

136.275, Assignment No. 4

January 14, 2005

The assignment is due Friday, January 21, 2005 in class. Late assignments receive a mark zero.

1. Let $f(x,y) = \frac{\sqrt{1-y^2}}{2x}$.

- a) Draw the domain of f. Explain. [4]
- b) Draw the level curves $k = -1, 0, 1$, for f. [4]

2. Let $f(x,y,z) = \frac{xz^2 \cos y}{\sqrt{x^2 + y^2 + z^2}}$ for $(x,y,z) \neq (0,0,0)$.

- a) Show that $\lim_{(x,y,z) \rightarrow (0,0,0)} f(x,y,z) = 0$ by using the ε, δ definition of the limit. [7]
- b) Can $f(x,y,z)$ be redefined at $(0,0,0)$ such that it is continuous on $\mathbb{R}^3$? Explain. [2]

3. Show that $\lim_{(x,y) \rightarrow (0,1)} \tan^{-1} \left(\frac{x}{x + (1-y)^2} \right)$ does not exist. [6]

4. Let $f(x,y) = \begin{cases} x^2 + y^2 & , y < 0 \\ 0 & , y \geq 0 \end{cases}$.

- a) Sketch the graph of f. [3]
- b) Find $f_x(a,0)$ and $f_y(a,0)$ wherever they exist. [7]
- c) Can f be differentiable at $(1,0)$? Explain by using b). [2]

5. Let $f(x,y) = x^2 \sin y$.

- a) Show that f is differentiable at $(1,0)$ by using the definition of differentiability. [7]
- b) State a theorem by which you know that f is differentiable everywhere. [2].