THE UNIVERSITY OF MANITOBA

December 10, 2014 FINAL EXAMINATION

DEPARTMENT & COURSE NO: Math 1500 COVER PAGE

EXAMINATION: Intro. to Calculus TIME: 2 HOURS EXAMINER: Various

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

☐	A01	10:30-11:20 MWF	B. Waters
☐	A02	9:30-10:20 MWF	S. Kalajdzievski
☐	A03	11:30-12:20 MWF	N. Harland
☐	A04	12:30-1:20 MWF	X. Zhao
☐	A05	11:30-12:45 TTh	D. Krepski
☐	A06	19:00-22:00 T	L. Menjivar Lopez
☐	A07	11:30-12:45 TTh	F. Ghahramani
☐	A08	8:30-9:20 MWF	X. Zhao
☐	Challenge for credit		

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. Show all your work and **justify** your answers. **Unjustified answers will receive LITTLE or NO CREDIT.**

No aids or electronic devices of any kind are permitted during the examination.

This exam has a title page, 8 pages of questions, including 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	13	
2	12	
3	21	
4	7	
5	9	
6	11	
7	10	
8	10	
9	15	
10	12	

TOTAL	120	

Values

[13] 1. Calculate each of the following limits if they exist. If the limit does not exist, determine whether the limit is ∞, $-\infty$ or neither.

[2] (a) $\lim_{x\to 0} \dfrac{x}{\cos x}$

[3] (b) $\lim_{x\to 1} \dfrac{\sin(x-1)}{x^2-1}$

[4] (c) $\lim_{x\to \infty} \dfrac{\sqrt{x^2-1}-\sqrt{x-1}}{x}$

[4] (d) $\lim_{x\to 0} x^4 \cos\left(\dfrac{1}{x}\right)$ [Hint: Squeeze theorem!]

[12] 2. Find $f'(x)$. Do NOT simplify your answer after you evaluate the derivative.

[3] (a) $f(x) = e^3 + 3^{\sqrt{x}} + \cot(x^3)$

[5] (b) $f(x) = x^{\ln x}$

[4] (c) $f(x) = \int_0^1 \sin(x^2)\,dx + \int_1^{\cos x} \sqrt{t^2+1}\,dt$

[21] 3. Consider the function $f(x)=\dfrac{1+x-x^2}{1+x}$. It can be shown (no need to check) that $f'(x)=\dfrac{-2x-x^2}{(1+x)^2}$ and that $f''(x)=\dfrac{-2}{(1+x)^3}$.

[16] (a) Compile the following information about $f(x)$ and its graph. (Give answers only: answer "none" if the function does not display a feature listed.)

[1] Domain _____

[1] Symmetry (is $f(x)$ even, odd or neither?) _____

[1] Equation(s) of vertical asymptote(s) _____

[1] Equation(s) of horizontal asymptote(s) _____

[1] Critical number(s) of $f(x)$ _____

[2] Interval(s) where $f(x)$ is increasing _____

[1] Interval(s) where $f(x)$ is decreasing _____

[2] x and y coordinates of each local maximum of $f(x)$ _____

[2] x and y coordinates of each local minimum of $f(x)$ _____

[2] Interval(s) where $f(x)$ is concave up _____

[1] Interval(s) where $f(x)$ is concave down _____

[1] x coordinates of the inflection points _____

[5] (b) Use the information obtained in part (a) of this question to make a clear sketch of $y=f(x)$, labeling extrema and any horizontal or vertical asymptotes.

[7] 4. Find the absolute minimum and the absolute maximum of the function $f(x) = x - x^3$ over the interval $[0,2]$.

[9] 5. Let $F(x)$ be an antiderivative of $f(x) = 4x^3 + \frac{2}{x}$. If $F(1) = 1$, find $F(e)$. Simplify your final answer.

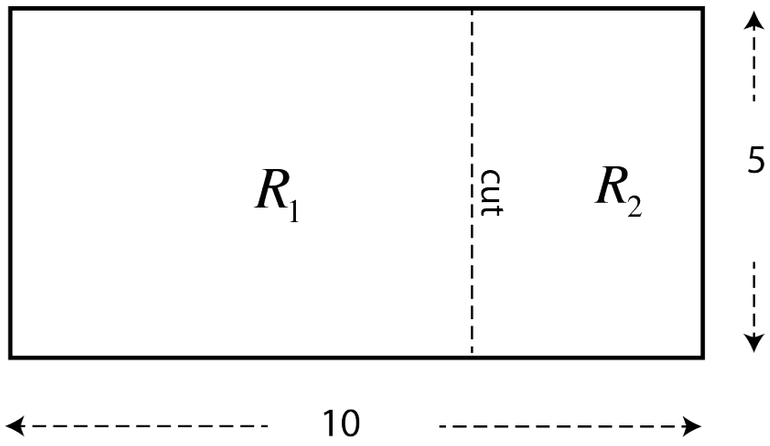
[11] 6.

[3] (a) State the Mean Value Theorem for a function $f(x)$ over an interval $[a,b]$.

[8] (b) Prove that if $f'(x) > 0$ for every x in an interval (a,b), then $f(x)$ is increasing over (a,b).

[10] 7. The curve defined by $y^3x^2 + y = 3$ and the curve defined by $y = 3e^x - 3x$ intersect at the point $(0,3)$. (This information is given for free; no need to check it.) Show that the two curves have the same tangent line at the point $(0,3)$.

[10] 8. A rectangle 10 metres long and 5 metres wide is to be cut perpendicularly to the base as indicated in the illustration below. The result is two smaller rectangles, denoted by R_1 and R_2 in the figure below. Where should the cut be done so that the product of the areas of R_1 and R_2 is the largest possible? Justify your answer!



[15] 9. [5] (a) Evaluate the integral $\int (1+3e^x+\sqrt{x}+\cos 2x)\,dx$.

[5] (b) Evaluate the integral $\int_{-1}^{2} x(1-x^2)\,dx$.

[5] (c) Let $f(x)$ be a function with continuous derivative. Suppose $f(1)=-2$ and $\int_{1}^{3} f'(x)\,dx=10$.

Find the value of $f(3)$.

[12] 10. We are given the function $g(x) = 36x - 4x^3$.

[3] (a) Is the function $g(x)$ even, odd, or neither? Justify!

[2] (b) Find the points where the graph of the function $g(x)$ intersects the x-axis. Make a rough sketch of the graph of $g(x)$.

[7] (c) Determine the area of the region bounded by the graph of the function $g(x)$ and the x-axis.

BLANK PAGE

BLANK PAGE