136.275, Assignment No. 5

February 4, 2005

The assignment is due Friday, February 11 , 2004 in class. Late assignments receive a mark zero.

1. a) Let f(x,y) be such that $\lim_{(x,y)\to(a,b)} f(x,y) = 0$ and let g(x, y) be bounded in a neighbourhood of the point (a, b), i.e. $\exists r > 0, \exists M > 0$, such that $\forall (x,y) \in D((a,b);r)$ we have $|g(x, y)| \le M$. Use the ε, δ definition of the limit to prove that $\lim_{(x,y)\to(a,b)} f(x,y)g(x,y) = 0$. [3]

   b) Prove that if a function f is differentiable at the point (x,y) and if $D_u f(x,y) = 0$ in two nonparallel directions , then $D_u f(x,y) = 0$ in all directions. [4]

2. a) A particle is moving along a metal plate in the xy plane with velocity **v**= <1,–4> at the point (3, 2). Given that the temperature of the plate is $T(x,y) = y^2 \ln x, x > 0$ **,** find $\frac{dT}{dt}$ at the point (3, 2). What is the direction of the greatest increase of the temperature at the point (3, 2)? [5]

   b)Let H(u,v) = f(uv, u/v). Find $\partial^2 H / \partial v^2$ in terms of u, v and partial derivatives of f in terms of (uv and u/v.) [5]

3. a) Find the equation of the tangent plane to the surface $z = x^2 e^{2y}$ at the point (1, ln2, 4). [4]

   b) Find all points on the surface z= 2– xy at which the normal line passes through the origin. [5]

4. Let d be the usual metric on $\mathbb{R}^2$, let A= $\{(x,y); x > 0, y = \sin\frac{1}{x}\}$, B= $\{(x,y); x \in \mathbb{Z}, y = x\}$ and C=D($\mathbf{0}$; 2)\{(–1, 0), (1, 0) }. For each of A, B and C determine by explaining your work:

   a) interior points and boundary points,
   b) are they open, closed or neither,
   c) are they bounded or not. [7]

5. Let f(x, y) = $xe^y - x^2 - e^y$.

   a) Determine the local extremes and saddle points of f on $\mathbb{R}^2$. [4]

   b) Find the absolute extremes of f over the closed region bounded by y= lnx, y= 0 and x= e. [6]

Total [43/42]