Last Name (Print)______________________________

First Name (Print)______________________________

I understand that cheating is a serious offense.

Signature:______________________________

Student Number______________________________

Room _______________Seat Number_______________

THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1300 Vector Geometry and Linear Algebra
Final Exam

Paper No: 109
Date: Friday, April 13, 2007
Time: 6:00–8:00 PM

Identify your section by marking an X in the box.

	Section	Instructor	Slot	Time	Room
☐	A01	E. Schippers	5	TTh 10:00–11:15am	208 Armes
☐	A02	N. Zorboska	8	MWF 1:30–2:20pm	204 Armes
☐	A03	D. Kelly	12	MWF 3:30–4:20pm	208 Armes
☐	A04	C. Platt	15	TTh 4:00–5:15pm	200 Armes
☐	A05	J. Sichler	E2	T 7:00–10:00pm	204 Armes
☐	Other	(challenge, deferred, etc.)			

DO NOT WRITE IN THIS COLUMN

1	/8
2	/5
3	/8
4	/12
5	/18
6	/18
7	/10
8	/15
9	/6
10	/8
11	/12
Total	/120

Instructions

*Fill in **all** the information above.*

This is a two-hour exam.

***No** calculators, texts, notes, or other aids are permitted.*

***Show your work clearly** for full marks.*

*This exam has 11 questions on 7 numbered pages, for a total of 120 points. **Check now** that you have a complete exam.*

*Answer all questions on the exam paper in the space provided. If you need more room, you may continue your answer on the **reverse** side, but **clearly indicate** that your work is continued there. There are two blank pages at the end for scratch work, but nothing on these pages will be marked. You may also use the backs of numbered pages for scratch work, but none of it will be marked unless clearly indicated otherwise.*

***Do not separate any pages**.*

THE UNIVERSITY OF MANITOBA

Friday, April 13, 2007, 6:00–8:00 PM Final Exam
PAPER NO: 109
COURSE: Mathematics MATH 1300 Vector Geometry and Linear Algebra TIME: 2 HOURS
EXAMINERS: Kelly, Platt, Schippers, Sichler, Zorboska

[Values]

[8] **1.** Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 0 & 2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$.

(a) Find the reduced row echelon form of the augmented matrix $[\,A \mid \mathbf{b}\,]$.

(b) Find the general solution of the system $A\mathbf{x} = \mathbf{b}$, entering your answer in the spaces provided:

$x_1 =$ ____________________

$x_2 =$ ____________________

$x_3 =$ ____________________

$x_4 =$ ____________________

$x_5 =$ ____________________

[Values]

[5] **2.** Let $A = \begin{bmatrix} 2 & 4 & 1 & 1 \\ 2 & 4 & 1 & 2 \\ 2 & 4 & 2 & 3 \\ 2 & 5 & 0 & 1 \end{bmatrix}$. Find $\det(A)$ by first reducing A to an upper triangular matrix.

[8] **3.** Assume A is a 4×4 matrix with determinant -3. Find the determinants of the inverse, A^{-1}, and of the adjoint, $\text{adj}(A)$, without calculating either A^{-1} or $\text{adj}(A)$. Justify your answers, making sure your reasoning applies to *all* such matrices A.

(a) $\det(A^{-1}) =$ __________. Reason:

(b) $\det(\text{adj}(A)) =$ __________. Reason:

[Values]

[12] **4.** **(a)** Find the area of the triangle in $\mathbb{R}^3$ with vertices $P(1, 1, 1)$, $Q(0, 2, 1)$, and $R(2, 1, 2)$.

(b) Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (0, -7, 5)$, $\mathbf{v} = (2, 0, 0)$ and $\mathbf{w} = (1, 0, 3)$.

[18] **5.**

(a) Write the parametric equations of the line l in $\mathbb{R}^3$ that contains the points $P(1, 0, 1)$ and $Q(-1, 2, 1)$.

(b) Show that the line l from (a) is parallel to the plane with equation $2x + 2y - 7z = 4$.

(c) Find the distance from the point $P(1, 0, 1)$ to the plane with equation $2x + 2y - 7z = 4$.

[Values]

[18] **6.** In $\mathbb{R}^4$, let $\mathbf{u} = (2, 0, k, -1)$, $\mathbf{v} = (-4, 0, -3, 2)$, and $\mathbf{w} = (0, 1, 0, 0)$. In each question, justify your answers, and if there are no such k, answer "none".

(a) Find all values of k (if any) for which $\mathbf{u}$ is orthogonal to $\mathbf{v}$.

(b) Find all values of k (if any) for which the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.

(c) Find all values of k (if any) for which the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis of $\mathbb{R}^4$.

[10] **7.** In the polynomial space P_2, let $p_1(x) = 1 - x$ and $p_2(x) = x + 3x^2$.

Determine whether $p_3 = 1 + 2x + 6x^2$ is in the space spanned by p_1 and p_2, and prove your answer.

[Values]

[15] **8.** In each question, determine whether the given set W is a subspace of the given vector space V. Justify your answers.

(a) $V = M_{2,2}$ and W consists of all matrices of the form $\begin{bmatrix} a & 3 \\ 0 & 2a \end{bmatrix}$ for a in $\mathbb{R}$.

(b) $V = M_{2,2}$ and W consists of all matrices of the form $\begin{bmatrix} a & 3a \\ 0 & b \end{bmatrix}$ for a and b in $\mathbb{R}$.

(c) Let $\mathbf{a} = (2, 0, -1)$. $V = \mathbb{R}^3$ and W is the set of all vectors $\mathbf{u}$ such that $\mathbf{u} \times \mathbf{a} = \mathbf{0}$.

 Continued ⟶

[Values]

[6] **9.** Let A and B be $n \times n$ matrices.

(a) Show that $A^2 - B^2 = (A + B)(A - B)$ if and only if $AB = BA$.

(b) If A satisfies $A^2 + A - I_n = 0$, write the inverse A^{-1} in terms of the matrix A.

[8] **10.** Let A be a 4×6 matrix. Answer the following questions by filling in the blanks:

(a) The largest possible dimension of the null space of A is ________.

(b) The smallest possible dimension of the null space of A is ________.

(c) Suppose now the rows of A are linearly independent.

1. The dimension of the null space of A is ________.

2. The dimension of the column space of A is ________.

[Values]

[12] **11.** The matrix $A = \begin{bmatrix} 1 & -2 & 3 & 0 & 2 & 0 \\ -3 & 6 & -9 & 1 & -7 & 0 \\ 2 & -4 & 6 & 0 & 4 & 0 \\ 5 & -10 & 15 & -1 & 11 & 1 \end{bmatrix}$ has reduced row echelon form

$$R = \begin{bmatrix} 1 & -2 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find the dimension and a basis of the row space of A.

(b) Find the dimension and a basis of the null space of A.

(c) Find the dimension and a basis of the column space of A.

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