UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH 1300 Vector Geometry & Linear Algebra
FINAL EXAMINATION
Thursday, April 17 2008 6 pm

FIRST NAME: (Print in ink) ______________________

LAST NAME: (Print in ink) ______________________

STUDENT NUMBER: (in ink) ______________________

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	slot 5	T, Th - 10:00 am	E. Schippers
☐	A02	slot 8	MWF - 1:30 pm	K. Kopotun
☐	A03	slot 12	MWF - 3:30 pm	D. Kelly
☐	A04	slot 15	T,Th - 4:00 pm	C. Platt
☐	A05	slot E2	T - 7:00 pm	J. Sichler
☐			challenge/deferred	

Question	Points	Score
1	10	
2	16	
3	12	
4	12	
5	12	
6	12	
7	12	
8	9	
9	15	
10	10	
Total:	120	

INSTRUCTIONS TO STUDENTS:

Fill in all the information above

This is a 2 hours exam.

No *calculators, texts, notes, cellphones or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has 10 questions on 10 numbered pages, for a total of 120 points. There are also 2 blank pages for rough work. You may remove the blank page if you want, but do not remove the staple. ***Check now*** *that you have a complete exam.*

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

If a question calls for a specific method, no credit will be given for other methods.

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[10] **1.** Given the following system of equations:

$$\begin{cases} x + y + 3z = 5 \\ \quad\; y + z = a \\ \quad\; by + z = 2 \end{cases}$$

(a) For what values of a and b does the system of equations have no solution?

(b) For what values of a and b does the system of equations have exactly one solution?

(c) For what values of a and b does the system of equations have infinitely many solutions?

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[16] **2.** Let

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 2 \\ 3 & 0 & -2 & -3 \end{bmatrix}$$

(a) Evaluate the missing $2,3$ entry x in the adjoint of A below:

$$adj(A) = \begin{bmatrix} 3 & 7 & 0 & 2 \\ -3 & 5 & x & 4 \\ 0 & 9 & 0 & 0 \\ 3 & 1 & 0 & -1 \end{bmatrix}$$

(b) The determinant of A is 9. Find A^{-1} by using Part (a).

(c) Let $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

Use A^{-1} from part (b) to find $\mathbf{x}$. *No credit will be given for any other method.*

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[12] **3.** State **clearly** whether each of the following statements is true or false. *No explanation is necessary.*

(a) $\det\left((2A)^{-1}(A^T)(2A^T)\right) = \det(A)$ for all square matrices A.

(b) If $\det(AB^{-1}) = \det(A^{-1}B)$, then $A = B$.

(c) The product of elementary matrices is always invertible.

(d) Let $A = (a_{ij})$ be the 2008×2008 matrix such that

$$a_{ij} = \begin{cases} 1\,, & \text{if } i \le j\,, \\ 0\,, & \text{if } i > j\,. \end{cases}$$

Then A is invertible.

(e) Let A be an $n \times n$ matrix. If A is invertible, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(f) The following augmented matrix is in reduced row echelon form.

$$\left(\begin{array}{ccccc|c} 1 & 2 & 3 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & -4 & 1 \end{array}\right)$$

[12] **4.** Let $\mathbf{u} = (2, -1, 3)$, $\mathbf{v} = (2, 3, -1)$, $\mathbf{w} = (4, 2, -2)$.

(a) Find the cosine of the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

(b) Find the area of the triangle with vertices $(0, 0, 0)$, $(2, 3, -1)$ and $(4, 2, -2)$.

(c) Find the volume of the parallelepiped with sides $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

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[12] **5.** Let l be the line $x = -2 + 2t$, $y = 1 - 2t$, $z = -3 + t$.

(a) Find an equation of the plane W perpendicular to l through the point $(-1, -4, 3)$.

(b) Find the point of intersection of l and W.

(c) Show that the plane $5x + 3y - 4z + 11 = 0$ is perpendicular to W.

[12] **6.** Let $\mathbf{u} = (2, -1, 2, 3)$, $\mathbf{v} = (4, 1, -1, 3)$.

(a) Find a unit vector in the direction of $\mathbf{v}$.

(b) Find all values of k such that $||k\mathbf{u} - k\mathbf{v}|| = 3$.

(c) For what values of s and t is $\mathbf{w} = (1, 2, s, t)$ orthogonal to both $\mathbf{u}$ and $\mathbf{v}$?

[12] **7.** The matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 4 & 0 & 0 & 0 \\ 1 & 2 & 1 & 8 & 1 & 6 & 7 \\ 0 & 0 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 6 & 7 \end{bmatrix}$$

has reduced row echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) The dimension of the null space of A is _____.

(b) Find a basis of the null space of A.

(c) The dimension of the row space of A is _____.

(d) Find a basis of the row space of A.

(e) The dimension of the column space of A is _____.

(f) Find a basis of the column space of A.

[9] **8.** Suppose that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal vectors in $\mathbb{R}^3$ with unit length.

(a) Give a reason why $\{\mathbf{a}, \mathbf{b}\}$ is not a basis of $\mathbb{R}^3$.

(b) Give a reason why $\{\mathbf{a}, \mathbf{b}, \mathbf{a}+\mathbf{b}, \mathbf{a}-\mathbf{b}\}$ is not a basis of $\mathbb{R}^3$.

(c) Give a reason why $\{\mathbf{a}, \mathbf{b}, 2\mathbf{a}-3\mathbf{b}\}$ is not a basis of $\mathbb{R}^3$.

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[15] **9.** For the vector spaces V and W given below, state whether W is a subspace of V. Justify your answer.

(a) $V = M_{2\times 2}$, the set of 2×2 matrices, and W consists of all 2×2 invertible matrices.

(b) $V = M_{2\times 2}$, and W consists of all 2×2 matrices with at least one zero row.

(c) $V = \mathbb{R}^3$ and W consists of all vectors in $\mathbb{R}^3$ of the form $(a, b, a - b)$.

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[10]**10.** Let $\mathbf{u}_1 = (1, 2, 0, 3)$, $\mathbf{u}_2 = (0, 1, 2, 1)$, $\mathbf{v}_1 = (1, 3, 2, 4)$ and $\mathbf{v}_2 = (1, 0, 0, 2)$.

Let $V = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

(a) Is $\mathbf{v}_1$ in V? Justify your answer.

(b) Is $\mathbf{v}_2$ in V? Justify your answer.

(c) What is the dimension of V? Find a basis for V and justify your answer.