THE UNIVERSITY OF MANITOBA

DATE: October 23, 2006 MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: 136.130

EXAMINATION: Vector Geometry & Linear Algebra TIME: 1 hour

EXAMINERS: Various

Values

[8] 1. Solve, by **Gauss-Jordan elimination**, the linear system:

$$\begin{array}{rcrcrcr} x & - & y & - & z & = & -1 \\ -x & + & y & + & 2z & = & 2 \\ 2x & - & 2y & + & z & = & 1 \end{array}$$

Values

[8] 2. Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 1 & -3 \end{bmatrix}$.

In each of the following cases, compute the given expression or briefly explain why the expression cannot be calculated:

a) AB

b) $A + B$

c) $B + 2C^T$

d) $AB - BA$

Values

[12] 3. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$.

a) Find A^{-1}.

b) Use (a) to solve the system $A\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, where $\mathbf{x}$ is a column matrix of variables.

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Values

[12] 4. Let $3A^{-1} = \begin{bmatrix} 0 & 3 \\ 3 & 6 \end{bmatrix}$.

a) Find A.

b) If B is derived from A by adding -2 times row two to row one $\left(A \xrightarrow{R_1 \to -2R_2 + R_1} B\right)$, find the elementary matrices E and F such that $B = EA$ and $A = FB$.

Values

[7] 5. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & a \\ 1 & 4 & 1 \end{bmatrix}$

(a) Evaluate $\det(A)$ by expansion along column 2. No other method will be awarded marks. **Show all your work.**

b) For what value of a is A invertible?

[6] 6. Evaluate $\det\begin{bmatrix} 0 & 2 & -2 \\ 1 & 2 & 4 \\ 1 & 3 & -1 \end{bmatrix}$ by row reduction to the determinant of an upper triangular matrix. No other method will be awarded marks. **Show all your work.**

Values

[7] 7. Let A be a 4x4 matrix, such that $\det(A) = 2$.

a) Write the reduced row echelon form of A.

b) Find all of the solutions of the linear system $A\mathbf{x} = \mathbf{0}$ (where $\mathbf{x}$ is a column matrix of variables, and $\mathbf{0}$ is a column matrix of zeroes).

c) Find $\det\left(-A^T\right)$.

d) Find $\det(B)$ if you know that $\det\left(BA^{-1}\right) = 1$.