UNIVERSITY OF MANITOBA
COURSE: MATH 1300
DATE & TIME: 14 December 2017, 18:00–20:00
DURATION: 2 hours

Name: ______________________________

Student Number: ______________________________

I understand that cheating is a serious offence: ______________________________
(Signature – *In Ink*)

Please place a check mark beside your section number and instructor:

☐ A01	L. Y. M. Lopez	☐ A02	K. R. Gunderson	☐ A03	M. F. Virgilio
☐ A04	Y. Zhang	☐ D01	J. Arino	☐ SJR	C. Bilyk

INSTRUCTIONS

I. No texts, notes, calculators, cellphones, translators or any other electronic devices are permitted.

II. This exam has 12 pages (including this cover page, 10 pages with questions and one blank page for rough work). Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 80 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. **Do not continue on the back of the page.** If you need more space, continue on the scrap page or other pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not make any marks on the QR codes, that would probably lead to the page not being recognized (and as a consequence, not being marked).

VI. Legibility and clarity of answers is very important: a poorly presented correct answer **will not** get full marks.

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[10] 1. Use Cramer's rule to solve the linear system

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 1 \\ -x_1 + 3x_2 - 5x_3 &= 2 \\ 2x_1 - x_2 + 6x_3 &= -1 \end{aligned}$$

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[12] 2. Suppose A, B and C are 3×3 matrices such that $\det(A) = 10$, $\det(B) = \dfrac{1}{3}$ and $\det(C) = 2$. Evaluate the following:

$$(a)\ \det(BAB^{-1}) \quad (b)\ \det(CBC^T) \quad (c)\ \det(5BC) \quad (d)\ \det(Adj(B))$$

and justify your work.

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3. Let $\vec{u} = (2, -2, 1)$, $\vec{v} = (4, 3, -5)$ and $\vec{w} = (-1, 3, 2)$.

[4] (a) Find the cosine of the angle θ between $\vec{u}$ and $\vec{v}$.

[5] (b) Find the vector component of $\vec{u}$ along $\vec{v}$.

[2] (c) Find the vector component of $\vec{u}$ orthogonal to $\vec{v}$.

[2] (d) Find the volume of the parallelepiped determined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

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[5] 4. Find a point-normal form equation of the plane containing two lines

$$\begin{aligned} x &= 6 - t \\ y &= -5 + 3t \\ z &= -4 + t \end{aligned} \quad \text{and} \quad \begin{aligned} x &= 3 + s \\ y &= 2 - s \\ z &= -4 + 2s \end{aligned}$$

where $t, s \in \mathbb{R}$.

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5. Let $\mathscr{L}$ be the line $x = 1 + 4t$, $y = 2 - 3t$, $z = 4 + t$ and Π be the plane $7x + y + 9z = 10$.

[3] (a) Find the point of intersection of the line $\mathscr{L}$ and the plane $2x - 3y + z = 36$.

[3] (b) Find the equation of the plane that is perpendicular to the line $\mathscr{L}$ and passes through the point $(3, 1, 2)$.

[3] (c) Find the distance from the point $(2, 3, -1)$ to the plane Π.

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[7] 6. Let $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 4 \\ 1 & -1 & -2 \end{bmatrix}$. Find the characteristic equation and all eigenvalues of A.

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[4] 7. Given that 2 is an eigenvalue for $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$. Find the eigenvectors for A corresponding to the eigenvalue 2.

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8. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that represents a counterclockwise rotation of each vector around the origin through an angle of $\theta = \dfrac{\pi}{6}$.

[3] (a) Find the standard matrix of T.

[2] (b) Evaluate $T(\sqrt{3}, 2)$.

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[10] 9. True-False Exercises

(a) Every elementary matrix is invertible.

(b) For all square matrices A and B, it is true that $\det(A + B) = \det(A) + \det(B)$.

(c) If A is a square matrix with two identical columns, then $\det(A) = 0$.

(d) If A and B are square matrices of the same size and A is invertible, then $\det(A^{-1}BA) = \det(B)$.

(e) Every vector in $\mathbb{R}^n$ has a positive norm.

(f) For all vectors $\vec{u}$ and $\vec{v}$, it is true that $||\vec{u} + \vec{v}|| = ||\vec{u}|| + ||\vec{v}||$.

(g) The vector equation of a plane can be determined from any point lying in the plane and a nonzero vector parallel to the plane.

(h) The vectors $\vec{v} + (\vec{u} + \vec{w})$ and $(\vec{w} + \vec{v}) + \vec{u}$ are the same.

(i) For all vectors $\vec{u}, \vec{v}$, and $\vec{w}$ in 3-space, the vectors $(\vec{u} \times \vec{v}) \times \vec{w}$ and $\vec{u} \times (\vec{v} \times \vec{w})$ are the same.

(j) If A is a square matrix and $A\vec{x} = \lambda\vec{x}$ for some nonzero scalar λ, then $\vec{x}$ is an eigenvector of A.

[5] 10. (Bonus Question) Prove that $\vec{u}$ is orthogonal to $\vec{u} \times \vec{v}$.

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If using this page to continue your work from a previous question. Clearly indicate on the original page that your work is continuing or this work will not be marked.