THE UNIVERSITY OF MANITOBA

DATE: February 24, 2005 **Midterm Examination**

DEPARTMENT & COURSE NO. 136.130 TITLE PAGE

EXAMINATION: Vector Geometry & Linear Algebra TIME: 1 Hour

EXAMINER: see below

NAME: (Print in ink) SOLUTIONS

STUDENT NUMBER: ______________________________________

SIGNATURE: (in ink)______________________________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

❑ Section L05	**V. Charette**	Tu, Th 10:00 am - 11:15 am	208 Armes
❑ Section L06	**N. Zorboska**	M, W, F 1:30 pm- 2:20 pm	204 Armes
❑ Section L07	**K. Doerksen**	M, W, F 1:30 pm- 2:20 pm	223 Wallace
❑ Section L08	**R.S.D. Thomas**	M, W, F 2:30 pm- 3:20 pm	208 Armes
❑ Section L09	**J. Sichler**	Tues. Evening	204 Armes
❑ Section L92	**Challenge for credit** (SJR)		

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.** Please justify your answers, unless otherwise stated.

No calculators or other aids are permitted.

This exam has a title page, 5 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60.

Answer all questions on the exam paper in the space provide beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN

1.	/12
2.	/ 6
3.	/ 9
4.	/ 4
5.	/ 7
6.	/11
7.	/11
TOTAL	/60

(12) 1. Solve the following system by Gauss-Jordan elimination:

$$\begin{aligned} 3x_1 + 6x_2 \quad\quad - 3x_4 &= 0 \\ 2x_1 + 4x_2 + x_3 + \ x_4 &= 0 \\ x_1 + 2x_2 - x_3 + 4x_4 &= 0 \end{aligned}$$

No marks will be given for any other method.

SOLUTION:

$\begin{bmatrix} 3 & 6 & 0 & -3 \\ 2 & 4 & 1 & 1 \\ 1 & 2 & -1 & 4 \end{bmatrix}$ is row equivalent to

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \end{aligned}$$

solution :

$$\begin{aligned} x_2 &= t \\ x_1 &= -2t \\ x_3 &= 0 \\ x_4 &= 0 \end{aligned}$$ where t is arbitrary

(6) 2. a) Find the determinant of $M = \begin{bmatrix} -2 & \sqrt{3} & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

SOLUTION:

$$\det M = -10$$

b) Suppose A is a 3×3 matrix that is **invertible**, and that it can be put into row-echelon form by the following sequence of elementary row operations:

1) add $\sqrt{2}$ times row 1 to row 2;
2) permute rows 2 and 3;
3) multiply row 3 by $\sqrt{5}$.

Find the determinant of A.

SOLUTION:

$$-1 \cdot \sqrt{5} \cdot \det A = 1$$
$$\det A = -\tfrac{1}{\sqrt{5}}$$

(9) 3. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Express A^{-1} as an explicit product of elementary matrices.

SOLUTION:

$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\qquad E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$r_2 \to r_2 - 2r_1 \quad \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$ $\qquad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix}$

$r_2 \to -1/3\, r_2 \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$r_1 \to r_1 - 2r_2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

So $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

(4) 4. Let A, B and C be $n \times n$ matrices and suppose that $2AB - 3AC = I_n$. Indicate how you can tell that A^{-1} exists, and find A^{-1} in terms of B and C.

SOLUTION:

$$2AB - 3AC = I_n$$
$$A(2B - 3C) = I_n$$
$$A^{-1} = 2B - 3C$$

(7) 5. Let $X = [x_{ij}]$ be a 2×2 matrix. Given that $X + X^T = 0$ and $x_{12} = 7$, find X.

SOLUTION:

$$\begin{bmatrix} x_{11} & 7 \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} x_{11} & x_{21} \\ 7 & x_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$2x_{11} = 0$$
$$7 + 2x_{21} = 0$$
$$2x_{22} = 0$$

so :

$$x_{11} = x_{22} = 0$$
$$x_{21} = -7$$
$$X = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}$$

(11) 6. Find the inverse of the following matrix by row reduction:

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

No marks will be given for any other method.

SOLUTION:

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ -2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \text{ is row equivalent to } \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 0 & -1/4 \\ 0 & 1 & 0 & 1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right]$$

thus :

$$A^{-1} = \begin{bmatrix} 1/4 & 0 & -1/4 \\ 1/2 & 1 & -1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

(11) 7. Let $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ -2 & 2 & 2 \end{bmatrix}$. The adjoint of A is partially computed as shown. Enter the two missing numbers in the boxes.

$$\text{Adj } A = \begin{bmatrix} -4 & -4 & \boxed{4} \\ \boxed{-8} & -4 & -4 \\ 4 & -8 & -4 \end{bmatrix}.$$

Find $\det A$. Find A^{-1}.

SOLUTION:

C_{31}=2*2-0=4
C_{12}=-(2*2-(-2*2))=-8

$$\det A = 2 \cdot -4 + 2 \cdot -8 = -24$$

$$A^{-1} = \frac{1}{\det A} Adj(A)$$
$$= \frac{1}{-24}\begin{bmatrix} -4 & -4 & 4 \\ -8 & 4 & -4 \\ 4 & -8 & -4 \end{bmatrix}$$