MATH2202, Assignment No. 1

September 23, 2013

The assignment is due Monday, September 30, 2013 in class. Late assignments receive a mark zero.

1. a) Prove that if $C=A\bigcup B$ and $A\bigcap B=\varnothing$, then $A=C\setminus B$. [5]

b) Let $A_n=\left[0,2-\dfrac{1}{n}\right)$, with $n\in\mathbb{N}$. Show that $\bigcup_{n=1}^{\infty}A_n\subseteq[0,2)$ and that $[0,1)\subseteq\bigcap_{n=1}^{\infty}A_n$.

Does $2\in\bigcup_{n=1}^{\infty}A_n$ and does $1\in\bigcap_{n=1}^{\infty}A_n$? Explain. [6]

2. Let f(x) = $\sqrt{1-x}$ and g(x) $=x^2-1$. Find the functions $f\circ g$ and $g\circ f$ if they exist, by first finding the domains, co-domains and ranges of f and g. Draw the graphs of all of functions which do exist (including f and g). [7]

3. a) Show that if f: $A\rightarrow B$ is surjective and $H\subseteq B$, then $f(f^{-1}(H))=H$. Give an example to show that the equality need not hold if f is not surjective. [6]

b) If f is a bijection from A onto B, use the definition of f^{-1} to show that f^{-1} is also a bijection (from B onto A). [6]

4. a) Show that there is a bijection from $\mathbb{N}$ into the set of all odd integers greater than 13. [3]

b) Find an example of a countable collection of finite sets such that their union is not finite, and find an example of a countable collection of finite sets such that their union is finite. [2]

c) Let A be countable infinite and let $A\subseteq B$. Prove that B is infinite. (You can use Theorem 1.3.3. that says that $\mathbb{N}$ is an infinite set.) [6]

d) Show that $|[0,1]|=|(0,1)|$. [4]

5. For a and b in $\mathbb{R}$ let $a\,\#\,b:=\dfrac{ab}{2}+1$. (Note: a # b is a binary operation mapping $\mathbb{R}\times\mathbb{R}\rightarrow\mathbb{R}$.) Check if for all *a, b* and *c* in $\mathbb{R}$:

a) $a\,\#\,b=b\,\#\,a$, [2]

b) $a\,\#\,(b\,\#\,c)=(a\,\#\,b)\,\#\,c$, [4]

c) there exists *e* in $\mathbb{R}$ such that, for all *a* in $\mathbb{R}$, $a\,\#\,e=e\,\#\,a=a$. (Note: the question asks if there is one *e* that would do for all of the *a*'s.) [4]

6. Use the field axioms of $\mathbb{R}$ to prove that for $a,b\in\mathbb{R}$:

a) $-(a+b)=(-a)+(-b)$, [3]

b) If $a\cdot a=a$, then either $a=0$, or $a=1$, [3]

c) If $a\neq 0$ and $b\neq 0$ then $(a\cdot b)^{-1}=a^{-1}\cdot b^{-1}$. [3]

Total [64]