1.2. GAUSSIAN ELIMINATION

RECALL FROM 1.1., EXAMPLE (5,6), REDUCED AUGMENTED MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right], \quad \left[\begin{array}{cc|c} 1 & \frac{1}{2} & 4 \\ 0 & 0 & 0 \end{array}\right]$$

REDUCED ROW-ECHELON FORM OF A MATRIX:

1. ANY ROWS CONSISTING ENTIRELY OF ZEROS ARE AT THE BOTTOM.
2. THE FIRST NONZERO NUMBER IN A ROW IS 1 (**LEADING**)
3. THE LEADING 1 IN A LOWER ROW OCCURS FARTHER TO THE RIGHT THAN THE LEADING 1 IN A HIGHER ROW.
4. EACH COLUMN WITH A LEADING 1 HAS ZEROS EVERYWHERE ELSE.

ROW-ECHELON FORM IF 1, 2 AND 3, BUT POSSIBLY NOT 4, SATISFIED.

EXAMPLE 1: a) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 & 7 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 3 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \end{bmatrix}$ e) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

GAUSS-JORDAN ELIMINATION: IF, USING ROW REDUCTION, WE STOP THE ELIMINATION PROCESS WHEN WE GET THE REDUCED ROW-ECHELON FORM OF THE AUGMENTED MATRIX.

GAUSS ELIMINATION: IF WE STOP THE ELIMINATION PROCESS WHEN WE GET A ROW-ECHELON FORM OF THE AUGMENTED MATRIX. (SOONER)

EXAMPLE 2: #5 c) p. 20: (GAUSS ELIMINATION)

$$\begin{array}{c} \begin{array}{cccccc} & x_2 & & & x_5 & \\ & \downarrow & & & \downarrow & \end{array} \\ \left[\begin{array}{ccccc|c} \boxed{1} & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & \boxed{1} & 1 & 6 & 5 \\ 0 & 0 & 0 & \boxed{1} & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

(BACK SUBSTITUTION)

EXAMPLE 3: SOLVE

$$\begin{aligned} 3x_1 - x_2 &= 7 \\ x_1 - 2x_2 &= -1 \\ 2x_1 - 4x_2 &= -2 \end{aligned}$$

BY GAUSS-JORDAN ELIM

EXAMPLE 4: #17 p. 21: FOR WHICH VALUE OF a WILL THE SYSTEM HAVE NO SOLUTIONS, ONE SOLUTION OR INFINITELY MANY SOLUTIONS?

$$\begin{aligned} x + 2y \quad - 3z &= 4 \\ 3x - y \quad + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

NOTE: THE REDUCED ROW-ECHELON FORM IS UNIQUE. (ROW-ECHELON FORM NOT UNIQUE)

HOMOGENEOUS LINEAR SYSTEM:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= 0 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= 0 \end{aligned}$$

TRIVIAL SOLUTION: $x_1 = x_2 = \dots = x_n = 0$ (ALWAYS)

WHEN DOES A HOMOGENEOUS LIN. SYSTEM HAVE INFINITELY MANY SOLUTIONS? (HOM. SYSTEM CAN NEVER BE INCONSISTENT.)

CASE: MORE UNKNOWNS THAN EQUATIONS.

EXAMPLE 5: #13 b) p. 20: $$\begin{aligned} 3x_1 + x_2 + x_3 + x_4 &= 0 \\ 5x_1 - x_2 + x_3 - x_4 &= 0 \end{aligned}$$

<u>THEOREM 1</u>: A HOMOGENEOUS LINEAR SYSTEM WITH MORE UNKNOWNS THAN EQUATIONS HAS INFINITELY MANY SOLUTIONS.

HOMEWORK p. 19: # 1-14 (EVEN), 17-19, 22, 26, 27, 31, 32.