THE UNIVERSITY OF MANITOBA

DATE: February 23, 2007 **MIDTERM EXAMINATION**

DEPARTMENT & COURSE NO: MATH 1500 TITLE PAGE

EXAMINATION: Introductory Calculus TIME: 1 hour

EXAMINER: Various

LAST (FAMILY) NAME : (Print) _____

FIRST (GIVEN) NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____
(I understand that cheating is a serious offense)

Please mark your section number.

- ❑ Section A01 MWF (10:30 – 11:20)
  T (10:00 – 10:50) G.I. Moghaddam
- ❑ Section A02 MWF (9:30 – 10:20) S. Kalajdzievski
- ❑ Section A03 T & R (8:30 – 9:45) A. Gerhard
- ❑ Section A04 T & R (11:30 – 12:45) Y. Zhang
- ❑ Section A05 T & R (4:00 – 5:15) R.S.D. Thomas
- ❑ Section A91 Challenge for Credit SJR

INSTRUCTIONS TO CANDIDATES:

This is a 1 hour exam. **Please show your work clearly**. Please justify your answers, unless otherwise stated.

No calculators or other aids are permitted.

This exam has a title page, 6 pages of questions and 1 blank page for rough work. Please check that you have all the pages.

The value of each question is indicated in the left- hand margin beside the statement of the question. The total value of all questions is 60.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN
1. _____ /10
2. _____ / 7
3. _____ /12
4. _____ / 9
5. _____ / 6
6. _____ / 8
7. _____ / 8
TOTAL _____ /60

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Values:

1. Find the limit or explain why the limit does not exist.

[2] a) $\lim_{x\to 2^+} \dfrac{2-x}{|2-x|}$,

[2] b) $\lim_{x\to -3} \dfrac{x+3}{3+\sqrt{3-2x}}$,

[3] c) $\lim_{x\to 0} \dfrac{\sin 2x}{3x}$,

[3] d) $\lim_{x\to 3} \dfrac{2-\sqrt{x+1}}{3-x}$.

Values:

[7] 2. Find the value or values of k such that the function

$$f(x)=\begin{cases} k^2x^2+kx & x<3, \\ 6 & x=3, \\ x^2-k^2x & x>3, \end{cases}$$

is continuous at $x=3$.

Values:

3. Find $\frac{dy}{dx}$. Do not simplify your answer.

[3] a) $y = \sin(\cos x)$,

[3] b) $y = \sqrt[4]{x^9} + \left(\frac{3}{2}\right)^2 - e^{x^2}$,

[3] c) $y = \frac{\cos x}{1+\sqrt{x}}$,

[3] d) $y = (\sin x)\sqrt{\pi - x}$.

Values:

4. a) When is a function $f(x)$ differentiable at $x = a$? (State the definition.) **[3]**

b) Use **only** the definition of the derivative (part (a) of this question) to compute $f'(a)$ if $f(x) = x^2 - 2x$. **[6]**

5. Suppose $f(x)$ and $g(x)$ are differentiable functions. Prove that $\big(f(x) + g(x)\big)' = f'(x) + g'(x)$. **[6]**

Values:

6. a) The equation $y^3 = \dfrac{4x-2y}{x+y}$ defines y implicitly as a function of x.

[6] Find the value of the derivative y' at the point $(1,1)$.

b) Find the equation of the tangent line to the curve determined by

[2] $y^3 = \dfrac{4x-2y}{x+y}$ at the point $(1,1)$.

Values:

[8] 7. The line segment AB is 5 meters long. The bottom A slides away from the origin O along the x-axis at the rate of $2\,{}^{m}/_{\text{sec}}$, while the top B slides down along the y-axis (see the illustration). How fast does B approach the origin O at the moment when A is 3 meters from O?