1.3. MATRIX OPERATIONS

MATRIX IS A RECTANGULAR ARRAY OF NUMBERS

EXAMPLE 1: $\begin{bmatrix} 0 & 2 \\ 0 & 7 \\ -3 & 5 \end{bmatrix}$, $[8]$, $[-1\ 0\ 1\ 0]$, $\begin{bmatrix} 1/4 \\ \pi \end{bmatrix}$

$A = [a_{ij}]_{m\times n}$, $(A)_{ij} = a_{ij}$ (ROW i, COLUMN j)

SQUARE MATRIX: MATRIX OF SIZE $n \times n$

MAIN DIAGONAL OF A SQUARE MATRIX: $a_{11}, a_{22}, \dots, a_{nn}$

IF A AND B ARE OF THE SAME SIZE $m\times n$, DEFINE **$A+B$** TO BE THE $m\times n$ MATRIX SUCH THAT:

$$(A+B)_{ij} = (A)_{ij} + (B)_{ij} \quad \text{(ADD CORRESP. TERMS)}$$

cA, FOR A CONSTANT c IS THE $m\times n$ MATRIX WITH ENTRIES: $(cA)_{ij} = c(A)_{ij}$

(MULTIPLY EACH TERM OF A BY c).

EXAMPLE 2: $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

FIND, IF POSSIBLE a) $2B+C$ b) $A-5C$

HOW ABOUT A PRODUCT OF MATRICES?

EXAMPLE 3: $2x_1 - x_2 + 5x_3 = 2$ (COEFFICIENT MATRIX)
$3x_1 + 4x_2 - x_3 = -1$

LET A BE AN $m \times r$ MATRIX AND B AN $r \times n$ MATRIX. THEN AB IS AN $m \times n$ MATRIX (**PRODUCT** AB) SUCH THAT:

$$(AB)_{ij} = (A)_{i1}(B)_{1j} + (A)_{i2}(B)_{2j} + \dots + (A)_{ir}(B)_{rj}.$$

(ROW i FROM A MULTIPLIED BY COLU. j OF B BY MULTIPLYING THE CORRESPONDING r ENTRIES AND THEN ALL ADDED UP.) $(m \times \underline{r}) \cdot (\underline{r} \times n) \text{"="} m \times n$

EXAMPLE 4: $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

FIND AB AND AC IF POSSIBLE.
HOW ABOUT BA?

IF A IS $m \times n$ MATRIX, THE **TRANSPOSE OF A** (A^T) IS THE $n \times m$ MATRIX WITH ROWS THE COLUMNS OF A (INTERCHANGE TRE ROWS AND COLUMNS OF A)

EXAMPLES 5: FIND A^T, FOR A IN EX. 4. ($[8]^T = ?$)

HOMEWORK: p.34 # 1-6, 12-14, 18, 21, 29, 32.