[12] 1. (a) Solve the following systems by the method indicated. *Show your work!*

i. $$\begin{array}{rrrcr} x & +3y & +2z & = & -1 \\ 2x & +6y & +8z & = & 2 \\ 3x & +9y & +10z & = & 1 \end{array}$$ (Gauss-Jordan elimination)

ii. $$\begin{array}{rrcr} 3p & +4q & = & 1 \\ p & +2q & = & 2 \end{array}$$ (Cramer's rule)

(b) Indicate all types of solutions for systems satisfying the following descriptions by putting "YES" or "NO" in each empty box of the table.

The following kind of system:	Can have solutions of the following type(s)		
	no solution	unique solution	infinitely many
having 3 equations in 4 unknowns			
having invertible coefficient matrix			

[6] 2. Suppose A is 2×2, B is 2×3, C is 3×2 and D is 3×3. If any of the following expressions is defined, write down the size of the resulting matrix. If it is not defined, put an "X" beside it.

(a) $CA + DB^T$;

(b) D^3;

(c) B^2;

(d) $AB - 3C$;

[8] 3. (a) Let $X = \begin{pmatrix} 1 & 0 & 2 & 4 \\ 2 & -1 & 1 & -2 \end{pmatrix}$. Find XX^T.

(b) Suppose Y is a 2×2 matrix whose $(2,1)$ entry is 3, and $Y + Y^T = 4I$. Find the matrix Y.

[10] 4. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{pmatrix}$

(a) Find the cofactor C_{32} associated to the (3, 2) entry of this matrix.

(b) Use any method to find A^{-1}.

(c) Use your answer from (b) to solve the system $AX = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.

5. The 3×3 matrix A can be reduced to I_3 by the following sequence of 3 elementary row operations:

Step 1: add -2 times row 1 to row 2;

Step 2: multiply row 2 by $-\frac{1}{3}$;

Step 3: interchange rows 2 and 3;

[8] Use this information to express A as a product of 3 elementary matrices.

[6] 6. A, B, C are 4×4 matrices such that $\det(A) = -3$, $\det(B) = 2$ and $\det(C) = 7$.

(a) Given that $AB^2Y = 2C$, find the value of $\det(Y)$.

(b) Is Y invertible? Explain briefly.

[10] 7. Let $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (2, -1)$.

(a) Find the components of the vector $\mathbf{x}$ such that $\mathbf{x} - 2\mathbf{u} = 3\mathbf{x} + \mathbf{v}$.

(b) Find all scalars c such that $||c\mathbf{u}|| = 4\sqrt{3}$.

(c) On the coordinate system given below (grid lines represent units) draw and label the follo

$\mathbf{u}$, $\mathbf{v}$, $\mathbf{u} + \mathbf{v}$, and $\mathbf{u} - \mathbf{v}$.