UNIVERSITY OF MANITOBA

DATE: April 16, 2013
COURSE: MATH 1500
EXAMINATION: Introduction to Calculus

FINAL EXAMINATION
TITLE PAGE
TIME: 120 Minutes
EXAMINER: Various

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SIGNATURE: (in ink) ______________________________
(I understand that cheating is a serious offense)

- ❑ A01 10:30-11:20 MWF & 10:00-10:50 T P. Mendelsohn
- ❑ A02 9:30-10:20 MWF X. Zhao
- ❑ A03 8:30-9:30 TR J. Sichler
- ❑ A04 11:30-12:45 TR N. Zorboska
- ❑ A05 1:00-2:15 TR O. Klurman
- ❑ A06 3:30-4:20 MWF V. Shepelska
- ❑ Challenge for credit

INSTRUCTIONS TO STUDENTS:

This is a 120 Minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. **DO NOT** remove the blank pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 96 points. The exam is out of 90 points and the extra **6 points are BONUS points**.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, or on a page at the back, but CLEARLY INDICATE that your work is continued and label the continuation.

Question	Points	Score
1	7	
2	7	
3	16	
4	9	
5	5	
6	18	
7	8	
8	13	
9	6	
10	7	
Total:	96	

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[3] 1. (a) Find $\lim_{x\to 1} \dfrac{2-\sqrt{3x+1}}{x^2-1}$.

[4] (b) Use the Squeeze theorem to show that $\lim_{x\to 0^+} \sqrt{e^x - 1}\cdot\cos\left(\dfrac{\pi}{x^2}\right) = 0$.

[7] 2. Find the absolute maximum and absolute minimum values of f over the given interval:

$$f(x) = x^3 + 3x^2 - 9x + 1 \qquad\qquad \text{on } [-1, 2].$$

3. Find the derivative $f'(x)$ for each of the following functions. DO NOT SIMPLIFY.

[4] (a) $f(x) = \cos\left(3^{x^2}\right) + \log_2(\tan x)$

[4] (b) $f(x) = \sqrt{\ln x} - \sec\left(e^2 - x^2\right)$

[5] (c) $f(x) = (x \sin x)^x$

[3] (d) $f(x) = x^5 \ln \pi + \displaystyle\int_x^2 \frac{1}{(1+t^6)^3}\, dt$

[3] 4. (a) State the Mean Value Theorem.

[6] (b) Prove that if $f'(x) > 0$ for all x in an interval (a, b) then $f(x)$ is increasing on (a, b).

[5] 5. Show that the equation $x^3 + 3x - 3 = 0$ has exactly one real root. (Hint: Apply the Intermediate Value Theorem to the function $f(x) = x^3 + 3x - 3$.)

[18] 6. Let $f(x) = \dfrac{x^2}{(1-x)^2}$. Then $f'(x) = \dfrac{2x}{(1-x)^3}$ and $f''(x) = \dfrac{2(2x+1)}{(1-x)^4}$.

(a) Compile the following information about $f(x)$ and its graph. (Give answers only. Answer "none" if the function does not display a feature listed.)

Domain__________

x and y-intercept(s)__________

Symmetry__________

Equation(s) of vertical asymptotes __________

Equation(s) of horizontal asymptotes __________

Critical number(s) of f __________

Interval(s) where f is increasing __________

Interval(s) where f is decreasing __________

Local maximum value(s) of f __________

Local minimum value(s) of f __________

Interval(s) where f is concave upward __________

Interval(s) where f is concave downward __________

Point(s) of inflection __________

(b) Using the above information sketch the graph of f, indicating key points and asymptotes.

[8] 7. A rectangular box with a square base is to be made from two kinds of materials. The material for the sides and the top costs \$10 per square meter and the material for the base costs \$20 per square meter. What are the dimensions of the least expensive box with a volume 12 m^3?

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[4] 8. (a) Find the most general antiderivative of $f(x) = 2e^x + \pi \sin x - \sqrt{5}x^{\frac{2}{3}}$.

[5] (b) Find $\displaystyle\int_0^{\frac{\pi}{4}} \left(\sec^2 x + 3 - \sec x \tan x\right) dx$. Do not leave any trigonometric functions in your final answer.

[4] (c) Find f given that $f'(t) = \dfrac{1}{\sqrt{t}} + 3\sqrt{t}$ and $f(1) = -3$.

[6] 9. Given that $2 + \int_a^{x^2} \frac{f(t)}{t^3} dt = 2x$, find the expression for the function $f(x)$ and the value of the positive constant a. (Hint: Differentiate the equation.)

[7] 10. Find the value of the positive constant a such that the area between the graph of the function $f(x) = \begin{cases} -x^2 + 1, & -1 \le x < 0 \\ \dfrac{-x}{a} + 1, & 0 \le x \le a \end{cases}$ and the x-axis equals $\frac{5}{3}$.

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