136.275, Assignment No. 3

November 26, 2004

The assignment is due Friday, December 3, 2004 in class. Late assignments receive a mark zero.

1. Find an equation for the surface consisting of all points P for which the distance from P to the y-axis is twice the distance from P to the xz- plane. Identify and sketch the surface by first drawing the traces in y= ± 1 planes , and in z=0 plane. [7]

2. Prove **by using the definition of a limit** that for $\mathbf{r}(t) = < 2t^2, t+3, 1-2t >$ $\lim_{t\to 0} \mathbf{r}(t) = < 0, 3, 1 >$. [6]

3. The vector valued function **r** is given by $\mathbf{r}(t) = < e^t, e^{-t}, \sqrt{2}t >$, $0 \le t \le 1$.
   a) Find $\lim_{t\to 1} (r(t) \times r''(t)) \cdot r'(t))$. [5]
   b) Find the arc length of the parametric curve **r**(t) . [4]

4. a) Find the arc length parametrization of the line segment x=2t, y=–2+3t, z=1–t with $-1 \le t \le 20$. [5]

   b) Use the parametrization equations obtained in a) to find the point on the line segment that is $2\sqrt{14}$ units away from the point (–2, –5 , 2) . [2]

   c) How long is the line segment (again, use the arc length parametrization from a))? [1]

5. The vector valued function **r** is given by $\mathbf{r}(t) = < t, -t, \cos t >$, $-2\pi \le t \le 2\pi$.

   a)Sketch the graph of **r** (t) and show the direction of increasing t. [6]

   b) Find the tangent vector and the tangent line to the graph of **r**(t) at t=π and draw the tangent vector on the graph of **r** (t). [4]

   c) Guess what is the osculating plane (**TN** plane) and what is the binormal vector **B**(t) for the curve given by **r**(t) (without calculating them the long way .) [3]

Total [43/42]