MATH 2202
Test 1

50 min October 28, 2009

Instructions: The total value of all questions is 39(+2). Values of individual questions are printed beside the statement of the question. If you need more space, use the reverse side of the page, but indicate clearly that you are doing so.
There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below.

Name (print)__________________________

Student No.__________________________

Signature__________________________

QUESTION#	MARK
1	/6
2	/8
3	/7
4	/10
5	/8
TOTAL	/39

Value

1. Let $f: A \rightarrow B$ and let G and H be subsets of B.

[1] a) State the definition of $f^{-1}(G)$.

b) Prove that if $G \subseteq H$, then $f^{-1}(G) \subseteq f^{-1}(H)$.

[3]

c) Give an example of f, domain A, codomain B, and sets G and H such that $G \subseteq H$, $f^{-1}(G) = f^{-1}(H)$, but $G \neq H$.

[2]

[3] 2. a) Prove that for every a and b in $\mathbb{R}$, $-(a + b) = (-a) + (-b)$. State which field axioms or theorems you are using in each step.

[5] b) Prove that if $a > 0$, then $a^{-1} > 0$. State the order and field axioms, or the theorems that you are using in each step.

[2] 3. a) Define when is a function f: A $\rightarrow$B an injection and when is it a surjection.

[5] b) Prove in details that the set A = { $3n^2 + \sqrt{2}$: n in ℕ } is denumerable (i.e. infinite countable). (You have to find a bijection f between ℕ and A and show that that f is a bijection.)

[4] 4. a) Prove, by using the Principal of Mathematical Induction, that $\frac{1}{2^n} < \frac{1}{n}$, for every n in ℕ.

[2] b) Define the infimum of a set A that is bounded from below.

c) Show that the infimum of the set A = $\{\frac{1}{2^n} : n \in \mathbb{N}\}$ is 0. (Hint: use a corollary of the Archimedean property and part a).)

[4]

d) **BONUS QUESTION**: Find the set A = $\bigcap_{n=1}^{\infty} I_n$, with $I_n = [-1, \frac{1}{2^n}]$, by using c)

[2] and the conclusion in the proof of the Nested Intervals Theorem, as done in class.

6. a) State the definition of $\lim_{n \to \infty} x_n = x$.

[2]

b) State the definition of a bounded sequence. State the Theorem on convergence and boundedness of sequences.

[2]

[4] c) Prove that the sequence with $x_n = n$ diverges. (Hint: use proof by contradiction, part b) and the Archimedean Theorem. State the Archimedean Theorem.)