5.6), REDUCED AUGMENTED MATRIX

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array} \right], \quad \left[ \begin{array}{cc|c} 1 & \frac{1}{2} & 4 \\ 0 & 0 & 0 \end{array} \right]$$

### REDUCED ROW-ECHELON FORM OF A MATRIX:

1. ANY ROWS CONSISTING ENTIRELY OF ZEROS ARE AT THE BOTTOM.
2. THE FIRST NONZERO NUMBER IN A ROW IS 1 (**LEADING**).
3. THE LEADING 1 IN A LOWER ROW OCCURS FARTHER TO THE RIGHT THAN THE LEADING 1 IN A HIGHER ROW.
4. EACH COLUMN WITH A LEADING 1 HAS ZEROS EVERYWHERE ELSE.

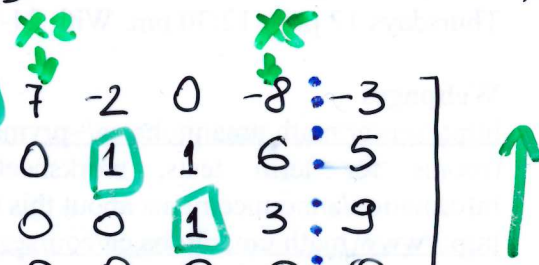
**ROW-ECHELON FORM** IF 1, 2 AND 3, BUT POSSIBLY NOT 4, SATISFIED.

**EXAMPLE 1:** a)  $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 2 & 0 & 15 \\ 0 & 1 & 0 & 02 \\ 0 & 0 & 1 & 37 \end{bmatrix}$

c)  $\begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 3 & 2 \end{bmatrix}$       d)  $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \end{bmatrix}$       e)  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

**GAUSS-JORDAN ELIMINATION**: IF, USING ROW REDUCTION, WE STOP THE ELIMINATION PROCESS WHEN WE GET THE REDUCED ROW-ECHELON FORM OF THE AUGMENTED MATRIX.

**GAUSS ELIMINATION**: IF WE STOP THE ELIMINATION PROCESS WHEN WE GET A ROW-ECHELON FORM OF THE AUGMENTED MATRIX. (SOONER)

**EXAMPLE 2:** #5 e) p. 20:  $\left[ \begin{array}{ccccc|c} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$  

(GAUSS ELIMINATION)

( BACK SUBSTITUTION)

**EXAMPLE 3:** SOLVE  $3x_1 - x_2 = 7$   
 $x_1 - 2x_2 = -1$  BY GAUSS-JORDAN ELIM  
 $2x_1 - 4x_2 = -2$

**EXAMPLE 4:** #17 p. 21: FOR WHICH VALUE OF  $a$  WILL THE SYSTEM HAVE NO SOLUTIONS, ONE SOLUTION OR INFINITELY MANY SOLUTIONS?

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

NOTE: THE REDUCED ROW-ECHELON FORM IS UNIQUE.  
 (ROW-ECHELON FORM NOT UNIQUE)

# HOMOGENEOUS LINEAR SYSTEM:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0$$

$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = 0$$

**TRIVIAL SOLUTION:**  $x_1 = x_2 = \dots = x_n = 0$  (ALWAYS)

WHEN DOES A HOMOGENEOUS LIN. SYSTEM HAVE INFINITELY MANY SOLUTIONS? (HOM. SYSTEM CAN NEVER BE INCONSISTENT.)

CASE: MORE UNKNOWN THAN EQUATIONS.

**EXAMPLE 5:** #13 b) p.20:

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

THEOREM 1: A HOMOGENEOUS LINEAR SYSTEM WITH MORE UNKNOWN THAN EQUATIONS HAS INFINITELY MANY SOLUTIONS.

**HOMEWORK p.19:** # 1-14 (EVEN), 17-19, 22, 26, 27, 31, 32.