136.275, Assignment No. 2

October 18, 2004

The assignment is due Monday, October 25 , 2004 in class. Late assignments receive a mark zero.

1. Suppose that the series $\sum a_n$ converges and the series $\sum b_n$ diverges.
   a) Show that the series $\sum (a_n + b_n)$ diverges. [5]

   b) Find examples to show that if $\sum a_n$ and $\sum b_n$ both diverge, then the series $\sum (a_n + b_n)$ may either converge or diverge. [3]

2. Use the Integral Test to determine for which values of p does the series
   $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\mathrm{p}}} \quad \text{converge.}$$
   (You have to show that you can use the Integral Test.) [7]

3. Determine if the following series converge or not:

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{-\frac{1}{2}}}{5+\cos^2 n}$, [3]

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n\sqrt{n}}$, [4]

   c) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2 2^n}{(2n+2)!}$ [3]

   d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{e^n}{2} - 3\right)^n e^{-n^2}$. [3]

4. Find the radius of convergence and the interval of convergence for the series
   $$\sum_{n=5}^{\infty} \frac{(-1)^{n+1} 2^n (x-2)^n}{n}. \qquad [6]$$

5. a) Find the power expansion of e^x about $x_0 = 1$. Show that it converges to e^x for all x in |R by using the Remainder Theorem. [6]

   b) Can we expand 1/x about $x_0 = 0$? Can the function 1/x be represented by a power series about $x_0 = 1$ on an interval with radius greater than 1? Explain. [4]

Total [44/42]