Real Numbers Axioms

Let + and $\cdot$ be two binary (closed) operations on R called addition and multiplication.

Field Axioms: (R, +, $\cdot$) satisfies the following properties for all a, b and c in R:

A1. $a+b=b+a$ (commutativity of addition)

A2. $(a+b)+c = a+(b+c)$ (associativity of addition)

A3. There exists an element 0 in R such that $a+0=a$ (and $0+a=a$) (existence of zero)

A4. For each a in R, there exists an element $-a$ in R such that
$a+(-a)=0$ (and $(-a)+a=0$) (existence of negative)

M1. $a\cdot b=b\cdot a$ (commutativity of multiplication)

M2. $(a\cdot b)\cdot c = a\cdot(b\cdot c)$ (associativity of multiplication)

M3. There exists an element 1 in R, $1\neq 0$, such that $a\cdot 1=a$ ($1\cdot a=a$) (existence of unit)

M4. For each $a\neq 0$ there exists an element a^{-1} in R such that
$a\cdot a^{-1}=1$ (and $a^{-1}\cdot a=1$) (existence of inverse)

D. $a\cdot(b+c) = a\cdot b+a\cdot c$ (and $(b+c)\cdot a = b\cdot a+c\cdot a$) (distributivity)

Order Axioms: There exists a nonempty subset P of R called the set of positive real numbers, such that:

O1. If a, b are in P, then $a+b$ is in P.

O2. If a, b are in P, then $a\cdot b$ is in P.

O3. For a in R, exactly one of the following is true: a is in P, $a=0$, or $-a$ is in P.

Completeness Axiom:
Every nonempty set of real numbers that has an upper bound has a supremum in R.