MATH 2202 Assignment No. 2, October 10, 2014

The assignment is due Monday, October 20, 2014, in class. Late assignments receive mark zero.

1. a) Let $S=\left\{\frac{1}{n}:n\in\mathbb{N}\right\}$. Show that $\sup S=1,\inf S=0$. (Do not use limits.) [3]

b) Let $S=\left\{\frac{1}{10^n}:n\in\mathbb{N}\right\}$. Use PMI to show that $\frac{1}{10^n}<\frac{1}{n},\forall n\in\mathbb{N}$, and then use this to show that inf S=0. (You cannot use limits.) [4]

2. a) Prove that a supremum of a bounded above nonempty set A is unique, i.e. prove that if s_1=sup$A$ and s_2=sup$A$, then $s_1=s_2$. [3]

b) Prove that every nonempty bounded below subset of $\mathbb{R}$ has an infimum. [3]

c) Prove that for a nonempty, bounded above subsets A and B of $\mathbb{R}$,

$\sup(A+B)=\sup A+\sup B$. [4]

d) Show that if $\varnothing\neq A\subseteq B$ and B is bounded below, then $\inf B\leq\inf A$. [3]

3. Show that if $I=[a,b],J=[c,d]$, both nonempty, then $I\subseteq J$ iff $c\leq a\leq b\leq d$. [5]

4. Find the set A (proofs, **without using limits** are required):

a) $A=\bigcap_{n=1}^{\infty}I_n$, for $I_n=(3,4+\frac{1}{n}]$, [4]

b) $A=\bigcap_{n=1}^{\infty}I_n$, for $I_n=(1,1+\frac{1}{n})$, [4]

c) $A=\bigcap_{n=1}^{\infty}I_n$, for $I_n=[n,\infty)$. [4]

5. Let $\{I_k\}$ be a sequence of closed and bounded intervals such that for any finite set $F\subset\mathbb{N}$, we have $\bigcap_{k\in F}I_k\neq\varnothing$. Show that then $\bigcap_{k=1}^{\infty}I_k\neq\varnothing$, by showing each of the following steps:

a) In general, if I and J are any two closed bounded intervals and $I\cap J\neq\varnothing$, then $I\cap J$ is a closed bounded interval. [4]

b) Use the Principal of Mathematical Induction to show that if $J_n=\bigcap_{k=1}^{n}I_k$, then J_n is a closed and bounded interval, for all $n\in\mathbb{N}$. [4]

c) Use the Nested Intervals Theorem to show that $\bigcap_{k=1}^{\infty}I_k\neq\varnothing$. [3]

Total [48 marks]