MATH2202, Assignment No. 1
September 22, 2014

The assignment is due Monday, September 29, 2014 in class. Late assignments receive a mark zero.

1. a) Prove that if $A \subseteq C$, $B \subseteq C$ and $C \setminus A \subseteq B$, then $C = A \bigcup B$. [4]

   b) Let $f(x) = \dfrac{-2x}{\sqrt{1+x^2}}$, $x \in \mathbb{R}$. Show that (the range of *f*) $R(f) = (-2,2)$. [4]

2. a) Show that if *f*: A$\rightarrow$ B is an injection and $E \subseteq \mathrm{A}$, then $f^{-1}(f(E)) = E$. Give an example of *f, A, B* and *E* to show that the equality need not hold if *f* is not injective. [4]

   b) If *f* is a bijection from A onto B then:
   $f^{-1}(f(a)) = a, \forall a \in A$ and $f(f^{-1}(b)) = b, \forall b \in B$. [4]

   c) If *f* is a bijection from A onto B, use the definition of f^{-1} to show that f^{-1} is also a bijection (from B onto A). [4]

3. Use the field axioms of $\mathbb{R}$ to prove that for $a, b \in \mathbb{R}$:

   a) $-(a+b) = (-a) + (-b)$, [3]

   b) If $a \neq 0$ and $b \neq 0$ then $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$. [3]

   c) Prove that for any $a, b \in \mathbb{R}$, $a>b$ if and only if $b<a$, by using the field axioms and the two facts proven in class: $-a = (-1)a, (-1)(-1) = 1$.
   (Recall that $a > b \Leftrightarrow a + (-b) \in P$ and $b < a \Leftrightarrow -(b + (-a) \in P$.) [3]

4. For *a* and *b* in $\mathbb{R}^+ = (0, \infty)$ let $a \oplus b = ab$, and $a \odot b = a + b + 1$. Show that:

   a) $\oplus$ and $\odot$ are binary operations mapping $\mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$. [3]

   b) The field axioms A1, A2, A3 and A4 hold for $\oplus$. [4]

   c) The field axiom M1 and M2 hold, but M3 does not hold for $\odot$. [4]

5. Use the field and order axioms of $\mathbb{R}$ and theorems proven in class to show that for $a, b \in \mathbb{R}$:

   a) If $a > 0$, then $a^{-1} > 0$. [3]

   b) If $a < b$, then $a < 2^{-1}(a+b) < b$. [4]

6. Use the Principal of Mathematical Induction to show that:

   a) There is no $n \in \mathbb{N}$ such that $0 < n < 1$. [3]

   b) $n^3 + 5n$ is divisible by 6 for all *n* in $\mathbb{N}$. [3]

Total [53]