UNIVERSITY OF MANITOBA
COURSE: MATH 1300
DATE & TIME: April 12, 2018, 6:00–8:00
FINAL EXAMINATION
DURATION: 2 hours

Name: ______________________________

Student Number: ______________________________

I understand that cheating is a serious offence: ______________________________
(Signature – *In Ink*)

INSTRUCTIONS

I. No calculators, texts, notes, cell phones, pagers, translators or other electronics are permitted. No outside paper is permitted.

II. This exam has a title page, 26 pages including this cover page and 1 scrap page for rough work. Please check that you have all the pages.

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 85 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

V. Work on pages without questions will only be marked if it is clearly linked to a question (e.g., "Question 3 (continued)").

VI. Do not make any marks on the QR codes, that would probably lead to the page not being recognized (and as a consequence, not being marked). Do not write too close to the staple, as the area near it will be chopped off for scanning.

VII. *Show all your work clearly and justify your answers using complete sentences.* **Unjustified answers will receive LITTLE or NO CREDIT.**

VIII. If a question calls for a *specific method*, no credit will be given for any other method.

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1. Suppose that the augmented matrix of a system of linear equations has been partially reduced using elementary row operations to

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 5 \\ 0 & 1 & 1 & a \\ 0 & b & 1 & 2 \end{array}\right).$$

[5] (a) Find all values (if any) of a and b for which the system is inconsistent.

[3] (b) Find all values (if any) of a and b for which the system has a unique solution.

[2] (c) Find all values (if any) of a and b for which the system has infinitely many solutions.

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[6] 2. Let $A = \begin{pmatrix} 2 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$. Given that $|BA + B| = 10$, find $\left|A^T B\right|$.

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3. Let $P(-1,1,1)$, $Q(1,2,3)$ and $R(2,1,0)$ be points in $\mathbb{R}^3$.

[6] (a) Find an equation of the plane in $\mathbb{R}^3$ containing P, Q and R. Write the equation in the form $ax + by + cz + d = 0$.

[4] (b) Find parametric equations of the line L in $\mathbb{R}^3$ passing through P and Q.

⇒ *Continued on the next page*

[4] (c) Find the point of intersection of L, found in part (b), and the xy-plane.

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[7] 4. Fill in the blanks. No justification required.

If A is an $n \times n$ matrix, then the following statements are equivalent.

1. A is invertible.
2. $A\mathbf{x} = \mathbf{0}$ has only ____________.
3. The reduced row echelon form of A is ____________.
4. A is expressible as a product of ____________.
5. $A\mathbf{x} = \mathbf{b}$ is ____________ for every $n \times 1$ matrix $\mathbf{b}$.
6. $A\mathbf{x} = \mathbf{b}$ has exactly ____________ for every $n \times 1$ matrix $\mathbf{b}$.
7. $\det(A) \neq$ ____________.
8. ____________ is not an eigenvalue of A.

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[4] 5. (a) Find $\text{proj}_{\mathbf{v}}\mathbf{u}$, the projection of $\mathbf{u}$ along $\mathbf{v}$, where $\mathbf{u} = (2, -2, 3)$ and $\mathbf{v} = (2, 1, 3)$.

[3] (b) Find the value of t so that the vector $(2, 5, -3, 6)$ is orthogonal to $(4, t, 7, 1)$.

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6. Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (-y, 2x + y, x).$$

[6] (a) Prove, by using the definition, that T is a linear transformation.

[3] (b) Find the standard matrix of the linear transformation T.

7. Consider the linear one-to-one transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T(1,0) = (0,1), \quad T(0,1) = (-1,0).$$

[3] (a) Find the image of $\mathbf{v} = (1,1)$ by T.

[2] (b) What is the effect of T on any vector $\mathbf{u} = (x,y)$ in $\mathbb{R}^2$?

[3] (c) Find the inverse of the standard matrix of the transformation T.

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8. Let $A = \begin{bmatrix} 2 & 0 & -2 & 1 & -1 \\ 0 & -2 & 1 & 3 & 9 \\ 0 & 0 & 0 & 6 & 6 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$.

[4] (a) Find the eigenvalues of A.

[2] (b) Find A^{-1} if possible. Justify.

[6] (c) Find the eigenvectors associated to the eigenvalue -1.

⇒ *Continued on the next page*

[2] (d) Is A diagonalizable? Justify.

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9. Let $A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$. The eigenvalues of A are $\lambda_1 = 4$ and $\lambda_2 = -1$.

[2] (a) Give the characteristic polynomial of A.

[8] (b) Find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$ **(There is no need to find** P^{-1}**).**

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