UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH 1300 Vector Geometry & Linear Algebra
Midterm Examination
February 28, 2008 5:30-6:30 PM

FIRST NAME: (Print in ink) ______________________

LAST NAME: (Print in ink) ______________________

STUDENT NUMBER: (in ink) ______________________

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offense)

Total	**/60**
1	/9
2	/8
3	/9
4	/9
5	/9
6	/8
7	/8

DO NOT WRITE IN THIS COLUMN

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	slot 5	T, Th - 10:00 am	E. Schippers
☐	A02	slot 8	MWF - 1:30 pm	K. Kopotun
☐	A03	slot 12	MWF - 3:30 pm	D. Kelly
☐	A04	slot 15	T,Th - 4:00 pm	C. Platt
☐	A05	slot E2	T - 7:00 pm	J. Sichler

INSTRUCTIONS TO STUDENTS:

Fill in all the information above

This is a 1 hour exam.

No *calculators, texts, notes, cellphones or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has 7 questions on 4 numbered pages, for a total of 60 points. There is also 1 blank page for rough work. You may remove the blank page if you want, but do not remove the staple. ***Check now*** *that you have a complete exam.*

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

If a question calls for a specific method, no credit will be given for other methods.

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[9] 1. Consider the linear system:

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & & & + & 6x_4 & = & 4 \\ x_1 & + & 3x_2 & + & 4x_3 & - & 2x_4 & = & 8 \end{array}$$

(a) Find the general solution to this system using Gauss-Jordan elimination.

(b) Find a solution to the above system with $x_2 = -2$ and $x_4 = 3$.

[8] 2. Let $A = \begin{bmatrix} 3 & 2 \\ -4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 2 \\ -2 & 0 \\ 0 & 4 \end{bmatrix}$.

In each part below, evaluate the expression or state that it does not exist. If the expression does not exist, give a reason.

(a) $AB + C$

(b) $AC + B$

(c) $BC + A$

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[9] 3. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$. Find A^{-1} by the method of row reduction. Show all your work. Write your final answer where indicated at the bottom of the page. (*No Credit* for any other method.)

Answer: $A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

[9] 4. Express $A = \begin{bmatrix} 0 & 3 \\ 1 & 5 \end{bmatrix}$ as a product of elementary matrices. Show all your work.

[9] 5. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -4 & 1 & 2 \end{bmatrix}$, and assume B is another 3×3 matrix with $\det(B) = 5$.

(a) Find $\det(A)$, by expansion along row 2. (*No Credit* for any other method.)

(b) Find the determinant of BAB^T.

(c) Find the determinant of $(2B)A^{-1}$.

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[8] 6. Use Cramer's rule to solve the following system. (*No Credit* for any other method)

$$\begin{aligned} 4x - 2y &= 4 \\ 3x + y &= 3 \end{aligned}$$

[8] 7. Assume that the augmented matrix of a certain linear system can be reduced to

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & a & b \end{array}\right]$$

with elementary row operations.

Determine all values of a and b (if any) for which this system

(a) has **no** solutions:

(b) has a **unique** solution:

(c) has **infinitely many** solutions:

(d) In case (c), determine the **general solution.**