Algebraic Properties of R On the set R of real numbers there are two binary operations, denoted by $+$ and $\cdot$ and called **addition** and **multiplication**, respectively. These operations satisfy the following properties:

(A1) $a + b = b + a$ for all a, b in R (*commutative property of addition*);

(A2) $(a + b) + c = a + (b + c)$ for all a, b, c in R (*associative property of addition*);

(A3) there exists an element 0 in R such that $0 + a = a$ and $a + 0 = a$ for all a in R (*existence of a zero element*);

(A4) for each a in R there exists an element $-a$ in R such that $a + (-a) = 0$ and $(-a) + a = 0$ (*existence of negative elements*);

(M1) $a \cdot b = b \cdot a$ for all a, b in R (*commutative property of multiplication*);

(M2) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all a, b, c in R (*associative property of multiplication*);

(M3) there exists an element 1 in R *distinct from* 0 such that $1 \cdot a = a$ and $a \cdot 1 = a$ for all a in R (*existence of a unit element*);

(M4) for each $a \neq 0$ in R there exists an element $1/a$ in R such that $a \cdot (1/a) = 1$ and $(1/a) \cdot a = 1$ (*existence of reciprocals*);

(D) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(b + c) \cdot a = (b \cdot a) + (c \cdot a)$ for all a, b, c in R (*distributive property of multiplication over addition*).

The Order Properties of R There is a nonempty subset P of R, called the set of **positive real numbers**, that satisfies the following properties:

(i) If a, b belong to P, then $a + b$ belongs to P.

(ii) If a, b belong to P, then ab belongs to P.

(iii) If a belongs to R, then exactly one of the following holds:

$$a \in P, \qquad a = 0, \qquad -a \in P.$$

(COMPLETENESS)

The Supremum Property of R *Every nonempty set of real numbers that has an upper bound has a supremum in R.*